Supplementary Material to the submission EGU25-1
EGU General Assembly 2025, Vienna, Austria

THEORETICAL REFERENCE ESTIMATE

for the components of the global energy balance

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2025

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Theoretical reference estimate for the components of the global energy balance

Abstract

Wild (2020), and Wild and Bosilovich (2024) provide estimates of global mean energy balance components as represented in climate models and reanalyses, with reference estimates from Loeb et al. (2018), Wild et al. (2015, 2019), L'Ecuyer et al. (2015) and Kato et al. (2018). Here we add a theoretical reference estimate (TRE) based on four radiative transfer equations and geometric considerations as detailed in Zagoni (2025). The equations do not refer to the atmospheric gaseous composition or the reflective properties of the surface or clouds. The first equation is a clear-sky constraint relationship on the net radiation at the surface (R_N), following from the two-stream approximation of Schwarzschild's (1906-Eq.11) radiative transfer equation as given in standard university textbooks on atmospheric physics and radiation (Goody, Oxford, 1964_Eq.2.115; Houghton, Cambridge, 1977_Eq.2.13; Hartmann, Academic Press 1994, Eqs. 3.51-3.54; Ambaum, Royal Met Soc, 2021_Eq.10.56), and in university lecture notes (Stephens 2003): R_N=OLR/2. The second equation is a clear-sky constraint relationship on the total radiation at the surface (R_T), following from the simplest greenhouse geometry (Hartmann 1994, Fig.2.3): R_T=2OLR. The third and fourth equations are all-sky versions of the first pair: R_N(all-sky) = (OLR−LWCRE)/2, and R_T(all-sky)=2OLR+LWCRE. Two decades of CERES observations (EBAF Ed4.1 April 2000−March 2022) give −2.33, −2.82, 2.71 and 2.44 [Wm^{-2}] deviations for the four equations, respectively, with a mean difference of 0.00. The all-sky equations are justified by an independent estimate of GEWEX within 0.1 Wm^{-2} (Zagoni 2024). The solution can be given in small integer ratios relative to LWCRE as the unit flux; the best fit is 1 unit = 26.68 Wm^{-2}, see Table1 (highres figures and other info about TRE available at TABLELINK). Some of the most remarkable precisions are in TOA SW up all-sky (=100) and clear-sky (=53). – Li, Li, Wild and Jones (2024) provide a global radiation budget from a surface perspective from 34 CMIP6 models for 2000-2022, with differences from the TRE integer positions less than 1 Wm^{-2} in SW down radiation, Thermal down Surface and the convective flux (Sensible heat + Latent heat); less than 2 Wm^{-2} in Thermal up Surface; and less than 3 Wm^{-2} in Reflect by surface; each within the noted ranges of uncertainty. Stackhouse et al. (2024) give Earth radiation budget at top-of-atmosphere; TRE differ from 2001-22 Climatological Mean in OLR, TSI and RSW by 0.23, 0.03 and 1.05 [Wm^{-2}], see details in TABLELINK in References.

References

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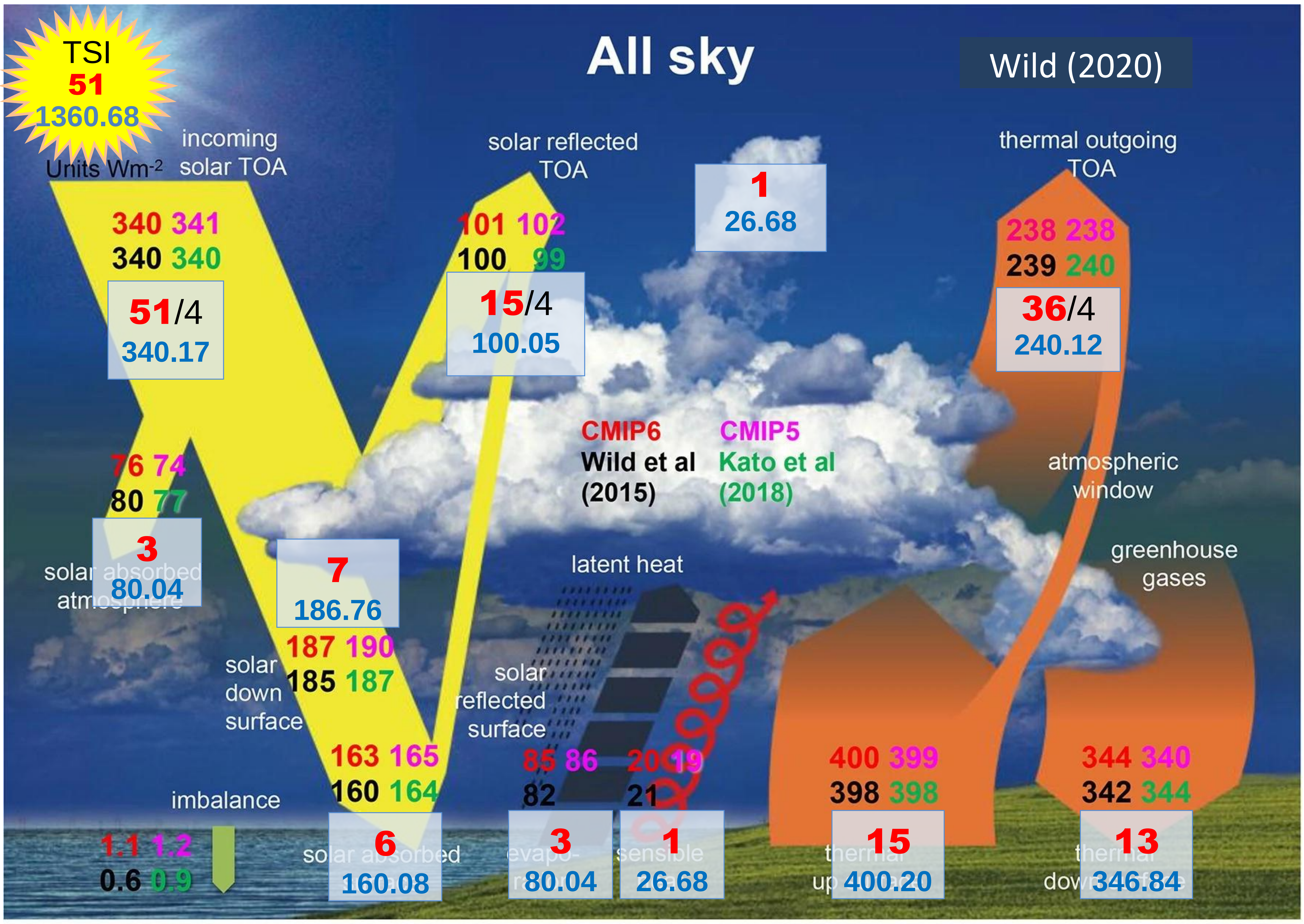
See also the updated Supplementary Material video: https://www.earthenergyflows.com/Zagoni-EGU2024-Trenberths-Greenhouse-Geometry_Full-v03-480.mp4

TABLELINK (this document): https://earthenergyflows.com/TRE20.pdf


All sky
Wild (2020)
TSI
51
1360.68
Units Wm-2
incoming solar TOA
340 341
340 340
51/4
340.17
solar reflected TOA
101 102
100 99
15/4
100.05
1
26.68
thermal outgoing TOA
238 238
239 240
36/4
240.12
CMIP6
Wild et al (2015)
CMIP5
Kato et al (2018)
atmospheric window
greenhouse gases
76 74
80 77
3
80.04
solar absorbed atmosphere
7
186.76
187 190
185 187
solar down surface
latent heat
solar reflected surface
163 165
160 164
imbalance
1.1 1.2
0.6 0.9
6
160.08
solar absorbed
85 86
82
3
80.04
20 19
21
1
26.68
sensible
400 399
398 398
15
400.20
thermal up
344 340
342 344
13
346.84
thermal

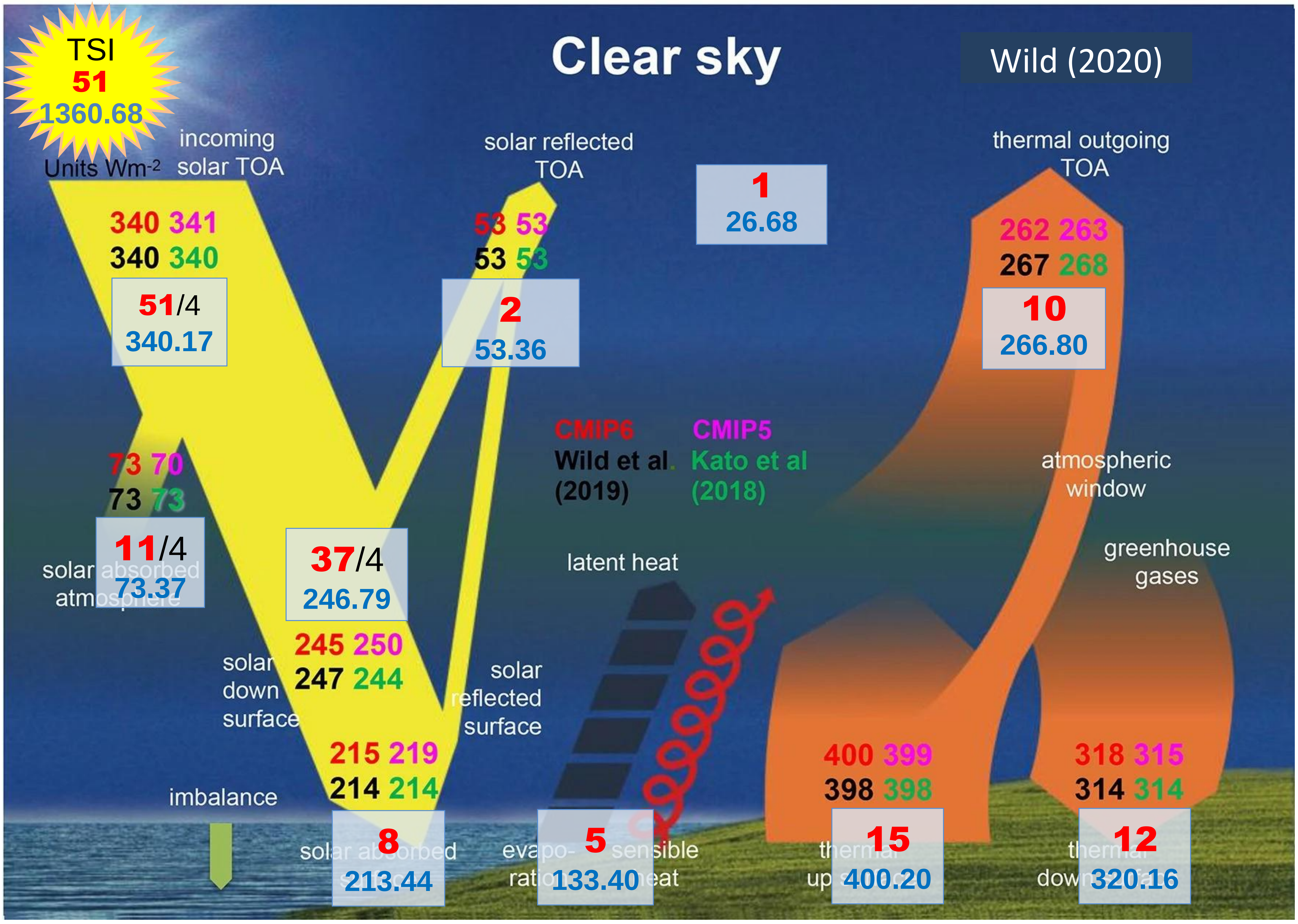

Clear sky
Wild (2020)
TSI
51
1360.68
Units Wm-2
incoming solar TOA
340 341
340 340
51/4
340.17
solar reflected TOA
53 53
53 53
2
53.36
1
26.68
thermal outgoing TOA
262 263
267 268
10
266.80
CMIP6
Wild et al (2019)
CMIP5
Kato et al (2018)
atmospheric window
greenhouse gases
73 70
73 73
11/4
73.37
solar absorbed atmosphere
37/4
246.79
245 250
247 244
solar down surface
latent heat
solar reflected surface
215 219
214 214
imbalance
8
213.44
solar absorbed
5
133.40
evapo-
sensible
400 399
398 398
15
400.20
thermal up
318 315
314 314
12
320.16
thermal

Fig.1a, b Theoretical reference estimate (TRE) projected on Wild (2020, Fig.13) all-sky and clear-sky, in textboxes. Values in upper rows (red bold typeface) are integer multiples of the unit flux of 26.68 Wm^{-2}; values in the lower rows (blue) are in Wm^{-2}. The colored numbers in the original diagram are reference estimates from four different sources as specified in Wild (2020). Total solar irradiance is shown as TSI = 51 units = 1360.68 Wm^{-2} [the most accurate community consensus value is 1360.8 ± 0.5 Wm^{-2}, Kopp and Lean 2011]. Recently, the solar minimum value was increased by 0.294 Wm^{-2} (G. Kopp, August 2023); the most current estimate of TSI for 2003-2024 mean from SORCE and TSIS-1 TIM is 1361.96 Wm^{-2}. This value, with a geodetic weighting factor of 4.0034 (as in CERES EBAF), corresponds to 1 unit = 26.682 Wm^{-2}. I use **1** = 26.68 Wm^{-2} throughout this study, belonging to TSI of 51 units = 1361.84 Wm^{-2}. — With this solar-based unit flux, there are several, remarkably accurate fits, one of the most unexpected is solar reflected TOA, both all-sky and clear-sky, having 100 Wm^{-2} and 53 Wm^{-2}, resp., without any reference to cloudy or surface reflective properties. Notice that in the all-sky, TRE albedo = 15/51 = 0.294; *c.f.* "The CERES flying on the Terra and Aqua satellites confirm that Earth's albedo is 29.4% (±0.3%)" [Ackerman, L'Ecuyer, Loeb et al. 2019, AMS Met Monographs].

Below: Theoretical reference estimate (**N** and **N** × unit) compared to Wild (2020, Table 1), and Wild and Bosilovich (2024, Table 1) (next page).

TOA	Reference estimates	N	N × unit	CMIP6 mean
SW down TOA	340[a], 340[b], 340[c]	**51**/4	340.17	**340.2**
SW up all-sky TOA	− 99[a], − 100[b], − 102[c]	**-15**/4	-100.05	**− 100.6**
SW absorbed all-sky TOA	241[a], 240[b], 238[c]	**36**/4	240.12	239.5
SW up clear-sky TOA	− 53[a], − 53[b]	**-8**/4	-53.36	− 53.0
SW absorbed clear-sky TOA	287[a], 287[b]	**43**/4	286.81	**287.3**
SW CRE TOA	− 46[a], − 47[b]	**-7**/4	-46.69	− 47.8
LW up (OLR) all-sky TOA	− 240[a], − 239[b], − 238[c]	**-9**	-240.12	− 238.3
LW up (OLR) clear-sky TOA	− 268[a], − 267[b]	**-10**	-266.80	− 262.4
LW CRE TOA	28[a], 28[b]	**1**	26.68	24.1
Net CRE TOA	− 18[a], − 19[b]	**-3**/4	-20.01	− 23.6
Imbalance TOA	0.7[a]	0		1.1
Atmosphere				
SW absorbed all-sky atmos.	80[b]. 74[c], 77[d]	**3**	80.04	**76.0**
SW absorbed clear-sky atmos.	73[b], 73[d]	**11**/4	73.37	**72.8**
SW CRE atmos.	7[b], 4[d]	**1**/4	6.67	**3.2**
LW net all-sky atmos.	− 183[b], − 180[c], − 187[d]	**-7**	-186.76	**− 182.1**
LW net clear-sky atmos.	− 183[b], − 184[d]	**-7**	-186.76	**− 180.9**
LW CRE atmos.	0[b], − 3[d]			− 1.3
Net CRE atmos.	7[b], 1[d]	**1**/4	6.67	**1.9**
Surface				
SW down all-sky surface	185[b], 186[c], 187[d]	**7**	186.76	**187.4**
SW up all-sky surface	− 25[b], − 22[c], − 23[d]	**-1**	-26.68	− 23.9
SW absorbed all-sky surface	160[b], 164[c], 164[d]	**6**	160.08	**163.4**
SW down clear-sky surface	247[b], 244[d]	**37**/4	246.79	**244.8**
SW up clear-sky surface	33[b], 30[d]	**5**/4	33.35	30.2
SW absorbed clear-sky surface	214[b], 214[d]	**8**	213.44	**214.6**
SW CRE surface	− 54[b], − 50[d]	**-2**	-53.36	**− 51.2**
LW down all-sky surface	342[b], 341[c], 344[d]	**13**	346.84	**343.8**
LW up all-/clear-sky surface	398[b], 399[c], 398[d]	**15**	400.20	− 399.9
LW net all-sky surface	− 56[b], − 58[c], − 54[d]	**-2**	-53.36	**− 56.2**
LW down clear-sky surface	314[b], 314[d]	**12**	320.16	**318.0**
LW net clear-sky surface	− 84[b], − 84[d]	**-3**	-80.04	**− 81.7**
LW CRE surface	28[b], 30[d]	**1**	26.68	25.5
Net CRE surface	− 26[b], − 20[d]	**-1**	-26.68	**− 25.4**
Net radiation surface	104[b], 106[c], 110[d]	**14**	106.72	107.2
Latent heat flux	− 82[b], − 81[c]	**-3**	-80.04	− 85.3
Sensible heat flux	− 21[b], − 25[c]	**-1**	-26.68	**− 20.1**
Surface Imbalance	0.6[b], 0.5[c]	0		1.5

The Global Energy Balance as Represented in Atmospheric Reanalyses

Martin Wild · Michael G. Bosilovich

Surveys in Geophysics 2024 https://doi.org/10.1007/s10712-024-09861-9

Energy balance component	Reference Estimates Wm^{-2}	Recent Reanal	Reanal r Wm^{-2}	N	N×unit
TOA					
SW down	340[a], 340[b], 340[c]	340.9	340.8	51/4	340.17
SW up all-sky	−99[a], −100[b], −102[c]	−101.3	−100.7	-15/4	-100.05
SW absorbed all-sky	241[a], 240[b], 238[c]	239.6	240.1	36/4	240.12
SW up clear-sky	−53[a], −53[b]	−51.3	−52.5	-8/4	-53.36
SW absorbed clear-sky	287[a], 287[b]	289.4	288.2	43/4	286.81
SW CRE	−46[a], −47[b]	−48.8	−47.7	-7/4	-46.69
LW up (OLR) all-sky	−240[a], −239[b], −238[c]	−243.4	−241.5	-9	-240.12
LW up (OLR) clear-sky	−268[a], −267[b]	−265.8	−265.2	-10	-266.80
LW CRE	28[a], 28[b]	22.4	23.9	1	26.68
Net CRE	−18[a], −19[b]	−26.5	−23.8	-3/4	-20.01
Imbalance	0.7 [a]	−3.9	−1.5	0	0
Atmosphere					
SW absorbed all-sky	80[b]. 74[c], 77[d]	76.1	75.9	3	80.04
SW absorbed clear-sky	73[b], 73[d]	73.6	72.8	11/3	73.37
SW CRE	7[b], 4[d]	2.7	3.1	1/4	6.67
LW net all-sky	−183 [b], −180[c], −187[d]	−184.2	−181.6	-7	-186.76
LW net clear-sky	−183[b], −184[d]	−182.0	−181.4	-7	-186.76
LW CRE	0[b], −3[d]	−1.4	0.3	0	0
Net CRE	7[b], 1[d]	1.3	3.4	1/4	6.67

Energy balance component	Reference Estimates Wm^{-2}	Recent Reanal	Reanal r Wm^{-2}	N	N×unit
Surface					
SW down all-sky	185 [b], 186 [c], 187[d]	187.8	189.2	7	186.76
SW up all-sky	−25 [b], −22[c], −23[d]	−24.4	−25.0	-1	-26.68
SW absorbed all-sky	160 [b], 164 [c], 164[d]	163.4	164.2	6	160.08
SW down clear-sky*	247[b], 244[d]	246.8	247.8	37/4	246.79
SW up clear-sky*	33[b], 30[d]	31.0	32.4	-5/4	-33.35
SW absorbed clear-sky	214[b], 214[d]	215.8	215.3	8	213.44
SW CRE	−54[b], −50 [d]	−51.5	−50.8	-2	-53.36
LW down all-sky	342[b], 341[c], 344[d]	339.1	338.2	13	346.84
LW up all-sky/ clear-sky	−398[b], −399[c], −398[c]	−398.3	−398.1	-15	-400.20
LW net all-sky	−56[b], −58[c], −54[d]	−59.3	−59.9	-2	-53.36
LW down clear-sky	314[b], 314[d]	314.7	314.3	12	320.16
LW net clear-sky	−84[b], −84[d]	−83.9	−83.9	-3	-80.04
LW CRE	28[b], 30[d]	23.7	23.6	1	26.68
Net CRE	−26[b], −20[d]	−27.7	−27.2	-1	-26.68
Net radiation	104[b], 106[c], 110[d]	104.1	104.3	4	106.72
Latent heat flux	−82[b], −81[c]	−88.7	−86.0	-3	-80.04
Sensible heat flux	−21[b], −25[c]	16.0	−17.5	-1	-26.68
Surface Imbalance	0.6 [b], 0.5[c]	−0.6	0.7	0	0

An intensification of surface Earth's energy imbalance since the late 20th century
X. Li, Q. Li, M. Wild, P. Jones (2024)

https://doi.org/10.1038/s43247-024-01802-z Article

Fig. 2 | The global Radiation Budget from a surface perspective in this study. The numbers indicate the best estimates and their uncertainties (at 95% confidence level) for the magnitudes of the globally averaged energy balance components, obtained using the BMA method constrained by observational data from 34 CMIP6 models during 2000–2022, representing present-day climate conditions since the start of the 21st century. Downward shortwave radiation is constrained by reconstructed SSR data from Jiao et al. 2023, while upward shortwave and longwave radiation are constrained using GEBA observational station data. Due to the lack of observational data for latent and sensible heat fluxes, SME results are used directly. Uncertainties are calculated from the weighted ensemble standard deviation of the models.

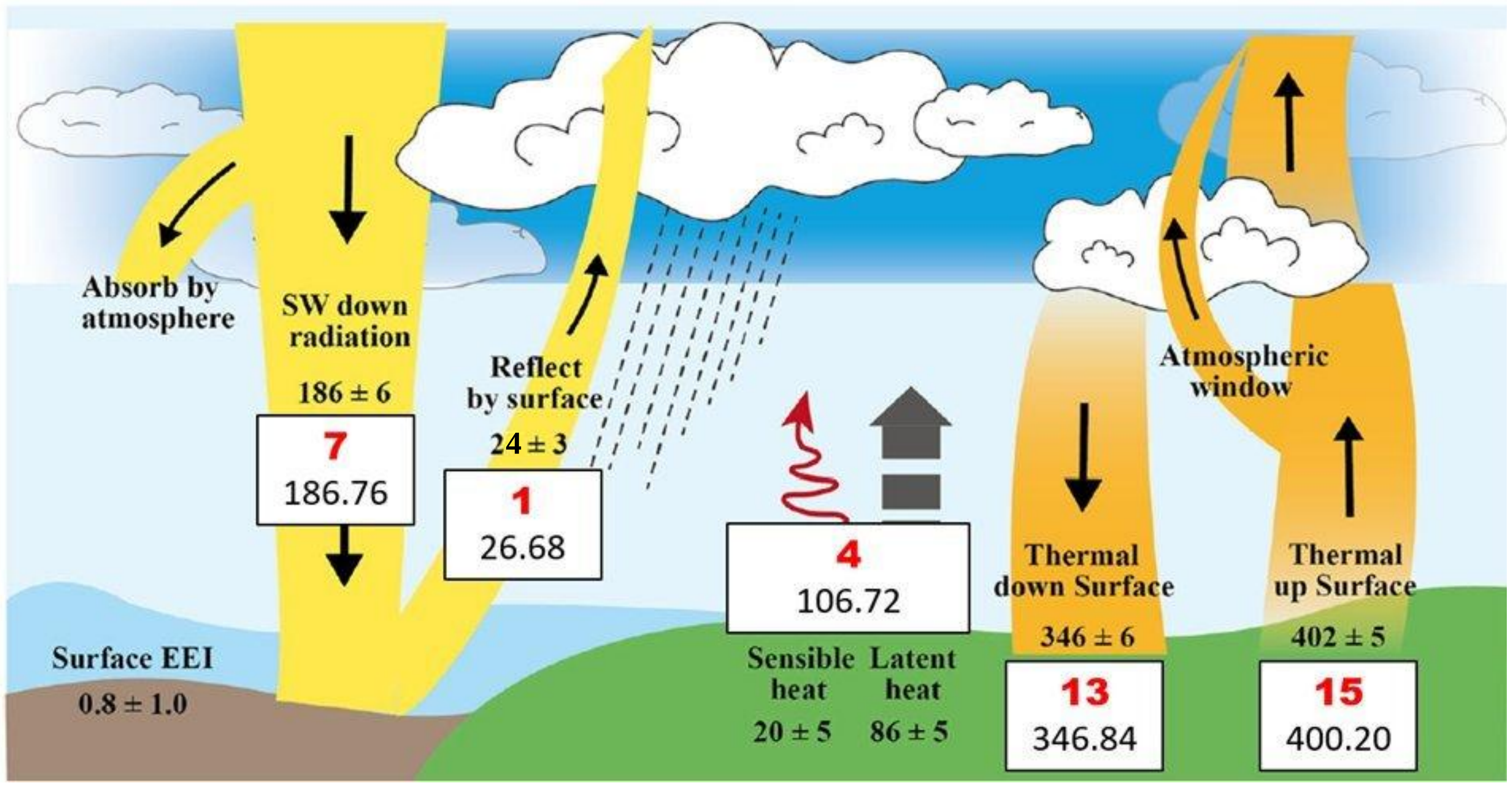
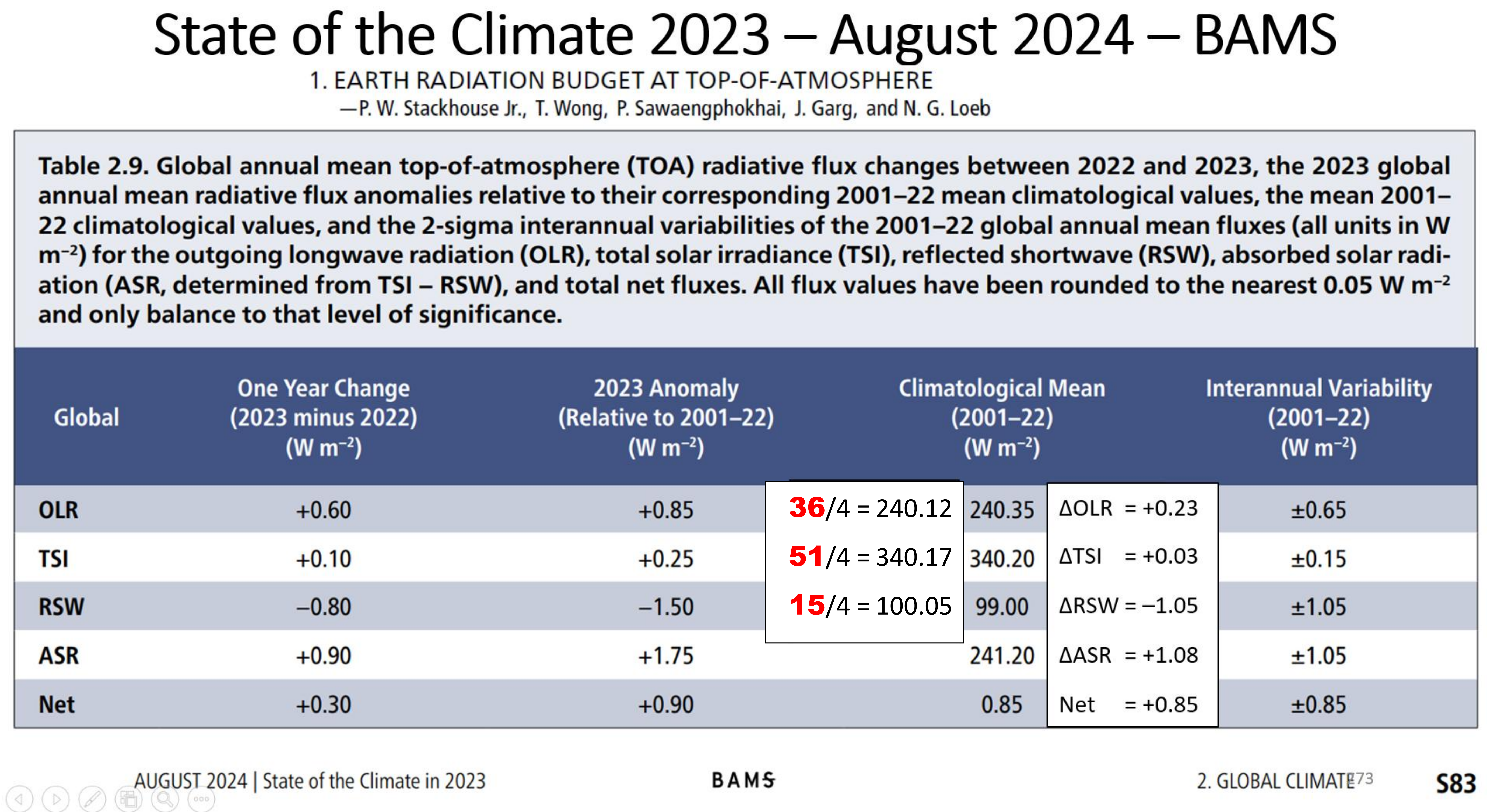


Fig. 2 Theoretical Reference Estimate (TRE) projected on the global radiation budget from a surface perspective (Li, Li, Wild and Jones, 2024) in form of integer positions and values in textboxes. The accuracy of their estimate to the GHG-independent theoretical expectation is within the acknowledged ranges of uncertainty; the differences are 0.76, 2.68, 0.72, 0.84 and 1.80 Wm^{-2}, respectively. The convective flux at the surface is 20 + 86 = 106 Wm^{-2}, the net radiation (R_N) is the same, 106 Wm^{-2}; the total absorbed radiation at the surface is R_T = 186–24+346 = 508 Wm^{-2}. My reference estimate gives R_N = **4** units which, with **1** unit = 26.68 Wm^{-2}, yields R_N (theory) = 106.72 Wm^{-2}, the difference is 0.72 Wm^{-2}; and my reference estimate for R_T = **6** + **13** = **19** units, yielding R_T (theory) = 506.92 Wm^{-2}; difference is 1.08 Wm^{-2}, showing the accuracy of the Earth's system sitting on its stationary geometric equilibrium position, far within the stated uncertainty. — Since the energy released by the surface is 402 + 20 + 86 = 508 Wm^{-2}, and its total absorption is the same, 186 – 24 + 346 = 508 Wm^{-2}, this surface energy balance represents a surface in equilibrium, that is, a zero EEI, being in evident contradiction to the indicated EEI = 0.8 and with the title of the article. [In a communication the Authors informed me about the values of the diagram in one decimal place, giving up the correct EEI = 0.8 Wm^{-2}, but these numbers are not presented in the paper.] — An easy solution could be to decrease 'Thermal up Surface' from 402 Wm^{-2} (which is unreasonably high, compared to Wild et al. (2015, 2019) = 398, CERES = 398.5 or CMIP6 = 399.9 for the examined time period) to 401 Wm-2, generating a positive (downward) 1 Wm-2 EEI, and satisfying the integer solution for this energy flow component by a difference of 0.8 Wm-2. Note that the most reliable assessment for components of the convective flux (Sensible heat, SH, and Latent heat, LH) is from the NEWS – NASA Energy and Water-cycle Study (Rodell et al. 2015, L'Ecuyer et al. 2015), proposing SH = 25 and LH = 81 (R_N = 106 Wm^{-2}), and, after a second optimization (Stephens and L'Ecuyer 2015), SH = 26 and LH = 82 (R_N = 108 Wm^{-2}). Notice that **1** = 26.68 Wm^{-2} and **3** = 80.04 Wm^{-2}; hence these components occupy integer positions separately.



State of the Climate 2023 – August 2024 – BAMS

1. EARTH RADIATION BUDGET AT TOP-OF-ATMOSPHERE

—P. W. Stackhouse Jr., T. Wong, P. Sawaengphokhai, J. Garg, and N. G. Loeb

Table 2.9. Global annual mean top-of-atmosphere (TOA) radiative flux changes between 2022 and 2023, the 2023 global annual mean radiative flux anomalies relative to their corresponding 2001–22 mean climatological values, the mean 2001–22 climatological values, and the 2-sigma interannual variabilities of the 2001–22 global annual mean fluxes (all units in W m^{-2}) for the outgoing longwave radiation (OLR), total solar irradiance (TSI), reflected shortwave (RSW), absorbed solar radiation (ASR, determined from TSI – RSW), and total net fluxes. All flux values have been rounded to the nearest 0.05 W m^{-2} and only balance to that level of significance.

Global	One Year Change (2023 minus 2022) (W m^{-2})	2023 Anomaly (Relative to 2001–22) (W m^{-2})		Climatological Mean (2001–22) (W m^{-2})		Interannual Variability (2001–22) (W m^{-2})
OLR	+0.60	+0.85	36/4 = 240.12	240.35	ΔOLR = +0.23	±0.65
TSI	+0.10	+0.25	51/4 = 340.17	340.20	ΔTSI = +0.03	±0.15
RSW	−0.80	−1.50	15/4 = 100.05	99.00	ΔRSW = −1.05	±1.05
ASR	+0.90	+1.75		241.20	ΔASR = +1.08	±1.05
Net	+0.30	+0.90		0.85	Net = +0.85	±0.85

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Fig. 3 Theoretical reference estimate (TRE) projected on TOA radiation budget of CERES by Stackhouse et al. (2024), Table 2.9. Differences from OLR (Outgoing Longwave Radiation), TSI (Total Solar Irradiance) and RSW (Reflected Shortwave Radiation) climatological means (2001-22) are 0.23, 0.03 and -1.05 Wm^{-2}, respectively; each within the interannual variability for the same period. TRE Unit Flux 1 = 26.68 Wm^{-2}.

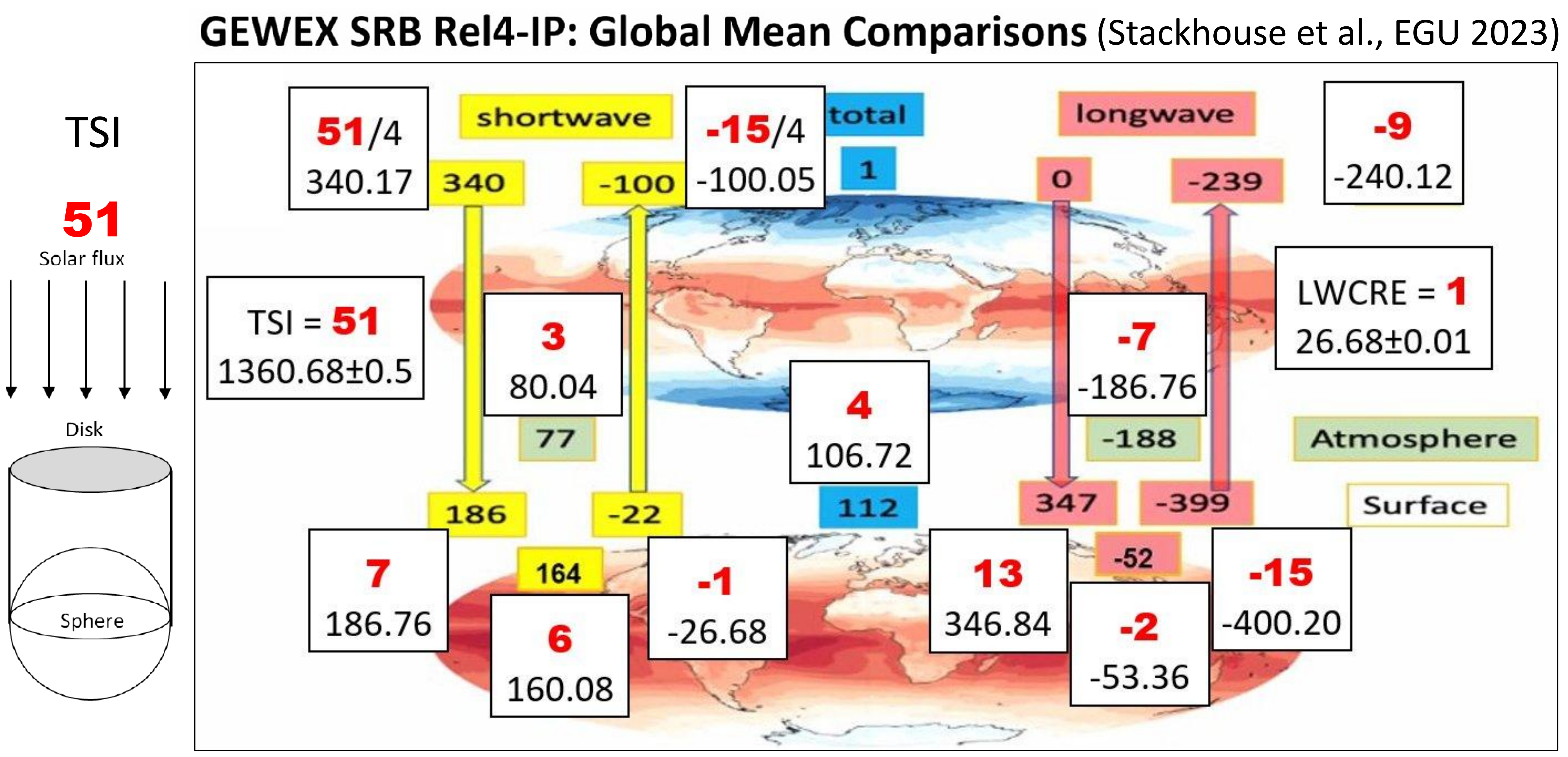


Fig. 4 TRE projected on SRB Surface Radiation Budget by GEWEX (Stackhouse et al. 2023, EGU)

Derivation of TRE Eqs. (1) and (2) from the simplest greenhouse model:

Dennis Hartmann: Global Physical Climatology (1994) Cambridge Univ. Press

2.5 Greenhouse Effect

One may illustrate the greenhouse effect with a very simple elaboration of the energy balance model used to define the emission temperature. An atmosphere that is assumed to be a blackbody for terrestrial radiation, but is transparent to solar radiation, is incorporated into the global energy balance (Fig. 2.3). Since solar radiation is mostly visible and near-infrared, and Earth emits primarily thermal infrared radiation, the atmosphere may affect solar and terrestrial radiation very differently. The energy balance at the top of the atmosphere in this model is the same as in the basic energy balance model that defined the emission temperature (2.9). Since the atmospheric layer absorbs all of the energy emitted by the surface below it and emits like a blackbody, the only radiation emitted to space is from the atmosphere in this model. The energy balance at the top of the atmosphere is thus

$$\frac{S_0}{4}\left(1-\alpha_p\right)=\sigma T_A^4=\sigma T_e^4 \tag{2.11}$$

Therefore we see that the temperature of the atmosphere in equilibrium must be the emission temperature in order to achieve energy balance. The surface tempera-

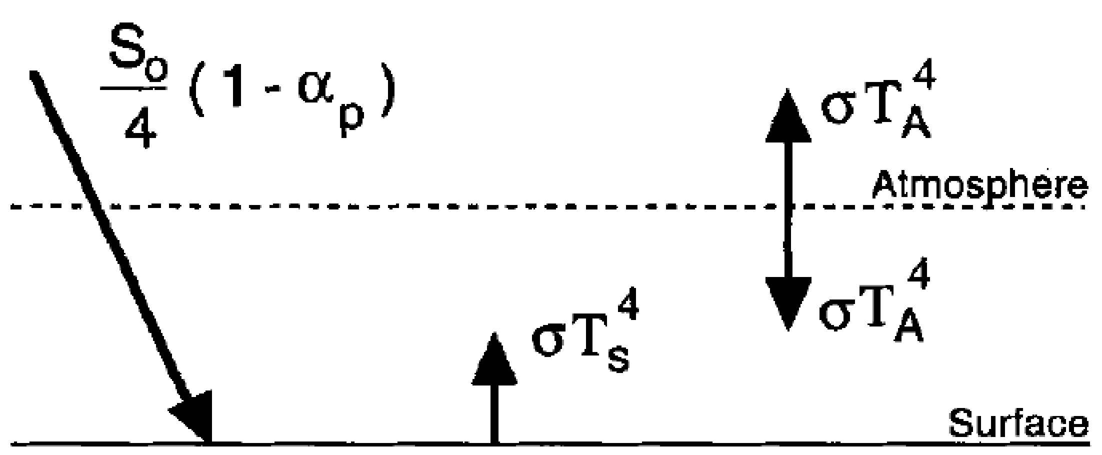


Fig. 2.3 Diagram of the energy fluxes for a planet with an atmosphere that is transparent for solar radiation but opaque to terrestrial radiation.

$$\sigma T_s^4=2\sigma T_A^4 \quad \Rightarrow \quad \sigma T_s^4=2\sigma T_e^4 \tag{2.12}$$

and the surface energy balance is consistent:

$$\frac{S_0}{4}\left(1-\alpha_p\right)+\sigma T_A^4=\sigma T_s^4 \quad \Rightarrow \quad \sigma T_s^4=2\sigma T_e^4 \tag{2.13}$$

Eq. (2) $\sigma T_S^4 = 2\sigma T_e^4$.

Then

A thin layer of atmosphere near the surface absorbs a fraction ε of the emission from above and below and emits in both directions. The temperature of the air adjacent to the surface, T_{SA}, may be derived from the energy balance there.

$$\varepsilon\sigma T_S^4 + \varepsilon\sigma T_A^4 = 2\varepsilon\sigma T_{SA}^4 .$$

From Eq. (2) $\sigma T_S^4 = 2\sigma T_e^4$ and $T_A = T_e$, we have $3\sigma T_e^4 = 2\sigma T_{SA}^4$,

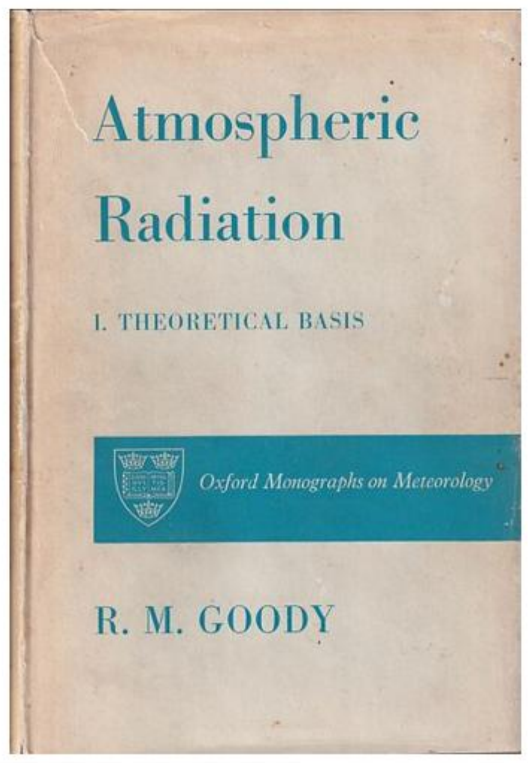
that is, $\sigma T_{SA}^4 = (3/2)\,\sigma T_e^4$,

and

Eq. (1) $\sigma T_S^4 - \sigma T_{SA}^4 = \sigma T_e^4/2$.

Some books with explanation of TRE Eq. (1):

R. Goody: Atmospheric radiation. Theoretical basis. Oxford, 1964

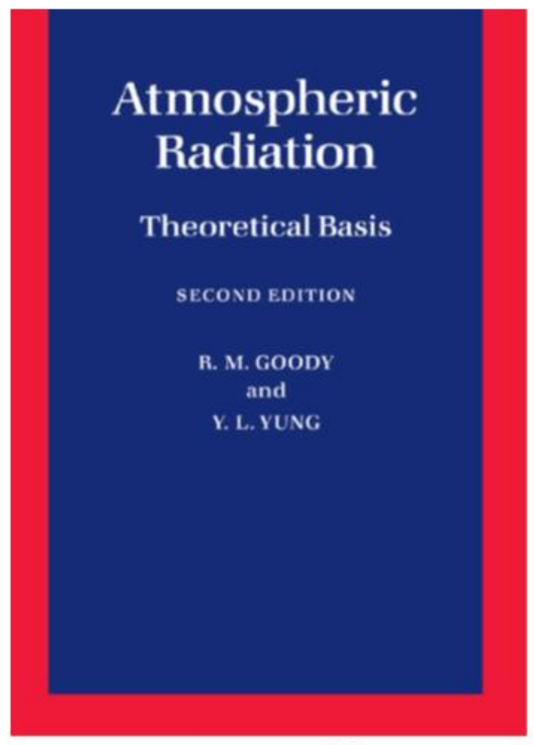


$$F_\nu/2\pi = J_\nu(0) - B_\nu^*(0) = B_\nu^*(\tau_\nu^*) - J_\nu(\tau_\nu^*). \qquad (2.115)$$

Note that if the medium is in thermodynamic equilibrium ($J_\nu = B_\nu$), equation (2.115) requires a discontinuity in B_ν (i.e. a *temperature discontinuity*) at both boundaries.

the step in B always equals $F/2\pi$

Goody – Yung: Atmospheric radiation. Theoretical basis. Oxford, 1989



$$F/2\pi = B(0) - B^*(0) = B^*(\tau_1) - B(\tau_1). \qquad (2.146)$$

Equation (2.146) requires a discontinuity in the Planck function, implying a discontinuity of temperature, at the boundary.

2. There are discontinuities,

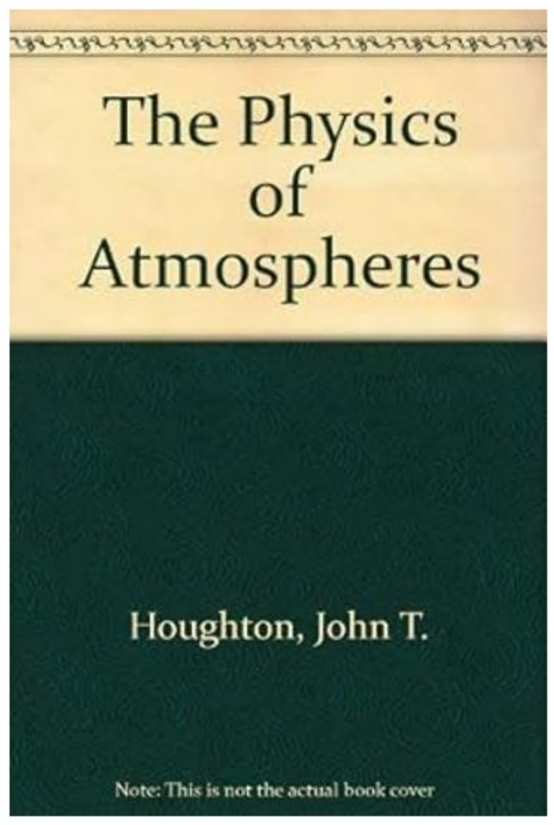
$$\Delta B = \frac{F_S}{2\pi}$$

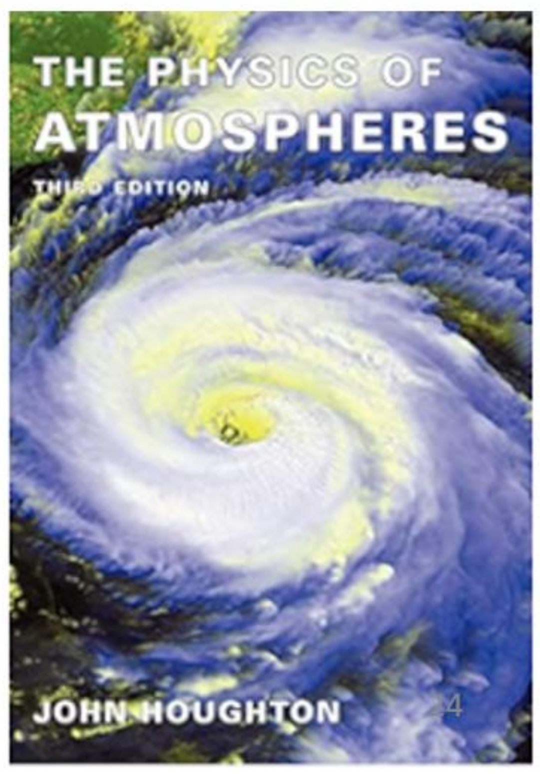
DISCONTINUITY = OLR/2

23

John Houghton: The Physics of Atmospheres

Cambridge (1977, 1986, 2002) Eq. 2.13

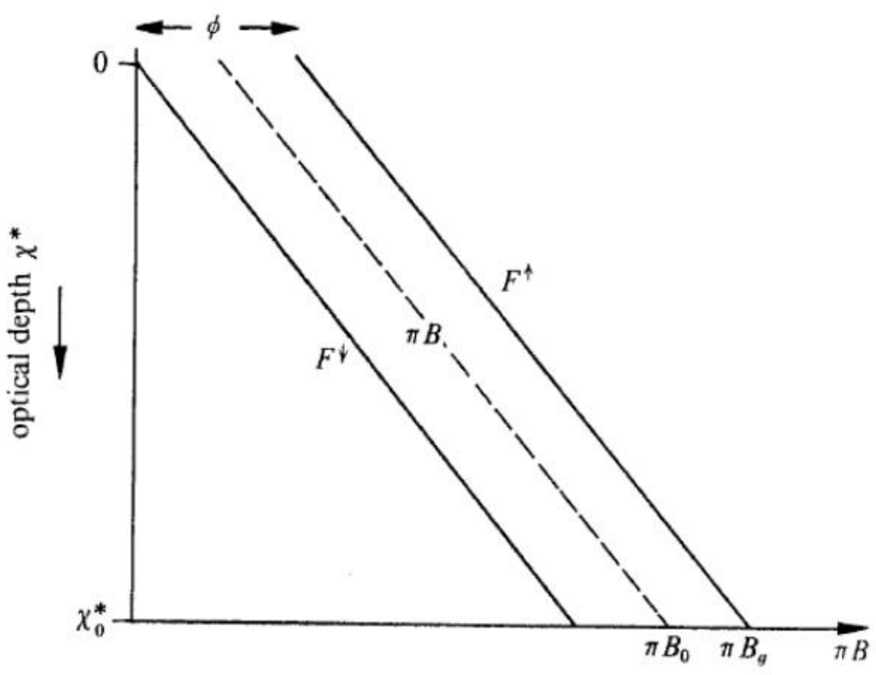




At the bottom of the atmosphere where $\chi^* = \chi_0^*$, $F^\uparrow = \pi B_g$, B_g being the black-body function at the temperature of the ground. It is easy to show that there must be a temperature discontinuity at the lower boundary, the black-body function for the air close to the ground being B_0, and

$$B_g - B_0 = \frac{\phi}{2\pi} \qquad (2.13)$$

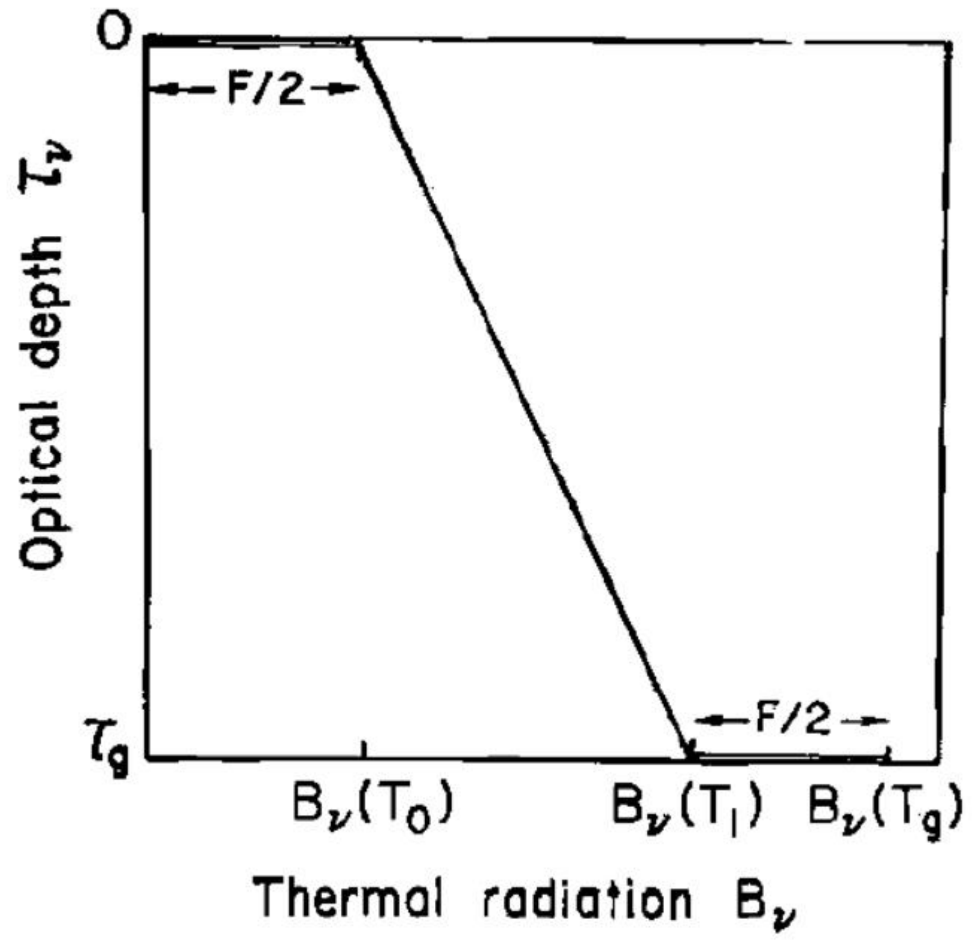
DISCONTINUITY = OLR/2
(independent of the optical depth)



such a steep lapse rate is very unstable with respect to vertical motion, and will soon be destroyed by the process of *convection*

Discontinuity = Convection = OLR/2

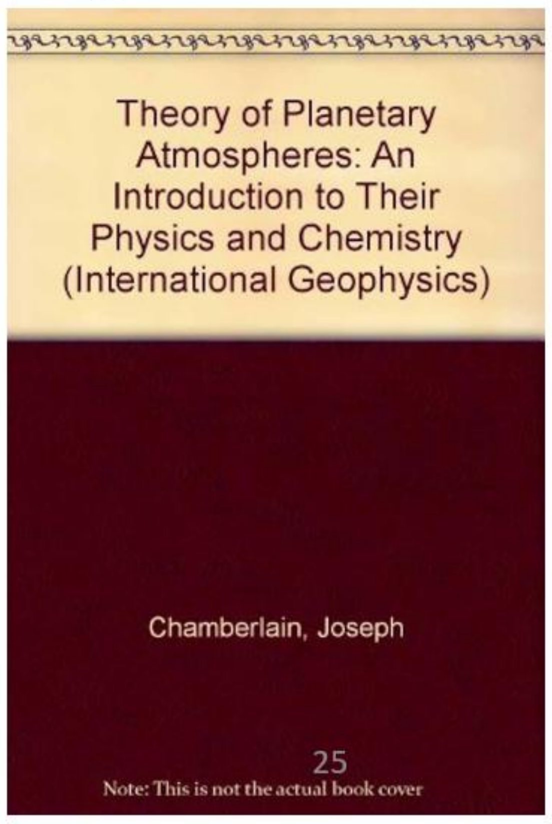
1.2 Radiative Equilibrium 9



Joseph Chamberlain
Theory of Planetary Atmospheres (1978, 1987)
Academic Press, Fig. 1.4, Eq. 1.2.29

Fig. 1.4 The MRE solution for $T(\tau)$, presented as $B_\nu(T)$ vs. τ. Note the discontinuity at the ground and the finite skin temperature at $\tau = 0$.

DISCONTINUITY = OLR/2
(independent of τ)



Hence the upward intensity at the ground is

$$I_g^+ \equiv B_\nu(T_g) = B_\nu(T_1) + \tfrac{1}{2}F_\nu \qquad (1.2.29)$$

With Hartmann's data, the equation is justified within 0.31 Wm^{-2}

Emission temperature T_e = 255 K
Air adjacent to surface T_{SA} = 320 K
Surface temperature T_S = 335 K

σT_S^4	**$-$ σT_{SA}^4**		**= $\sigma T_e^4 / 2$**
5.67 [$(3.35)^4 - (3.20)^4$]			= 5.67$(2.55)^4$/2
714.11	$-$ 594.54	= 119.56	= 119.87 – **0.31** Wm^{-2}

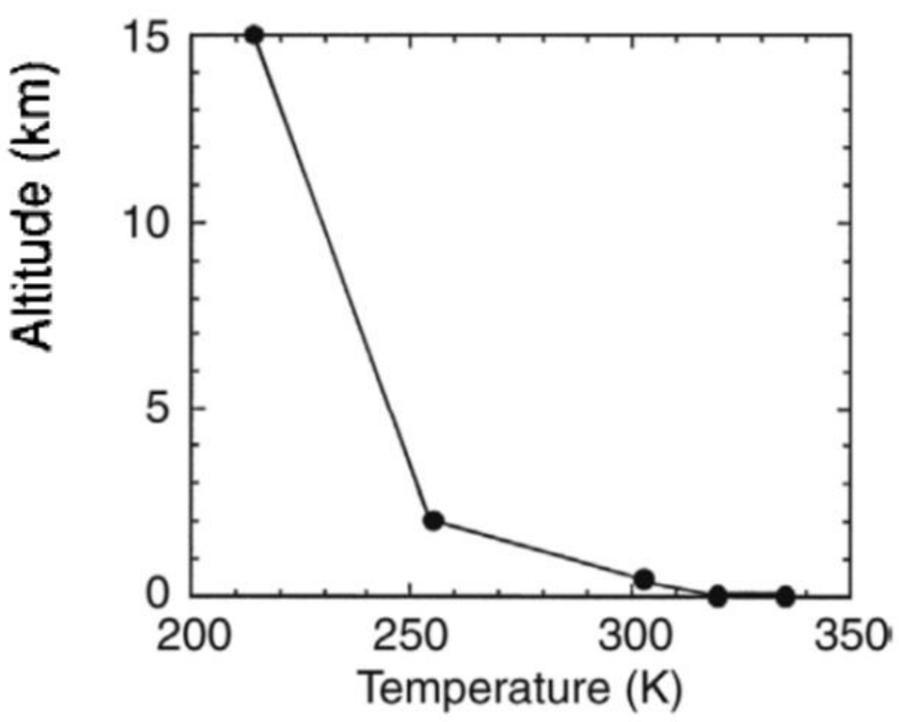
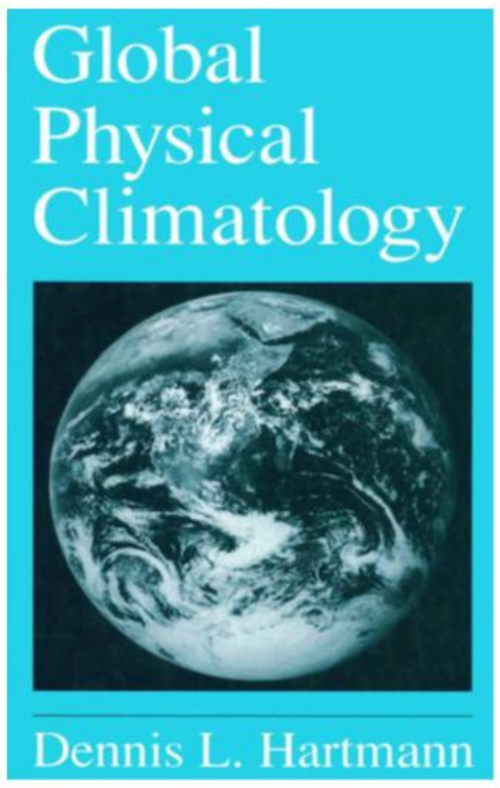
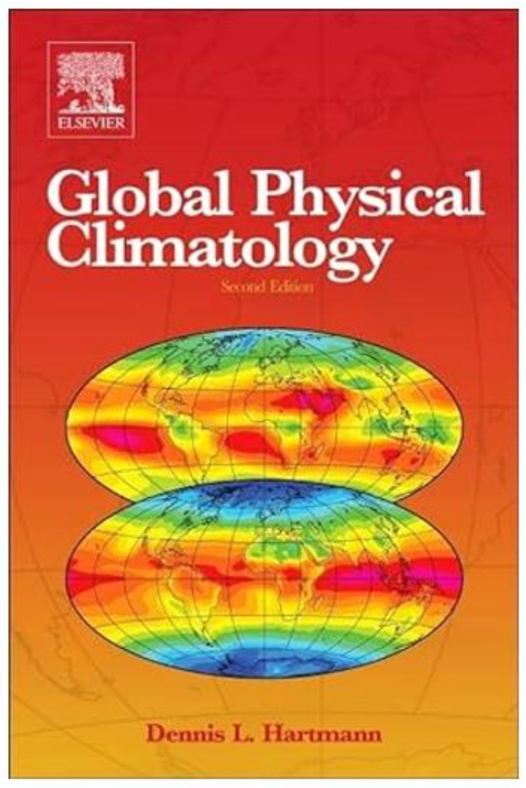


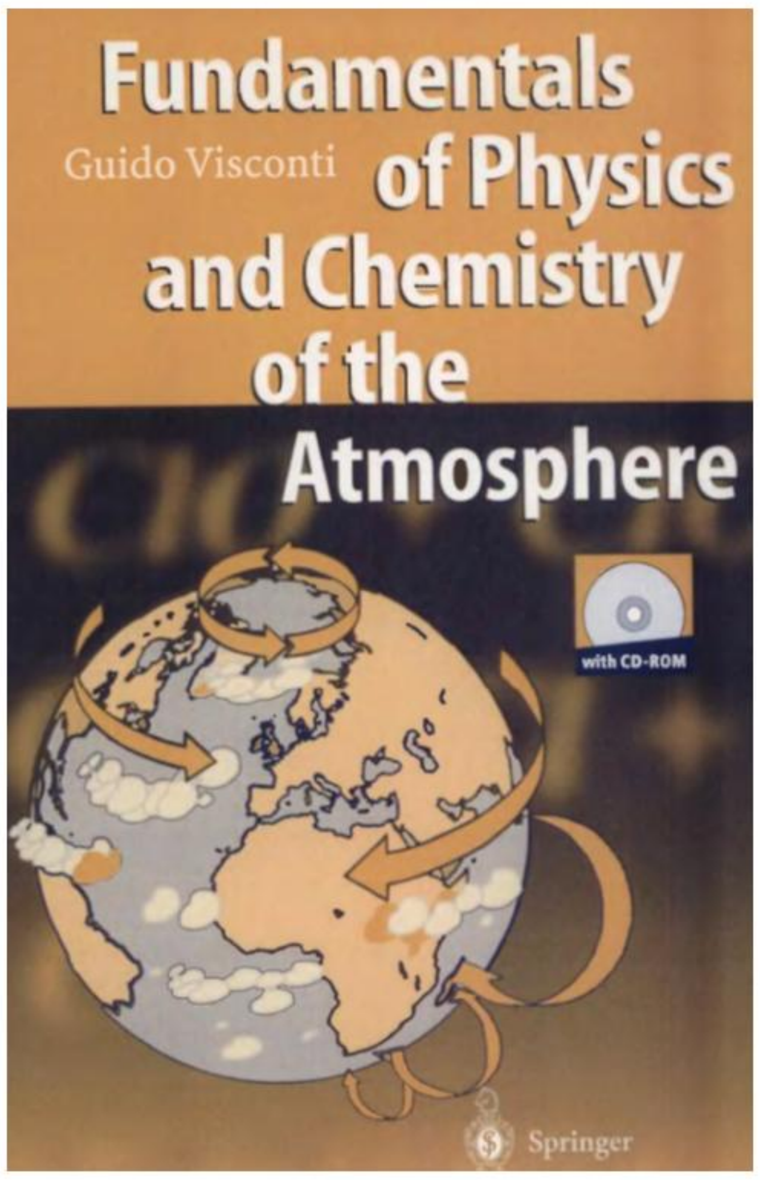
Fig. 3.11

Dennis Hartmann:
Global Physical Climatology
Academic Press
(1994, 2016)
Fig. 3.11





Visconti: Fundamentals of Physics and Chemistry of the Atmospheres — Springer (2001)



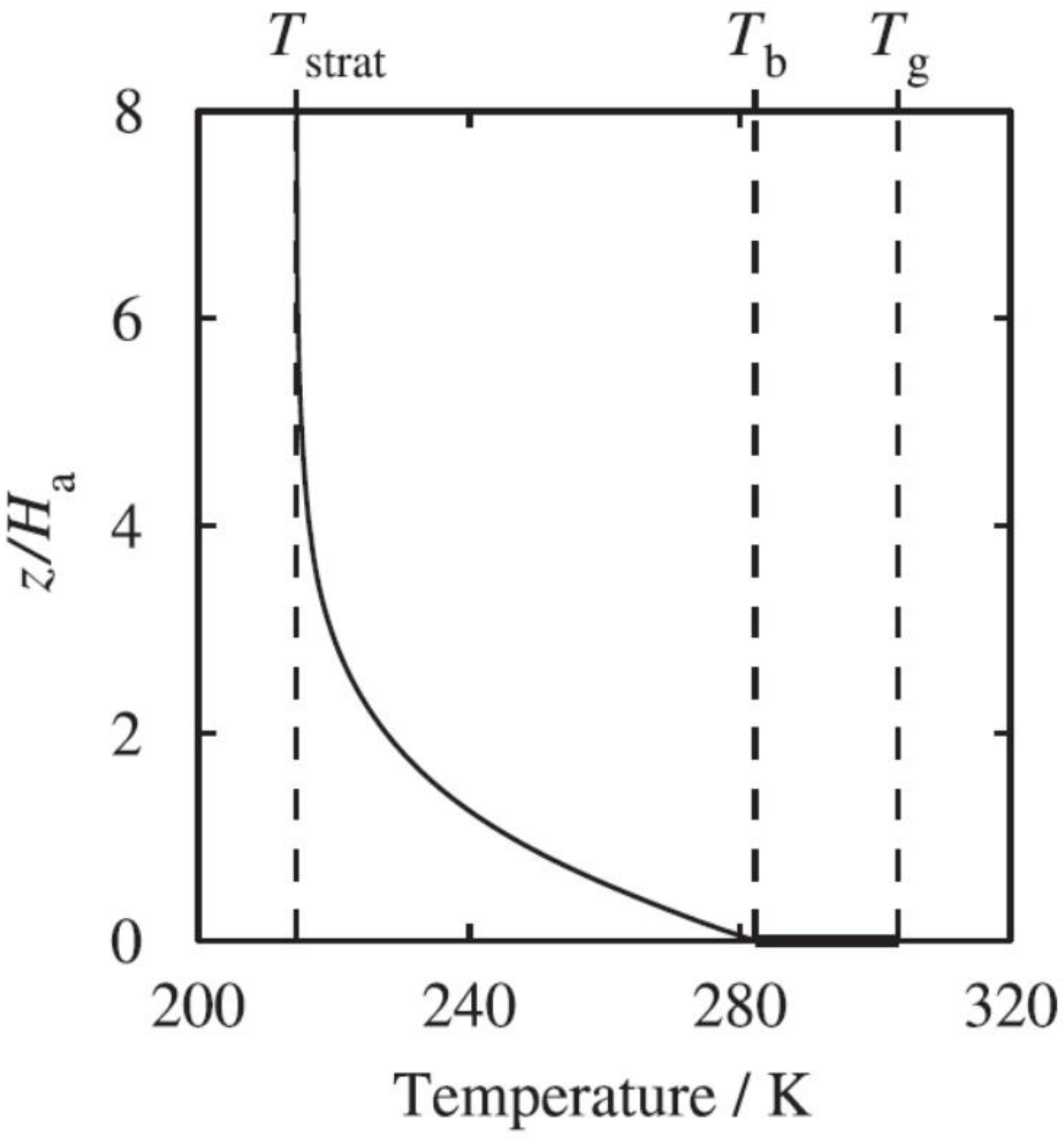
$$T^4 = \frac{T_e^4}{2}\left(\frac{3}{2}\tau + 1\right) \quad (3.47)$$

where τ has been replaced by its effective value. An interesting consequence of this solution is to discover what happens at the surface, where the optical thickness is τ^* and the temperature is T_s; we have

$$F^{\uparrow}(\tau^*) = \sigma T_s^4 = \frac{1}{2}\sigma T_e^4\left(\frac{3}{2}\tau^* + 2\right) \quad (3.48)$$

The temperature at the bottom of the atmosphere at τ^* is given by Equation (3.47), so that we have a discontinuity between the air temperature and that of the surface

$$T_s^4 - T(\tau^*) = T_e^4/2 \quad (3.49)$$

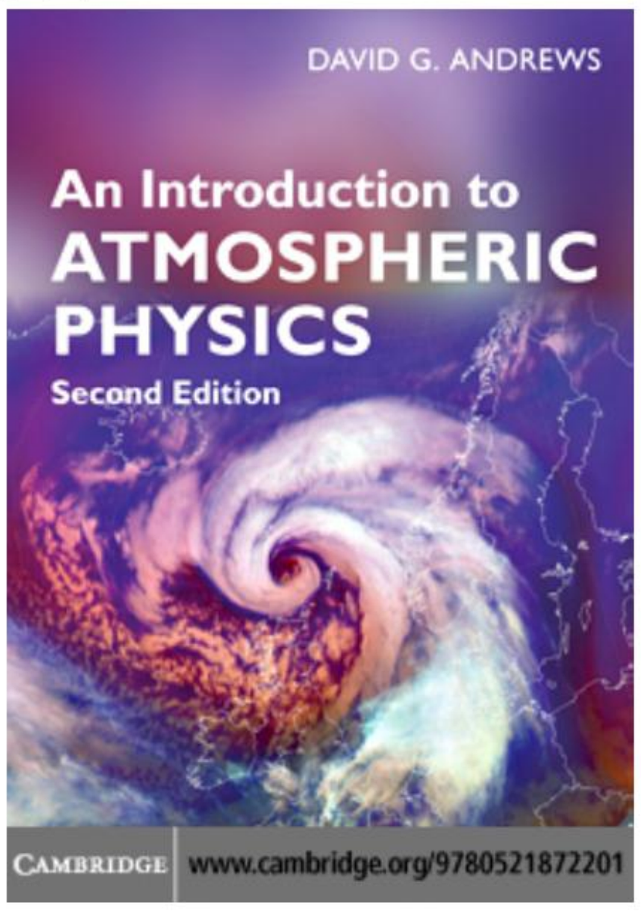


Andrews: An Introduction to Atmospheric Physics
Cambridge (2010), pp 85-86.

$$T_b \equiv T_e\left(\frac{1+\chi_g^*}{2}\right)^{1/4}$$

$$T_g \equiv T_e\left(\frac{2+\chi_g^*}{2}\right)^{1/4}$$

$$T_g^4 - T_b^4 = T_e^4/2$$



DISCONTINUITY = OLR/2 (independent of the optical depth)

discontinuity between the bottom of the atmosphere and the ground.

Inclusion of convection in the model removes the temperature discontinuity

8.5 Thermal equilibrium 235

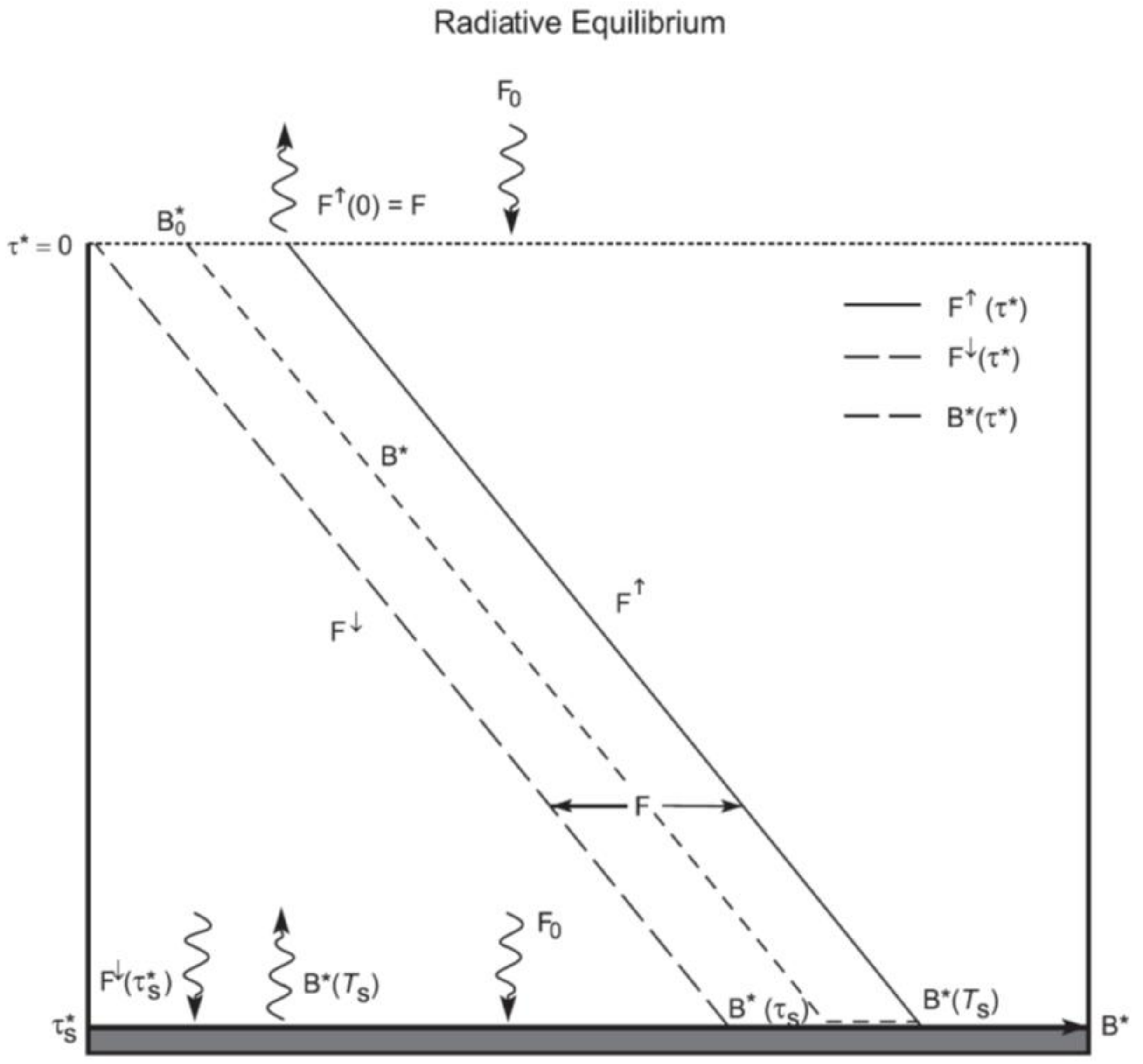
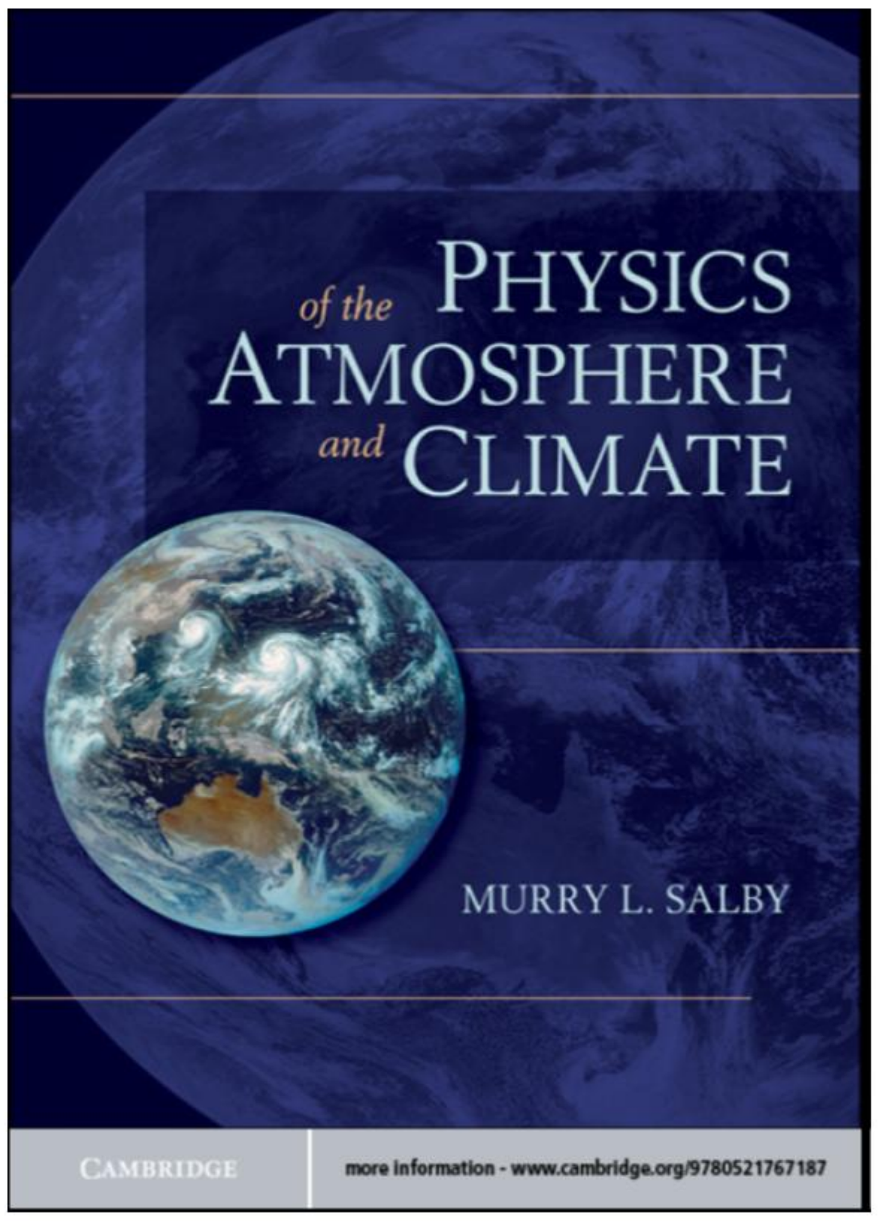


Figure 8.20 Upwelling and downwelling LW fluxes and LW emission in a gray atmosphere that is in radiative equilibrium with an incident SW flux F_0 and a black underlying surface. Note: the emission profile is discontinuous at the surface.



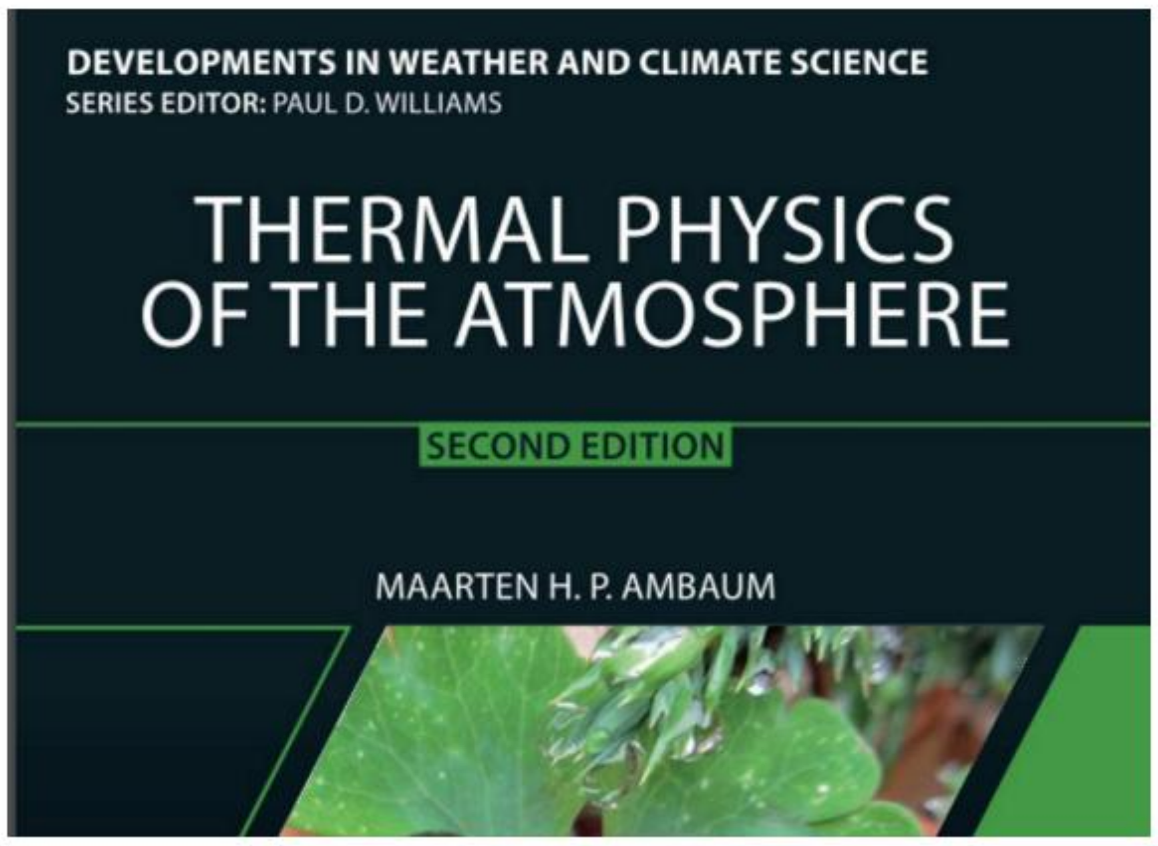
M. Salby: Physics of the Atmosphere and Climate
Cambridge, 2012

$$B^*(T_s) = B^*(\tau_s^*) + \frac{F_0}{2}. \quad (8.67)$$

According to (8.67), the temperature predicted by radiative equilibrium is discontinuous at the surface, the ground being warmer than the overlying air.

ELSEVIER

RMetS
Royal Meteorological Society



These two budget equations can be combined by eliminating $L_{\downarrow 0}$ to find

$$\blacktriangleright \qquad \sigma T_E^4 = \sigma T_S^4 + S_{n0}/2. \qquad (10.56)$$

So we find that under radiative equilibrium the surface temperature is higher than the temperature of the adjoining atmosphere. This temperature discontinuity is unstable in practice and there will be turbulent heat exchange which will remove the temperature discontinuity. This instability of the radiative state is a driver of surface layer turbulence: the radiation will force the lower part of the boundary layer, the surface layer, towards instability and this tendency is compensated by turbulent fluxes near the surface.

University lecture notes with TRE Eq. (1)

G. Stephens: Radiative transfer notes. Colorado State University

[G. L. Stephens: Radiative transfer notes AT 622 Colorado State Univ. (1992-2013)]

Example 6.3: Skin temperatures and temperature discontinuities

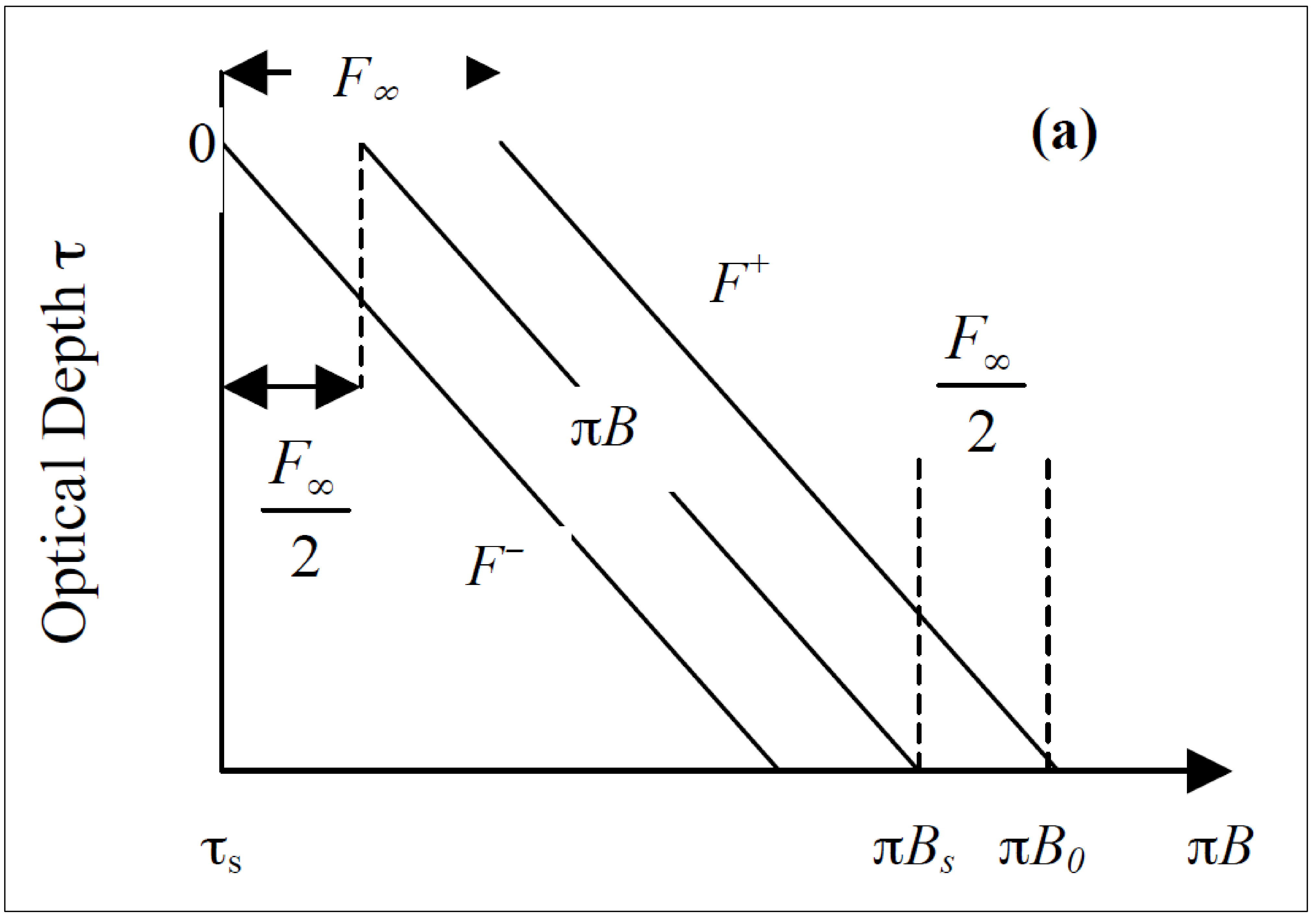
The solutions represented by Eqns. (6.10a) and (6.10b) provide rather interesting insights into the temperature profiles that are predicted by these equations. One of the results of this model is an estimate of the 'skin' temperature, which we think of as a measure of the stratospheric temperature. We obtain this using Eqn. (6.10a) with $\tilde{\tau} = 0$

$$\sigma T^4(\tilde{\tau} = 0) = \frac{F_\infty}{2}$$

and with $F_\infty \approx 235$ Wm^{-2}, it follows that this temperature is $T_{skin} = [117.5/5.68 \times 10^{-8}]^{0.25} = 213$ K.

The solutions in Eqns. (6.10a) and (6.10b) predict a discontinuity between the surface temperature T_s and the air temperature just above the ground $T(\tilde{\tau}_s)$. Differencing these equations and with $\tilde{\tau} = \tilde{\tau}_s$,

$$\sigma T_s^4 - \sigma T^4(\tilde{\tau}_s) = \frac{F_\infty}{2}.$$



Kerry Emanuel: Elements of Radiation Transfer

GFD / MIT / Woods Hole, June 16, 2014

https://gfd.whoi.edu/wp-content/uploads/sites/18/2018/03/Lecture_1_Emanuel_218144.pdf

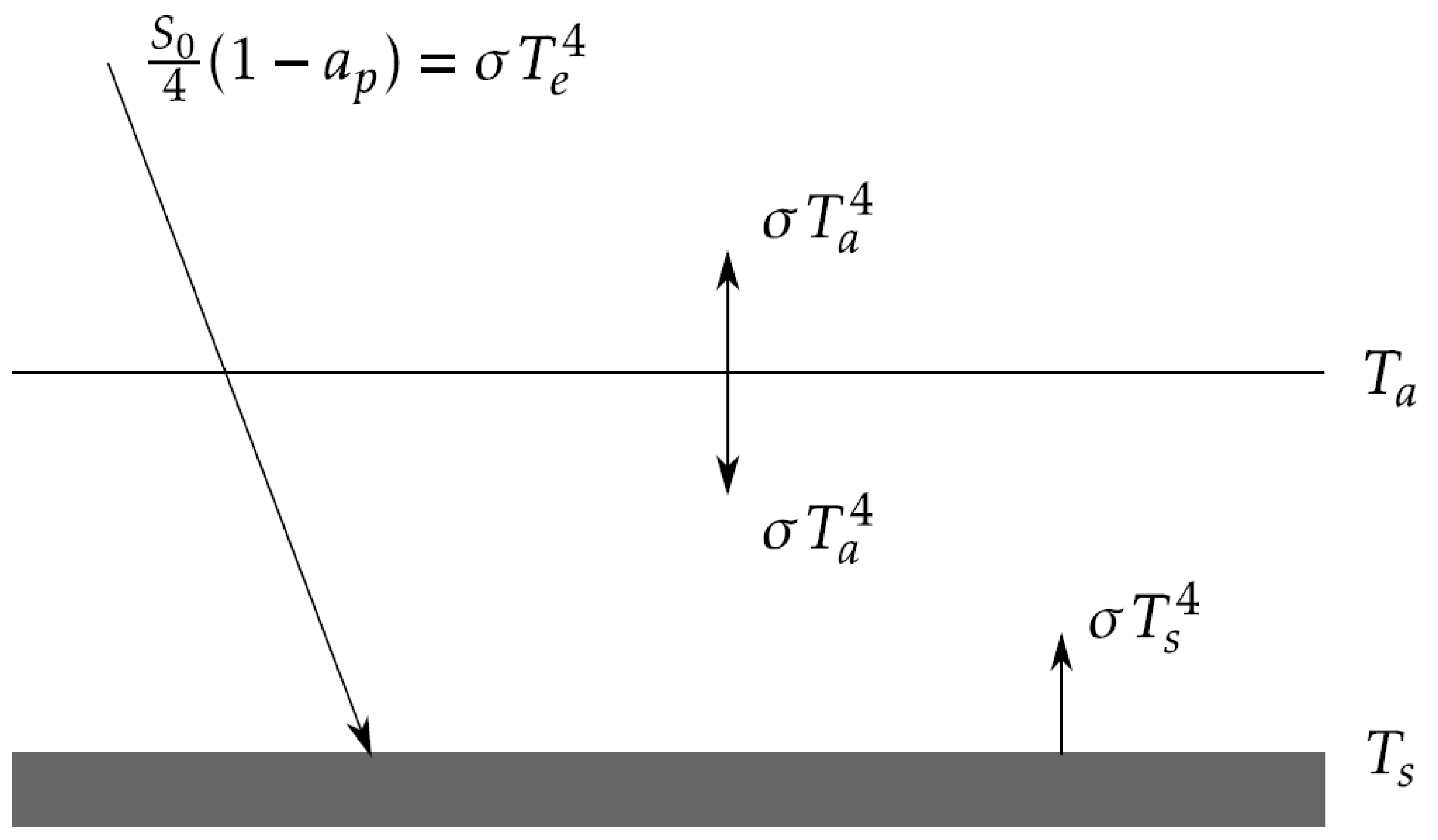


Figure 7: Radiative fluxes in a model with an atmosphere that consists of a single layer that is completely transparent to solar radiation and completely opaque to terrestrial radiation

$$\sigma T_e^4 = \sigma T_a^4 \text{ , } \sigma T_s^4 = 2\sigma T_a^4 = 2\sigma T_e^4 \text{ (This is our Eq. 2)}$$

Consider adding a thin layer of gas just above the surface. Let its temperature be T_{sa} and its emissivity ε tend to zero. The balance of this layer is then

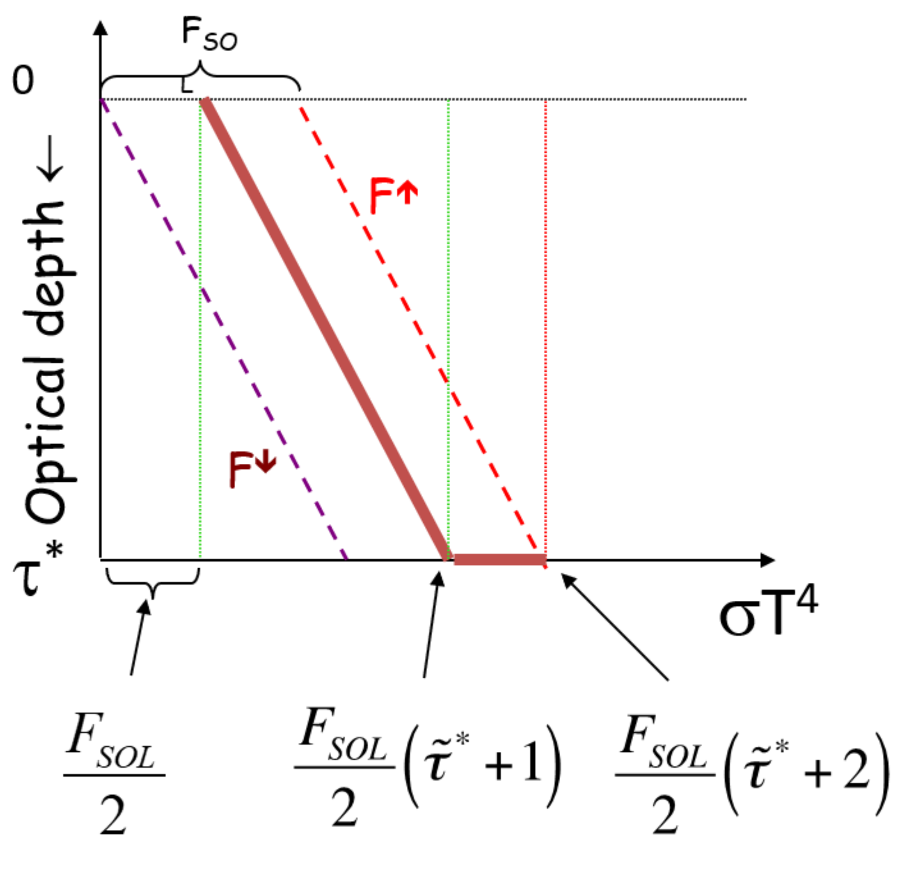
$$\varepsilon\sigma T_s^4 + \varepsilon\sigma T_a^4 = 2\varepsilon\sigma T_{sa}^4 \Rightarrow \sigma T_s^4 + \sigma T_a^4 = 2\sigma T_{sa}^4$$

"This layer therefore does not have the same temperature as the surface. This result is independent of ε so long as it is sufficiently small, and illustrates that a discontinuous emissivity entails a discontinuity in temperature. In radiative equilibrium, the surface atmospheric temperature is generally different from the temperature of the surface. Radiation drives the system into thermodynamic disequilibrium, which in reality is counteracted by heat diffusion or fluid motion."

It follows that

$\sigma T_s^4 - \sigma T_{sa}^4 = \sigma T_e^4/2$ which is our Eq. (1). It also follows that $\sigma T_s^4 = (3/2)\,\sigma T_e^4$

Chris O'Dell, Colorado State University (2013)



$$\sigma T^4(\tilde{\tau}) = \frac{F_{SOL}}{2}(\tilde{\tau}+1)$$

$$\sigma T_s^4 = \frac{F_{SOL}}{2}(\tilde{\tau}^*+2)$$

Manchester University (UK)

Consider the boundary conditions at the surface. We must balance the upward flux of radiation emitted by the Earth at a temperature T_s, $B(T_s)$, with the downwelling short and longwave radiation.

$$\pi B(T_s) = F_s + F^{\downarrow}(\chi_s)$$

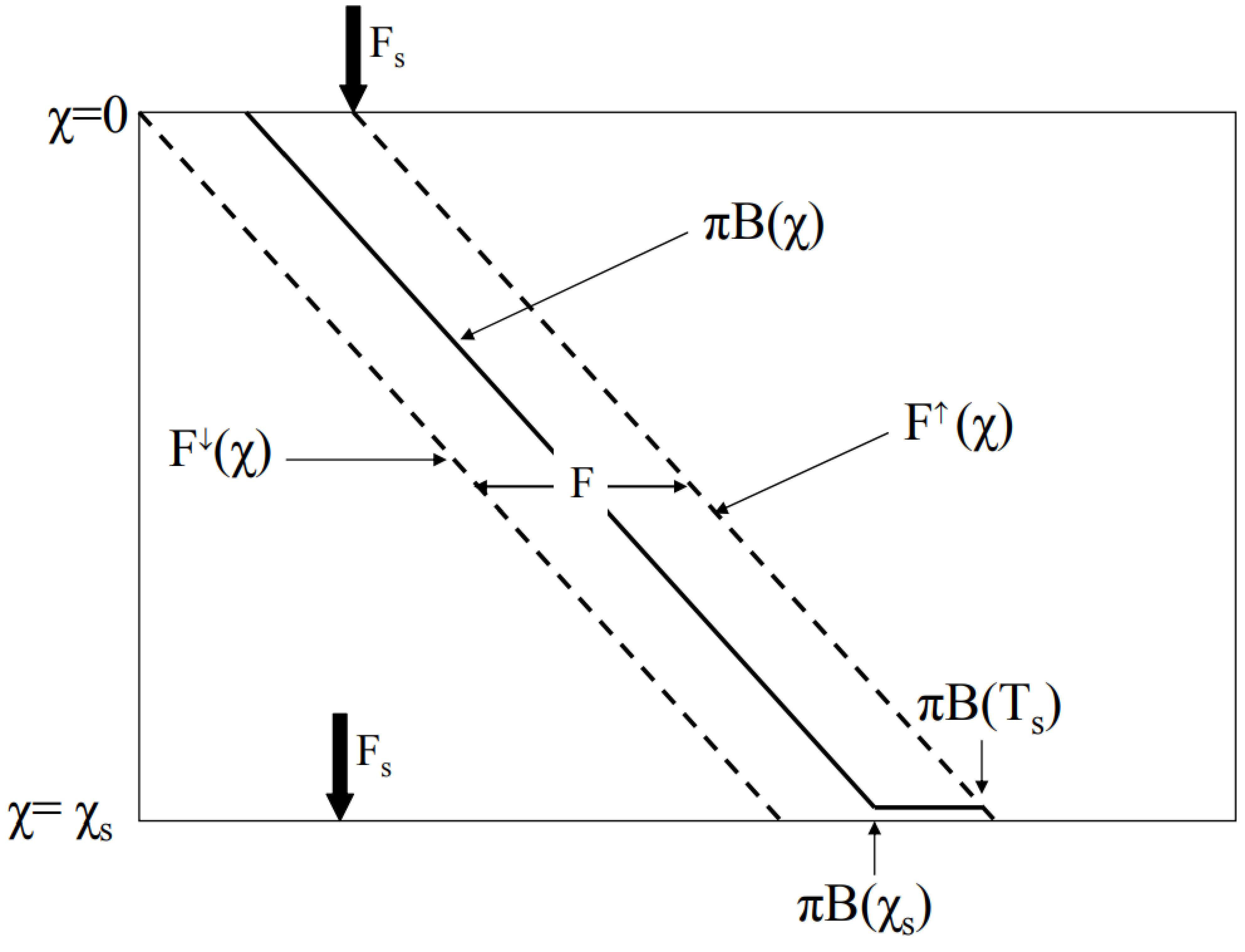
where χ_s is the optical depth of the lowest layer of the atmosphere. However,

$$\overline{F} - F = 2F^{\downarrow} = 2\pi B - F_s$$

which when evaluated at the surface gives

$$\pi B(T_s) = \pi B(\chi_s) + F_s/2.$$

This expression implies that there is a temperature discontinuity between the surface and the cooler lowest layer of the atmosphere.



P607 Climate and Energy Lecture 3

(Dr Hugh Coe, University of Manchester, UK 2008)

Tokyo University

惑星大気学_放射(2022).docx

From (3.4)(3.5) and the definition of F^{net} and F^{sum}, we have

$$F^{\downarrow} = \frac{F^{sum} - F^{net}}{2} = B^{*} - \frac{F^{0}}{2} \qquad (3.10)$$

Emission from the surface is equal to the sum of the solar flux reaching the surface and the downward emission from the atmosphere:

$$B^{*}(T_S) = F^{0} + F^{\downarrow}(\tau_s^{*}) \qquad (3.11)$$

T_S： surface temperature τ_S： optical depth at the surface

From (3.10)(3.11)

$$B^{*}(T_S) = B^{*}(\tau_s^{*}) + \frac{F^{0}}{2}$$

surface　bottom of atmosphere

A temperature discontinuity exists at the surface.（Note that $B^{*} = \sigma T^{4}$）

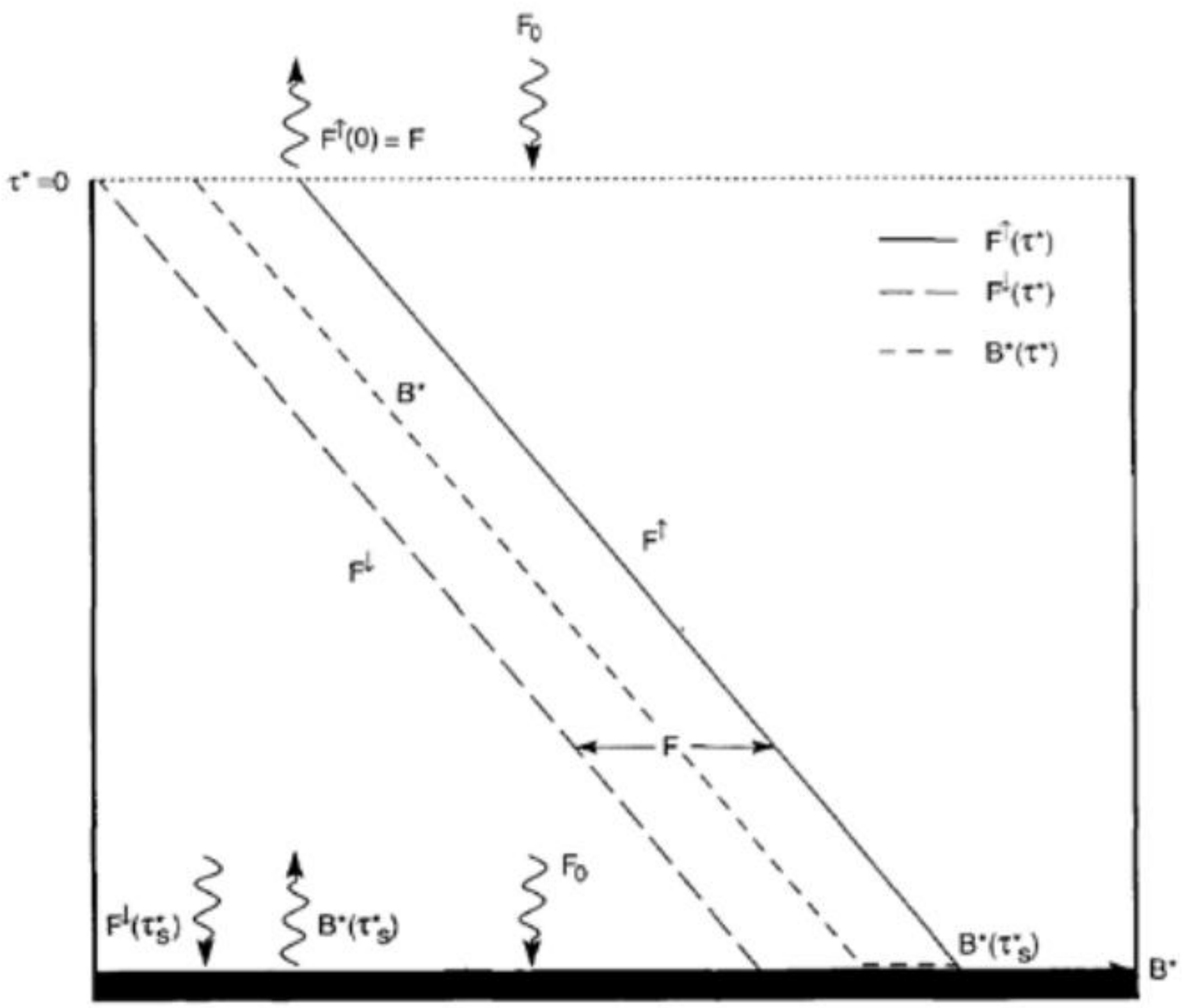


Figure 8.20 Upwelling and downwelling fluxes and emission in a gray atmosphere that is in radiative equilibrium with an incident SW flux F_0 and a black underlying surface. *Note:* the emission profile is discontinuous at the surface.

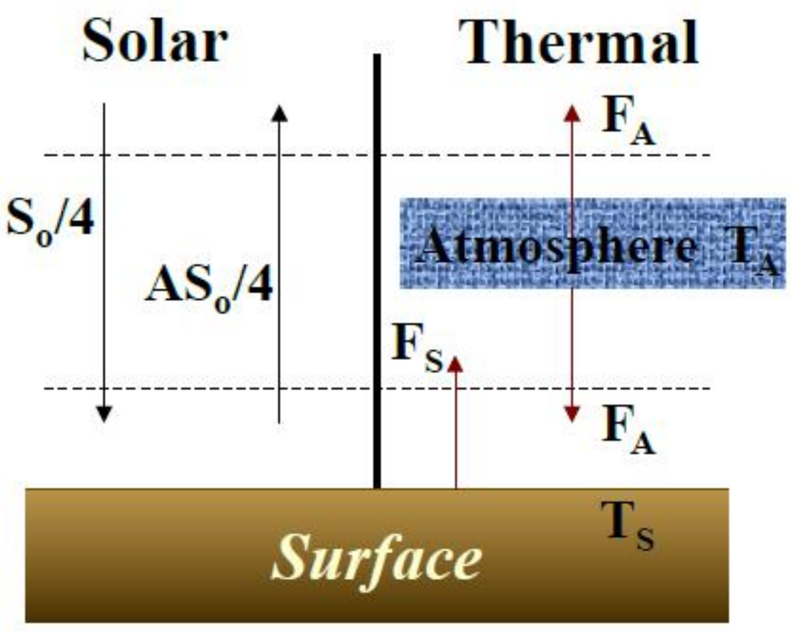
Toronto University

PHY2505S
Atmospheric Radiative Transfer and Remote Sounding

Lecture 6

- A Simple Greenhouse Model
- Terrestrial Fluxes: Schwarzchild's Equation Revisited
- The Two-Stream Model

Same Diagram, Slightly Different Symbols



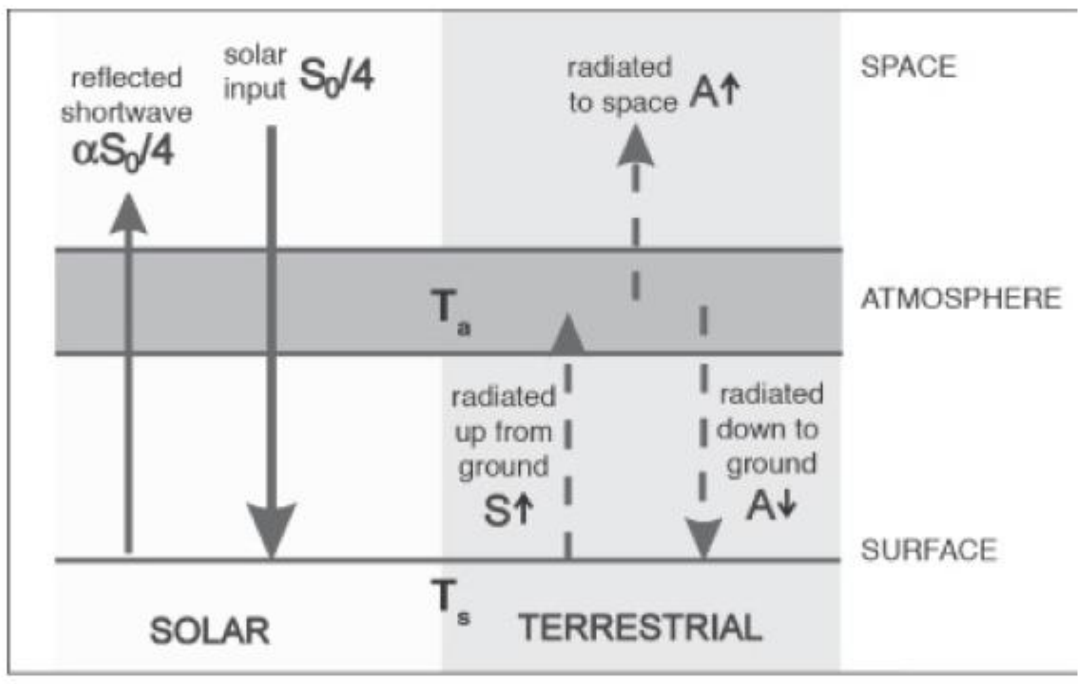


Figure 2.7: The simplest greenhouse model, comprising a surface at temperature T_s, and an atmospheric layer at temperature T_a, subject to incoming solar radiation $S_0/4$. The terrestrial radiation upwelling from the ground is assumed to be completely absorbed by the atmospheric layer.

A Simple Greenhouse Model - 3

- Writing down the equation for vertical energy transfer <u>above</u> the atmosphere and stating that the atmosphere/planet system is in radiative equilibrium:

$$S_o/4 = AS_o/4 + F_A$$

IR flux density emitted by the atmosphere (F_A)

Planetary albedo (A)

Average solar flux density ($S_o/4$)

- Below the atmosphere:

$$S_o/4 + F_A = AS_o/4 + F_S$$

$$(AS_o/4 + F_A) + F_A = AS_o/4 + F_S$$

$$F_S = 2F_A$$

$$\sigma T_S^4 = 2\sigma T_e^4$$

"We will use "Schwarzchild's Equation for Fluxes" in our simple greenhouse atmospheric model."

Two-Stream Model Solution - 1

- We have thus solved for the upward and downward flux densities:

$$F_{up} = \sigma T_e^4 (\chi^* + 2)/2$$
$$F_{down} = \sigma T_e^4 \chi^* / 2$$

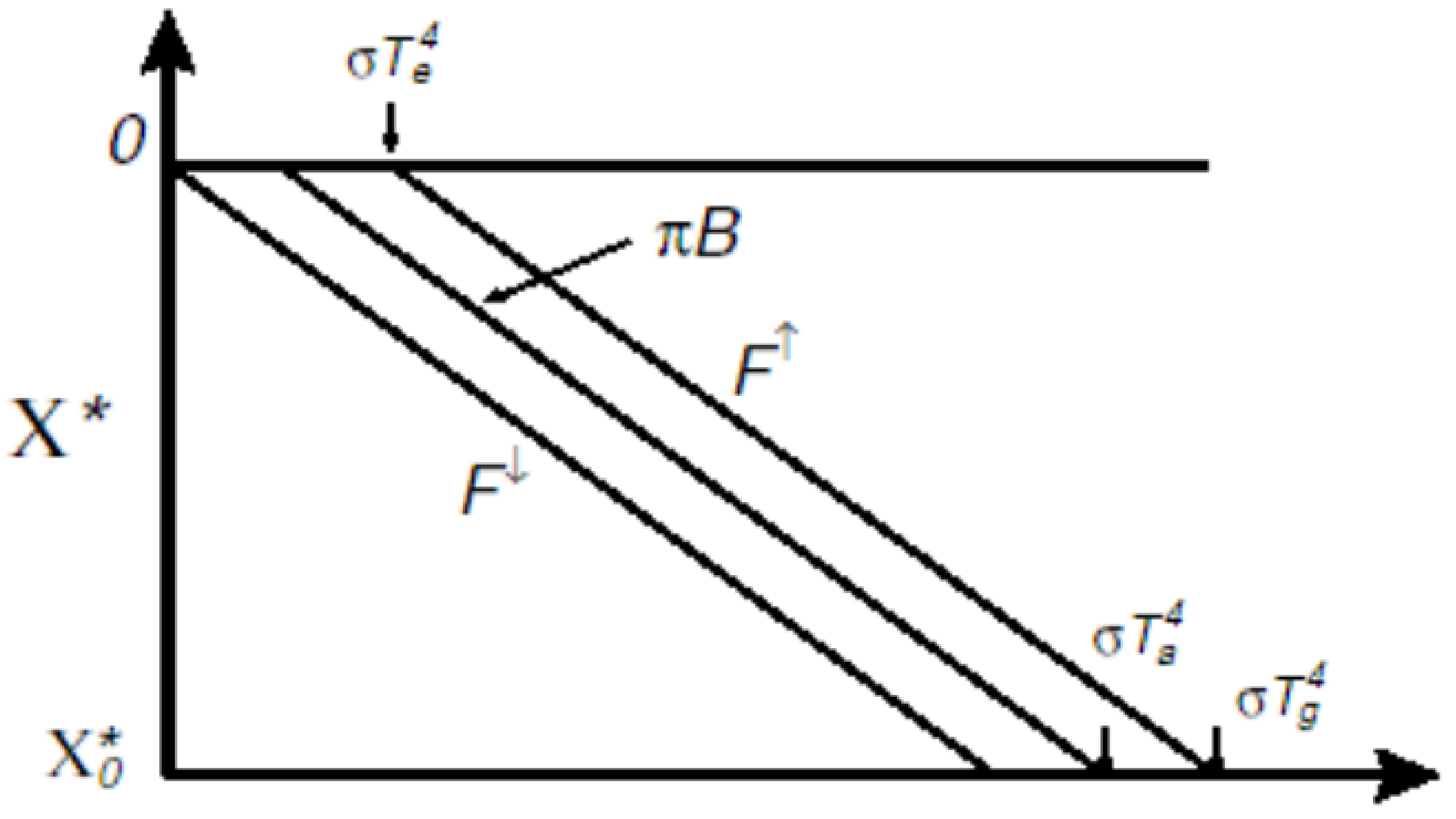
- Finally, we can use the original equations to solve for:

$$\pi B = \sigma T_a^4 = \frac{\sigma T_e^4 (\chi^* + 1)}{2}$$

  where T_a is the atmospheric temperature.

- These three solutions can be plotted against χ^*.
  → Gives three parallel lines that show how the flux densities and the blackbody function vary with χ^* and hence with altitude.

Two-Stream Model Solution - 3



- If the central line represents the atmospheric temperature and the line for F_{up} must match the ground temperature, then there is a temperature discontinuity at the ground.

In order to determine the temperature at the ground surface (T_g) for this purely radiative equilibrium, we need to consider the upward flux of infra-red radiation, since

$$[F_{up}]_{z=0} = \sigma T_g^4 \qquad (13)$$

Since $F_{up} = (F_{tot} + F_{net})/2 = (F_{tot} + F_0)/2$, we find using equation (10) that

$$F_{up} = \{F_0 (\tau + 1) + F_0\}/2 = F_0 (1 + \tau/2) \qquad (14)$$

Finally therefore, we also deduce that

$$F_{dn} = F_{up} - F_{net} = F_{up} - F_0 = F_0 (\tau/2) \qquad (15)$$

Thus in the special case of pure radiative equilibrium, F_{net} is constant and equal to F_0, and both F_{up} and F_{dn} increase linearly with optical thickness. This is illustrated in Figure 2 (see also [Salby, 1992 #3107][Houghton, 1997 #3186]).

However, it is very important to notice that the (ground) surface temperature is set by F_{up} through equation (13), i.e.

$$\sigma T_g^4 = [F_{up}]_{z=0} = F_0 (1 + \tau/2) \qquad (16)$$

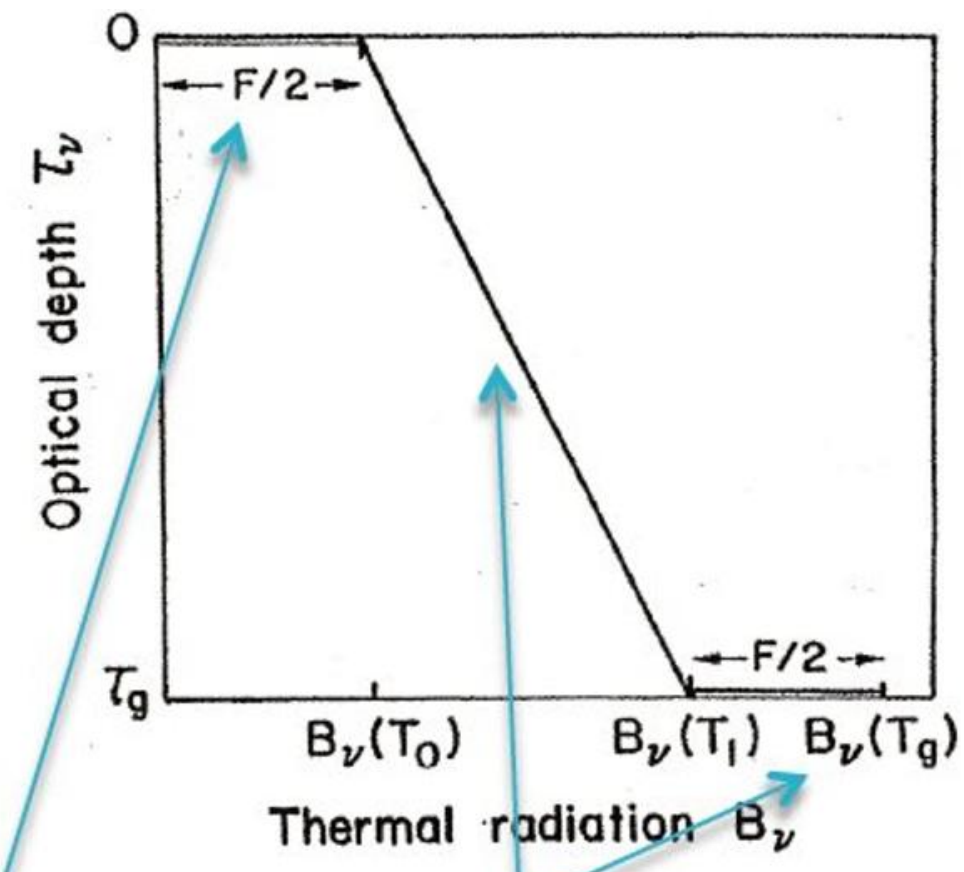
whereas the air temperature just above the ground is set by F_{tot} through equation (11) so that

$$\sigma T_0^4 = B(\tau) = F_0 (\tau + 1)/2 \qquad (17)$$

ground surface temperature derived above exceeds that of the overlying air in this model, by an amount corresponding to an extra heat flux of $F_0/2$. This calculated ground-air temperature discontinuity may be substantial (10 or 20 °K, or more). It only occurs because we have assumed that the only heat fluxes are those due to radiation, so there is no conduction and no turbulent convection. In the real atmosphere these would operate together, as conduction will transfer heat into the air near the ground, creating an unstable

University of Arizona (2015)

Final TP Profile



Expressions for intensity:

$$I^+ = B(T) + \frac{1}{2\pi}F$$

$$I^- = B(T) - \frac{1}{2\pi}F$$

Boundary Conditions:

At the Ground: $I_g^+ = B(T_g) = B(T_1) + \frac{1}{2\pi}F,$

Discontinuity

* * *

Harvard (2018)

We have got the temperature structure in the atmosphere as a function of τ. Now consider energy balance at the surface (looks familiar?),

$$B(T_s) = F_0 + F^{\downarrow}(\tau_s)$$

From their definitions, we have

$$F^{\downarrow} = \frac{1}{2}\left(\bar{F} - F\right)$$

As the net flux F is constant and equal to F_0, and use Eq. (14), we have:

$$B(T_s) = B(\tau_s) + \frac{F_0}{2}$$

Note the jump at the surface.

Everybody knows everything

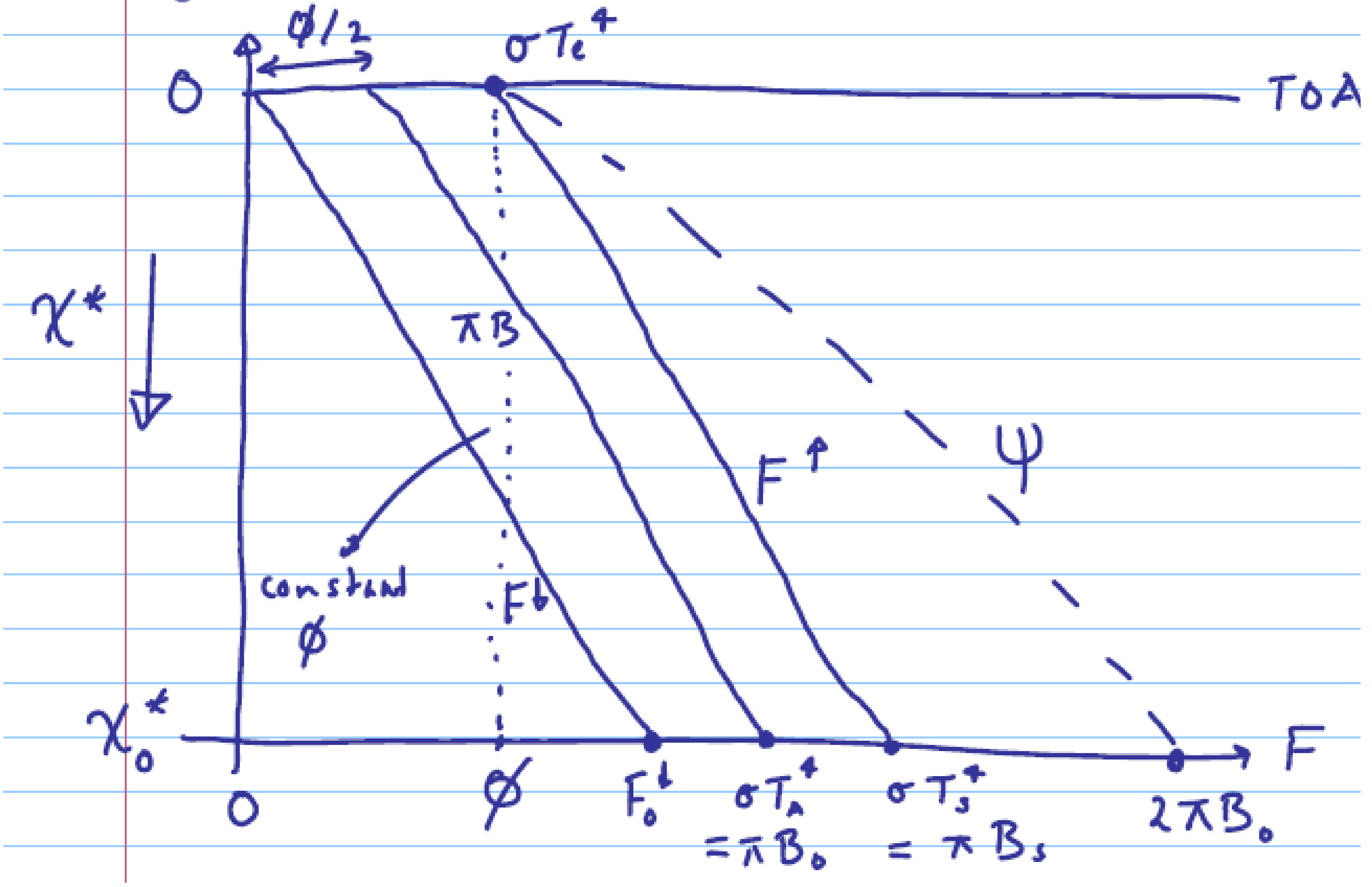
$$\phi = F^{\uparrow} - F^{\downarrow} = \sigma T_e^4$$

$$F^{\uparrow} = \frac{\phi}{2}(\chi^* + 2)$$

$$F^{\downarrow} = \frac{\phi}{2}\chi^*$$

$$\pi B = \frac{\phi}{2}(\chi^* + 1)$$

Let's plot these with χ^* on the y axis.



Theoretical Reference Estimate Equation (1) is there everywhere:

Ambaum, M. (2021): Thermal physics of the atmosphere. Royel Meteorological Society. (Eq. 10.56)
Andrews, D. (2010): An introduction to atmospheric physics. Cambridge University Press (pp. 85-86)
Chamberlain, J. (1978): Theory of planetary atmospheres. Academic Press (Eq. 1.2.29 and Fig. 1.4.) (2nd edition: 1987)
Goody, R. (1964) Atmospheric radiation: Theoretical basis. Oxford University Press (Eq. 2.115) (2nd edition 1989, Eq. 2.146)
Hartmann, D. (1994) Global physical climatology. Academic Press. (Eqs. 3.48-3.54). (2nd edition: 2016)
Houghton, J. (1977) The physics of atmospheres. Cambridge University Press (Eq. 2.13). (2nd edition: 1986, 3rd edition: 2002)
Pierrehumbert, R. (2008): Principles of planetary climate. Cambridge University Press (Eq. 4.45)
Salby, M. (1996): Fundamentals of atmospheric physics. Academic Press. (Eq. 8.67)
Salby, M. (2012): Physics of the atmosphere and climate. Cambridge University Press (Eq. 8.67)
Vardavas, I. and Taylor, F. (2007): Radiation and Climate. Oxford University Press. (Eqs. 11.7-11.8)
Visconti, G. (2001): Fundamentals of physics and chemistry of the atmospheres. Springer Verlag (Eq. 3.49)
Zdunkowski, Trautmann and Bott (2008): Radiation in the atmosphere. Cambridge University Press (Fig. 6.7)

University Lecture Notes: Arizona, Columbia, Harvard, Manchester UK, MIT, Southampton UK Tyndall Centre, Tokyo, Toronto …

Except:

The Charney Report (1979)
The Villach Statement (1980) (UNEP/WMO/ICSU/WCRP)
Theory of Climate (1983) (Academic Press)
Climate Change 1990 – The IPCC Scientific Assessment. Cambridge University Press
Climate Change 1992 – The Supplementary Report. Cambridge University Press
Climate Change 1995 – The Science of Climate Change. Cambridge University Press
Climate Change 2001 – The Physical Science Basis. Cambridge University Press
Climate Change 2007 – The Physical Science Basis. Cambridge University Press
Climate Change 2013 – The Physical Science Basis. Cambridge University Press
Climate Change 2021 – The Physical Science Basis. Cambridge University Press

Missing from essential journal articles:

Mamane and Möller (1961)
Manabe and Strickler (1964)
Manabe and Wetherald (1967)
Manabe and Wetherald (1975)
Ramanathan and Coakley (1976)
Ramanathan, Lian and Cess (1979)
Raval and Ramanathan (1989)
…

TRE basics: The four equations

Schwarzschild, K. (1906)

Ueber das Gleichgewicht der Sonnenatmosphäre, Eq. (11)

E emission of a layer, *A* upward beam, *B* downward beam, A_0 emerging flux at TOA, τ optical depth:

$$E = \frac{A_0}{2}(1 + \bar{\tau}), \qquad A = \frac{A_0}{2}(2 + \bar{\tau}), \qquad B = \frac{A_0}{2}\bar{\tau}. \qquad (11)$$

Eq. (1) $A - E = \Delta A = A_0/2$ Net radiation at the surface, independent of τ

Eq. (1)	**SFC Net = A – E =** $\mathbf{A_0/2}$		(clear-sky, net)
Eq. (2)	**SFC Tot =**	$\mathbf{A = 2A_0}$	(clear-sky, total at τ = 2)

Separating atmospheric radiation transfer from the longwave cloud effect (LWCRE):

Eq. (3)	**SFC Net = A – E =** $\mathbf{(A_0 - L)/2}$		(all-sky, net, incl LWCRE)
Eq. (4)	**SFC Tot =**	$\mathbf{A = 2A_0 + L}$	(all-sky, total at τ = 2 incl LWCRE)

Eq. (1) From Schwarzschild (1906, Eq. 11), net, clear-sky:
$\mathbf{A - E = \Delta A = A_0 /2}$

Eq. (2) From Schwarzschild (1906, Eq. 11) at $\tau = 2$, total, clear-sky:
$\mathbf{A = 2A_0}$

Eq. (3) From Schwarzschild (1906, Eq. 11), including LWCRE, net, all-sky:
$\mathbf{A - E = \Delta A = (A_0 - L) /2}$

Eq. (4) From Schwarzschild (1906, Eq. 11) at $\tau = 2$, including LWCRE, total, all-sky:
$\mathbf{A = 2A_0 + L}$

Eq. (1) Houghton (1977, 1986, 2002, Eq. 2.13), net, clear-sky:
$\mathbf{B_g - B_0 = \phi/2\pi}$

Eq. (2) Houghton (1977, 1986, 2002, Eq. 2.15) at $\chi_0^* = 2$, total, clear-sky:
$\mathbf{B_g = 2\phi/\pi}$

Eq. (3) Houghton (1977, 1986, 2002, Eq. 2.13), including LWCRE, net, all-sky:
$\mathbf{B_g - B_0 = (\phi - L)/2\pi}$

Eq. (4) Houghton (1977, 1986, 2002, Eq. 2.15) at $\chi_0^* = 2$, including LWCRE, total, all-sky:
$\mathbf{B_g = (2\phi + L)/\pi}$

TRE basics: The simplest greenhouse geometry

Four equations, coupling surface fluxes to TOA fluxes, without referring to GHG-s

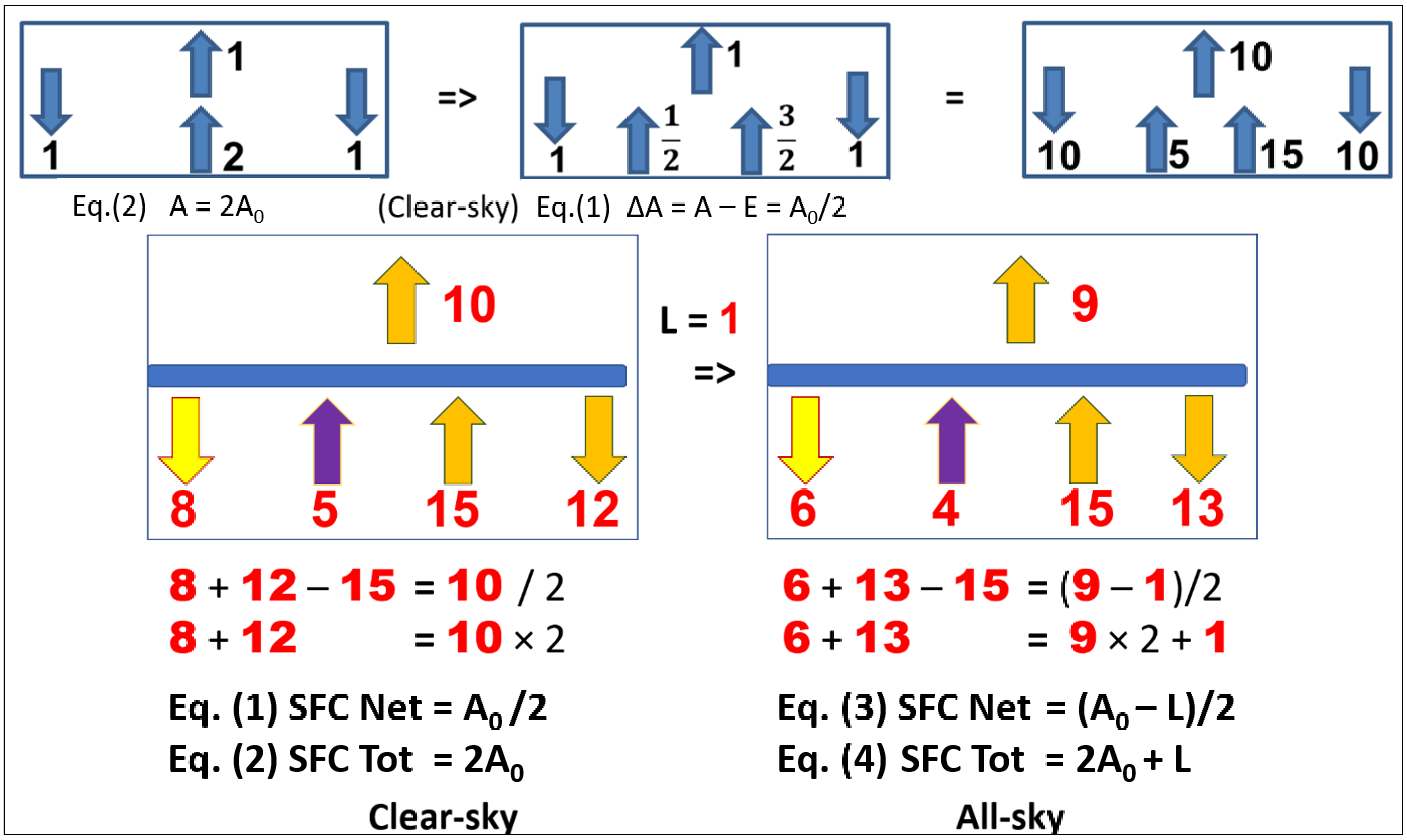


Fig. 5 Theoretical background for the reference estimate. Upper left panel: The simplest greenhouse geometry represents Eq. (2). Upper middle panel: The constraint on the net radiation at the surface (Eq.1) included. Upper right panel: since the unit is not specified yet, multiply it by 10. Lower left panel: The clear-sky structure in red units, with **1** unit = L, representing LWCRE. Lower right panel: The all-sky structure. Integer solution and the four equations are indicated.

The flux components with LWCRE = 1

TOA LW	clear-sky =	**10**	TOA LW	all-sky =	**9**
SFC LW up	clear-sky =	**15**	SFC LW up	all-sky =	**15**
SFC LW down	clear-sky =	**12**	SFC LW down	all-sky =	**13**
SFC LW net	clear-sky =	**–3**	SFC LW net	all-sky =	**–2**
SFC SW net	clear-sky =	**8**	SFC SW net	all-sky =	**6**
SFC SW+LW net	clear-sky =	**5**	SFC SW+LW net	all-sky =	**4**
SFC SW+LW total	clear-sky =	**20**	SFC SW+LW total	all-sky =	**19**
G greenhouse effect	clear-sky =	**5**	G greenhouse effect	all-sky =	**6**
SWCRE (surface)	=	**–2**	LWCRE (surface. TOA)	=	**1**

TRE as a completed, coherent set of the integer solution. LWCRE = **1** = 26.68 Wm^{-2} corresponds to TSI = **51** units = 1360.68 Wm^{-2} (*c.f.* the most accurate value is 1360.8 ± 0.5 Wm^{-2}, Kopp and Lean 2011 if spherical weighting is applied; with geometric weighting factor of 4.0034, as in CERES EBAF, TSI = 1361.84 Wm^{-2}).

Early verification of Eqs. (1) and (2)

CERES_EBAF-Surface_Ed2.8 Data Quality Summary (March 27, 2015)

Table 4-1. Global annual mean fluxes using data from March 2000 through February 2010 (W m^{-2}).

	Flux Component	Ed3A SYN1deg -Month	EBAF-Surface Ed2.6r	EBAF-Surface Ed2.7	EBAF-Surface Ed2.8	EBAF-TOA Ed2.8
TOA	Incoming solar	339.9	339.9	339.9	339.8	339.8
	LW (all-sky)	237.3	239.7	239.6	239.6	239.6
	SW (all-sky)	98.5	99.6	99.6	99.6	99.6
	Net (all-sky)	4.06	0.64	0.69	0.63	0.59
	LW (clear-sky)	263.7	265.8	265.7	265.7	265.8
	SW (clear-sky)	52.5	52.5	52.6	52.6	52.6
	Net (clear-sky)	23.6	21.6	21.6	21.6	21.5
Surface	LW down (all-sky)	341.8	343.7	345.1	345.1	
	LW up (all-sky)	397.6	398.1	398.1	398.0	
	SW down (all-sky)	187.2	186.7	186.5	186.4	
	SW up (all-sky)	23.3	24.1	24.1	24.1	
	Net (all-sky)	108.1	108.3	109.4	109.4	
	LW down (clear-sky)	313.5	314.1	315.8	316.0	
	LW up (clear-sky)	396.6	398.3	398.4	398.0	
	SW down (clear-sky)	242.4	243.4	244.1	243.9	
	SW up (clear-sky)	28.7	29.6	29.7	29.7	
	Net (clear-sky)	130.6	129.6	131.8	132.2	

Eq. (1) Surface (SW down – SW up + LW down – LW up) (clear) = TOA LW (clear) / 2

243.9 – 29.7 + 316.0 – 398.0 = 265.7 / 2 – 0.65 Wm^{-2}

Eq. (2) Surface (SW down – SW up + LW down) (clear) = 2 × TOA LW (clear)

243.9 – 29.7 + 316.0 = 2 × 265.7 – 1.2 Wm^{-2}

and Surface LW up = (3/2) TOA LW (clear) 398.0 = (3/2) × 265.7 – 0.55 Wm^{-2}.

F. Rose et al. (16 May, 2017) CERES 27th STM (Langley Research Center)

Clear Sky	Ed2.8
TOA SW Insolation	339.87
TOA SW Up	*52.50*
TOA LW Up	*265.59*
SFC SW Down	244.06
SFC SW Up	29.74
SFC LW Down	316.27
SFC LW Up	398.40

Eq. (1) SFC (SW dn – SW up + LW dn – LW up) (clear) = TOA LW (clear) /2

244.06 – 29.74 + 316.27– 398.40 = 265.59 /2 – 0.60 Wm^{-2}

Eq. (1) SFC (SW dn – SW up + LW dn) (clear) = 2 × TOA LW (clear)

244.06 – 29.74 + 316.27 = 2 × 265.59 – 0.59 Wm^{-2}

and SFC LW up (clear) = (3/2) TOA LW (clear)

398.40 = (3/2) 265.59 – 0.015 Wm^{-2}

Verification of the four equations

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})

Eq. (1)	SFC SW down – SW up + LW down – LW up	(clear)	= TOA LW (clear)/2	
	240.8680 – 29.0724 + 317.4049 – 398.5211		= 266.0122 /2	– 2.3267
Eq. (2)	SFC SW down – SW up + LW down	(clear)	= 2 × TOA LW (clear)	
	240.8680 – 29.0724 + 317.4049		= 2 × 266.0122	– 2.8238
Eq. (3)	SFC SW down – SW up + LW down – LW up	(all)	= [TOA LW (all) – LWCRE]/2	
	186.8544 – 23.1629 + 345.0108 – 398.7550		= (240.2450 – 25.7672)/2	+ 2.7083
Eq. (4)	SFC SW down – SW up + LW down	(all)	= 2 × TOA LW (all) + LWCRE	
	186.8544 – 23.1629 + 345.0108		= 2 × 240.2450 + 25.7672	+ 2.4450
			Mean	0.0007

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})
CERES EBAF Ed4.2 Version 4, 22 years (April 2000 – March 2022) (Wm^{-2})
CERES EBAF Ed4.2 Version 4, 24 years (April 2000 – March 2024) (Wm^{-2})

Eq. (1)	SFC SW down – SW up + LW down – LW up	(clear)	= TOA LW (clear)/2	
	240.8680 – 29.0724 + 317.4049 – 398.5211		= 266.0122 /2	– 2.3267
	241.0969 – 29.7521 + 317.8744 – 398.5890		= 265.9594 /2	– 2.3495
	241.0514 – 29.7043 + 318.0984 – 398.7742		= 265.9748 /2	– 2.3161
Eq. (2)	SFC SW down – SW up + LW down	(clear)	= 2 × TOA LW (clear)	
	240.8680 – 29.0724 + 317.4049		= 2 × 266.0122	– 2.8238
	241.0969 – 29.7521 + 317.8744		= 2 × 265.9594	– 2.6996
	241.0514 – 29.7043 + 318.0984		= 2 × 265.9748	– 2.5042
Eq. (3)	SFC SW down – SW up + LW down – LW up	(all)	= [TOA LW (all) – LWCRE]/2	
	186.8544 – 23.1629 + 345.0108 – 398.7550		= (240.2450 – 25.7672)/2	+ 2.7083
	187.1451 – 23.4950 + 346.1057 – 398.4220		= (240.3317 – 25.6277)/2	+ 3.9818
	187.1756 – 23.4607 + 346.3158 – 398.6162		= (240.3894 – 25.5854)/2	+ 4.0126
Eq. (4)	SFC SW down – SW up + LW down	(all)	= 2 × TOA LW (all) + LWCRE	
	186.8544 – 23.1629 + 345.0108		= 2 × 240.2450 + 25.7672	+ 2.4450
	187.1451 – 23.4950 + 346.1057		= 2 × 240.3317 + 25.6277	+ 3.4647
	187.1756 – 23.4607 + 346.3158		= 2 × 240.3894 + 25.5854	+ 3.6665
			Mean	0.0007
				0.5994
				0.7147

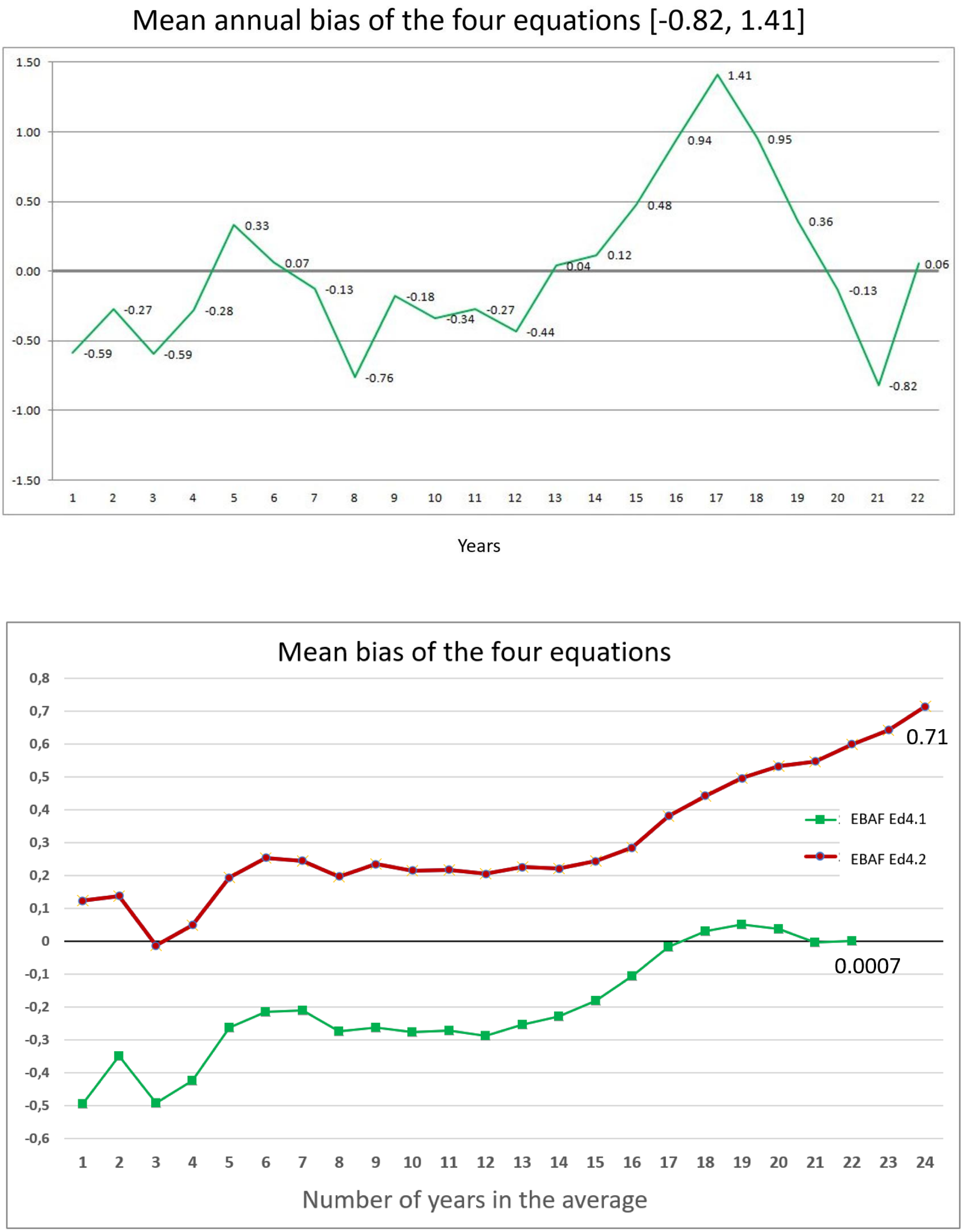


Fig. 6 Annual mean bias of the four equations (above) and mean bias as a function of the number of years (below)

241		95,353	239,685	54,535	265,303	-40,818	25,618	-15,200	337,434	190,746	239,062	24,855	30,177	343,153	317,451	400,769	400,410
242		95,447	241,783	54,721	267,445	-40,726	25,662	-15,064	332,585	186,328	235,238	24,873	31,085	347,424	321,464	405,103	404,095
243		94,748	243,653	52,368	268,842	-42,380	25,190	-17,190	329,614	179,961	230,369	21,661	27,821	351,881	326,080	408,133	407,299
244		93,229	244,372	49,661	269,916	-43,569	25,544	-18,025	329,266	177,576	228,671	18,938	24,551	354,238	328,435	408,966	408,587
245		91,200	244,796	48,524	270,265	-42,677	25,469	-17,208	331,600	180,825	230,967	18,070	23,493	353,598	327,599	408,248	407,728
246		93,522	243,406	50,144	268,342	-43,378	24,936	-18,443	336,209	184,379	235,043	19,457	24,764	349,969	323,257	405,377	405,008
247		99,960	240,666	53,206	266,197	-46,753	25,531	-21,222	342,033	188,506	243,908	23,298	28,984	344,441	316,277	399,994	399,627
248		105,153	239,328	56,926	264,350	-48,227	25,021	-23,206	347,485	193,210	251,215	27,127	33,319	339,377	310,844	395,008	394,438
249		107,742	237,758	57,036	262,951	-50,706	25,193	-25,513	350,953	193,344	253,883	27,058	33,152	335,309	307,389	390,669	390,449
250		105,590	237,448	55,347	263,216	-50,243	25,768	-24,475	351,193	193,375	252,863	24,786	30,641	333,255	306,333	389,512	389,499
251		100,909	238,551	54,053	263,572	-46,856	25,021	-21,835	348,320	193,167	249,184	23,027	29,049	334,525	308,038	390,523	390,833
252		97,347	238,327	53,761	263,778	-43,586	25,450	-18,136	343,350	192,217	244,177	23,100	28,775	337,431	311,242	394,143	394,461
253		96,018	238,578	54,536	264,364	-41,482	25,786	-15,696	337,529	190,544	239,745	24,841	30,368	341,326	315,414	399,249	399,052
254		96,465	240,493	54,783	266,702	-41,683	26,210	-15,473	332,568	185,198	235,392	24,634	31,124	346,845	320,182	404,380	403,388
255		94,443	242,463	52,122	268,077	-42,322	25,614	-16,708	329,597	180,391	230,664	21,678	27,871	351,224	325,245	407,640	406,783
256		93,235	244,019	50,193	269,585	-43,042	25,566	-17,476	329,253	177,040	227,387	18,976	24,485	354,971	329,091	409,170	408,753
257		91,248	244,527	49,343	269,337	-41,906	24,809	-17,096	331,630	180,035	228,828	17,951	23,211	354,051	327,911	408,525	408,038
258		93,310	242,530	49,731	267,824	-43,580	25,295	-18,285	336,223	184,936	235,997	19,395	24,734	350,749	323,572	405,223	404,610
259		98,633	240,874	52,761	265,929	-45,871	25,055	-20,816	342,035	189,649	244,424	22,766	28,571	345,043	317,252	400,266	399,765
260		104,357	238,192	55,983	263,800	-48,374	25,608	-22,766	347,511	193,334	251,464	26,307	32,442	339,903	311,451	395,210	394,979
261		107,262	237,329	56,519	262,954	-50,743	25,624	-25,119	350,948	193,126	253,603	26,343	32,380	336,003	308,218	390,950	390,797
262		104,735	237,315	55,233	262,965	-49,501	25,650	-23,851	351,238	193,858	252,529	24,629	30,463	333,918	307,249	389,882	389,981
263		100,422	238,699	54,249	263,628	-46,172	24,929	-21,243	348,479	193,638	248,699	23,076	28,839	335,035	308,781	391,151	391,254
264	month	97,714	238,489	54,196	264,088	-43,509	25,599	-17,910	343,513	191,770	243,601	23,128	28,777	338,861	312,673	394,885	394,891
265	name	sw_all	lw_all	sw_clr	lw_clr	cre_sw	cre_lw	cre_net	solar	sw_dn_all	sw_dn_cl	sw_up_all	sw_up_cl	lw_dn_all	lw_dn_cl	lw_up_all	lw_up_cl
266	**mean**	**98,96**	**240,25**	**53,72**	**266,01**	**-45,24**	**25,77**	**-19,48**	**340,02**	**186,85**	**240,87**	**23,16**	**29,07**	**345,01**	**317,40**	**398,75**	**398,52**
267	**N**	**15**/4	**36**/4	**8**/4	**40**/4	**−7**/4	**1**	**−3**/4	**51**/4	**7**	**9**	**1**	**1**	**13**	**12**	**15**	**15**
268	N × unit	100,05	240,12	53,36	266,8	-46,69	26,68	-20,01	340,17	186,76	240,13	26,68	26,68	346,84	320,16	400,2	400,2
269	diff	-1,09	0,13	0,36	-0,79	1,45	-0,91	0,53	-0,15	0,09	0,74	-3,52	2,39	-1,83	-2,76	-1,45	-1,68
270		TOA max diff -1.09 Wm^{-2} sw all								Surface max diff 3,52 Wm^{-2} sw up all							
271																	
272		ΔEq1= -2,3267		ΔEq2= -2,8238													
273		ΔEq3= 2,7083		ΔEq4= 2,4451		mean = 0,00070			g(clear)= 0,333					g(all) = 0,398			

CERES EBAF Ed4.1 data table, 22 full running years (264 monthly means) (only the last 24 months are displayed); The largest differences at TOA and at SFC; the four equations and their mean bias; and the greenhouse factors.

The all-sky integer structure and Eqs. (3) and (4) on Hartmann (2016)

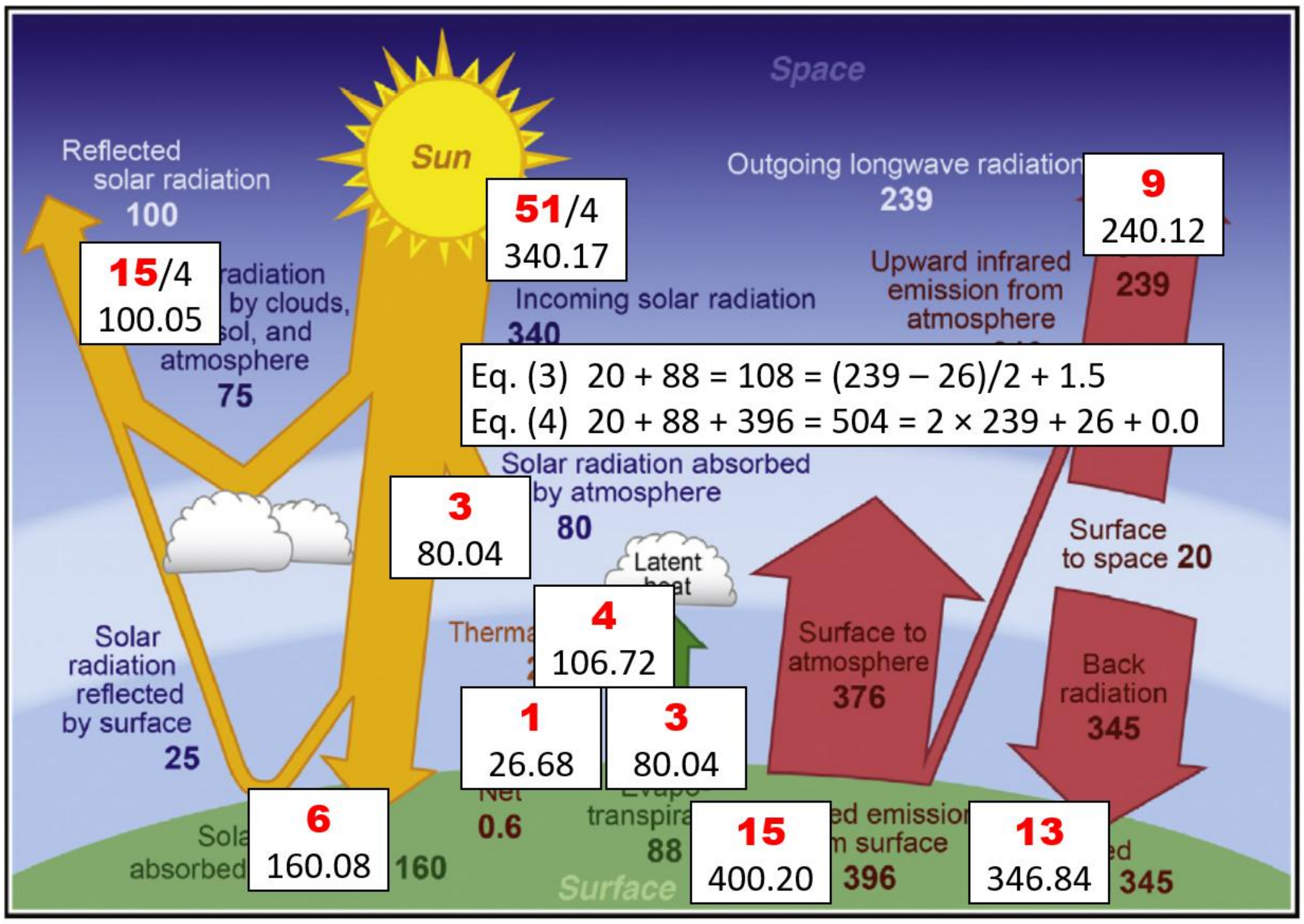


FIGURE 2.4 Global and annual average radiative and nonradiative energy-flow diagram for Earth and its atmosphere. Units are Wm^{-2}.

Figure 2.4 of D. Hartmann (IPCC AR5 2013 WGI Chapter 2 Coordinating Lead Author) Global Physical Climatology, 2nd Ed. (2016). With LWCRE = 26 Wm^{-2} of the book, Eq. (3) differs by 1.5 Wm^{-2}; Eq. (4) is exact.

The all-sky integer structure on Stephens et al. (2012)

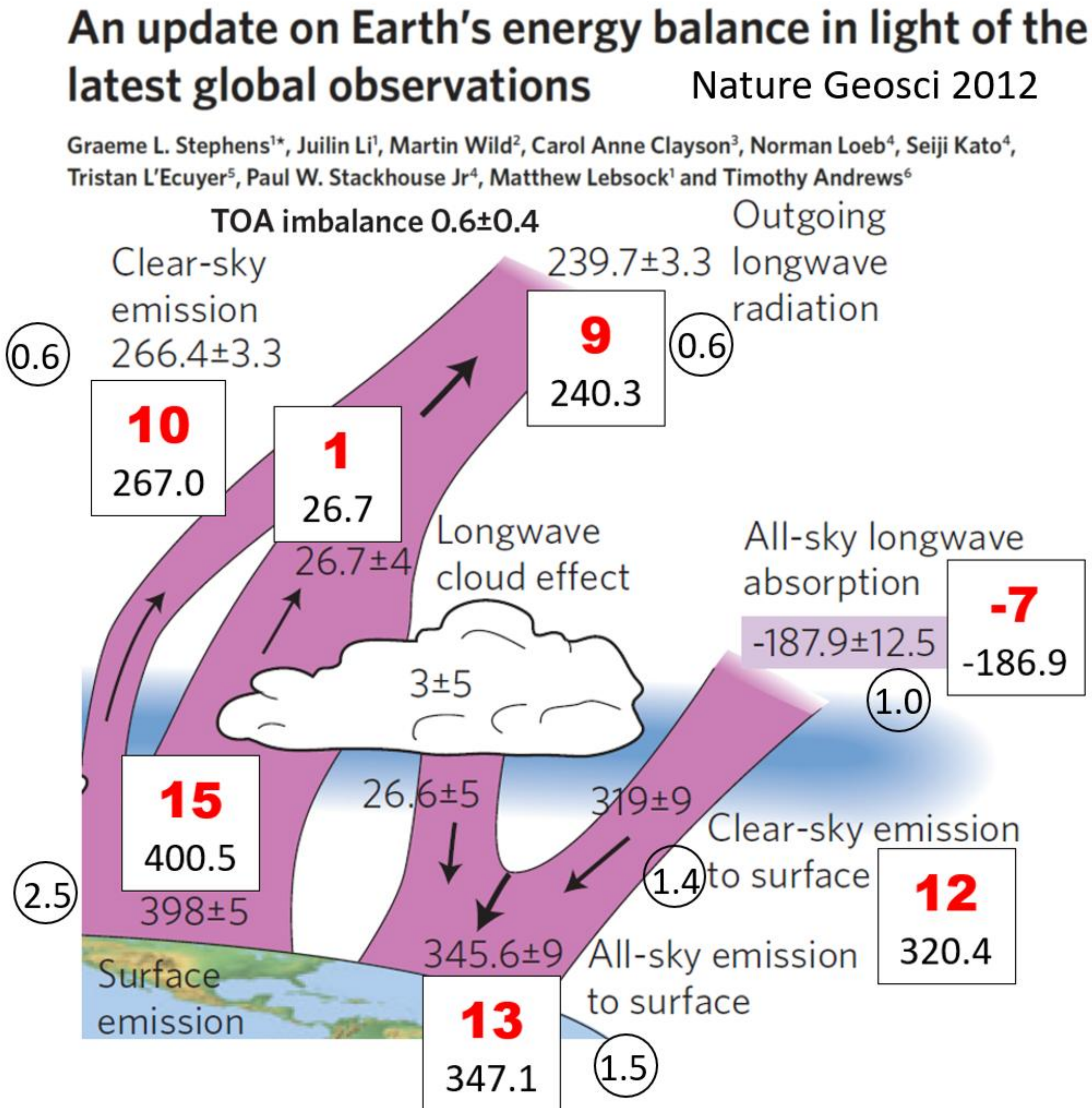


Fig. 7a Theoretical reference estimate projected on the LW part of the updated energy balance of Stephens et al. (2012) with unit flux LWCRE at TOA = 26.7 Wm^{-2}. Deviation from the integer position at TOA equals the TOA imbalance (0.6 Wm^{-2}); the largest difference at the surface is 2.5 Wm^{-2}, still within the noted range of uncertainty.

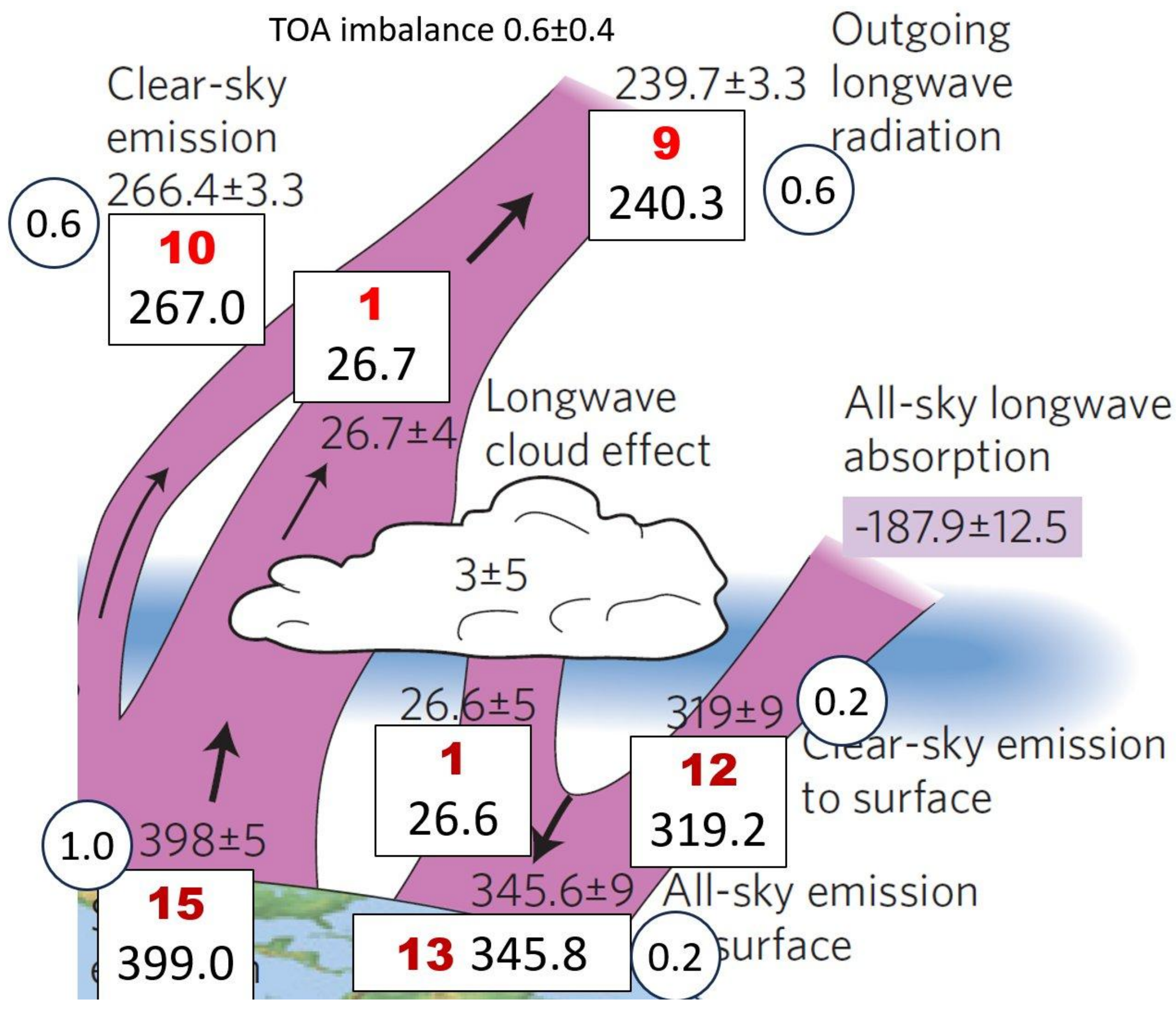


Fig. 7b When LWCRE at the surface is used as unit for the surface fluxes (shown in purple with 26.6 Wm^{-2}), the difference from the integer positions for the downward emissions drop to 0.2 Wm^{-2}.
A short video explaining this figure is available here:
https://earthenergyflows.com/Stephens2012-iPoster-1080.mp4

All-sky equations on Stephens et al. (2023)

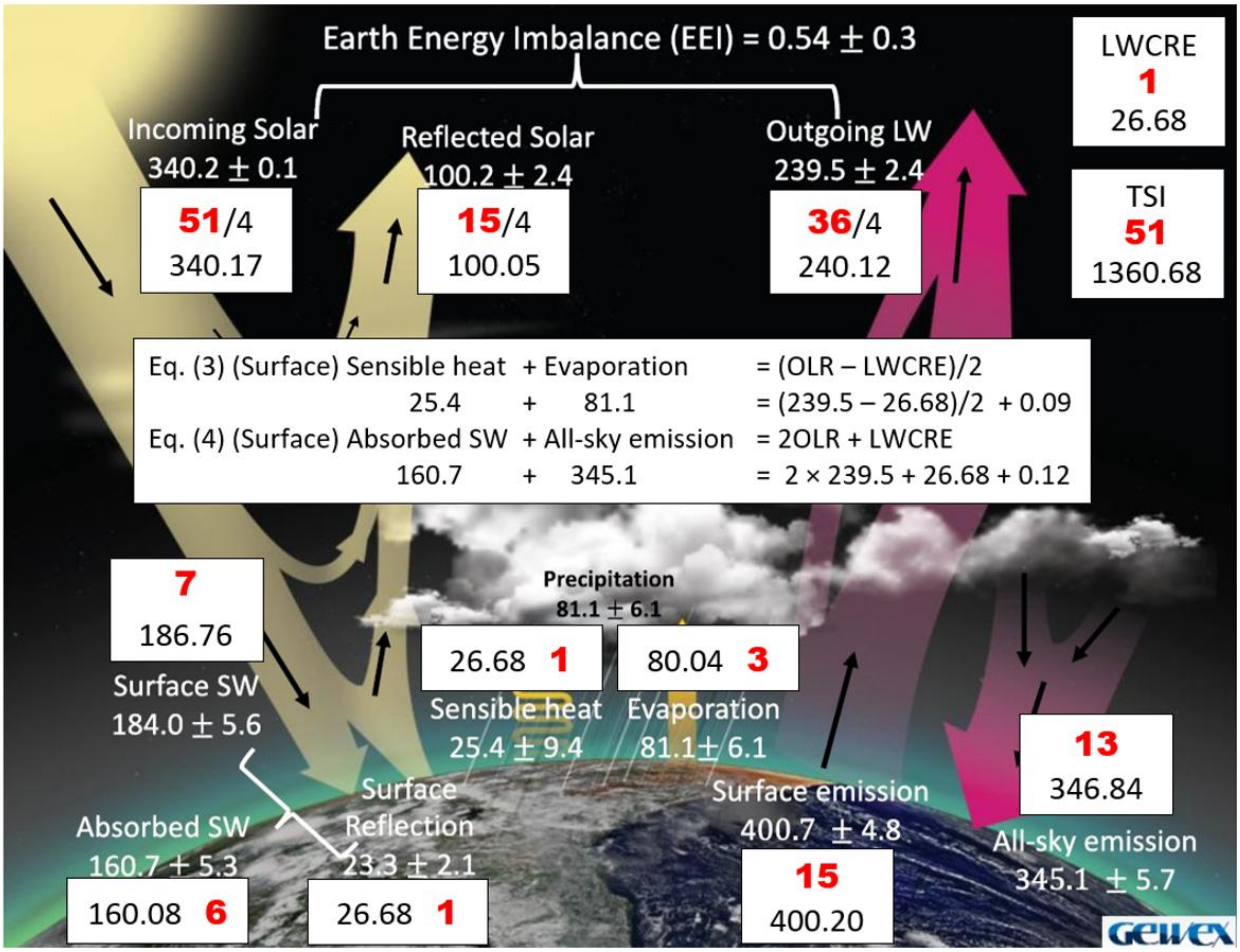


Fig. 8 Theoretical reference estimate projected on 30 years of GEWEX data (Stephens et al. 2023, BAMS). The two all-sky equations are satisfied in the magnitude of 0.1 Wm^{-2}. The value of LWCRE is the theoretical (1 units = 26.68 Wm^{-2}); compare it to that of 26.7 Wm^{-2} of Stephens, Li, Wild, Clayson, Loeb, Kato, L'Ecuyer, Stackhouse et al. (2012). The largest difference from integer position at TOA is 0.62 Wm^{-2} (Outgoing LW), in the magnitude of EEI; at surface is 3.38 Wm^{-2} (Surface Reflection).

"The CERES flying on the Terra and Aqua satellites confirm that Earth's albedo is 29.4% (±0.3%)" [Ackerman, L'Ecuyer, Loeb et al. 2019, AMS Met Monographs, Chapter 4]. With Incoming Solar of the GEWEX estimate, TSI = 340.2 Wm^{-2}, the corresponding Reflected Solar Radiation is 0.294 × 340.2 = 100.02 Wm^{-2}; our theoretical reflected solar is RSR = 15/51 × 340.17 = 100.05 Wm^{-2}, since our TRE albedo is α_{TRE} = 15/51 = 0.294. As 15/51 = 5/17, OLR on the disk is 12; after spherical weighting OLR = 3, ULW = 5 and G = 2 in this unit (80.04 Wm^{-2}), with all-sky values of 240.12 Wm^{-2}, 400.20 Wm^{-2} and 160.08 Wm^{-2}, respectively. (For further details, see 'magic numbers' below.)

It deserves to mention that the unique accuracy of the separate components of hydrological cycle (Sensible heat and Evaporation) in the GEWEX assessment is based on the NASA Energy and Water-cycle Study (NEWS) methodology.

An independent estimate: All-sky Equations on L'Ecuyer et al. (2015)

"Radiative Forcing of Climate", Meteorological Monographs 2019

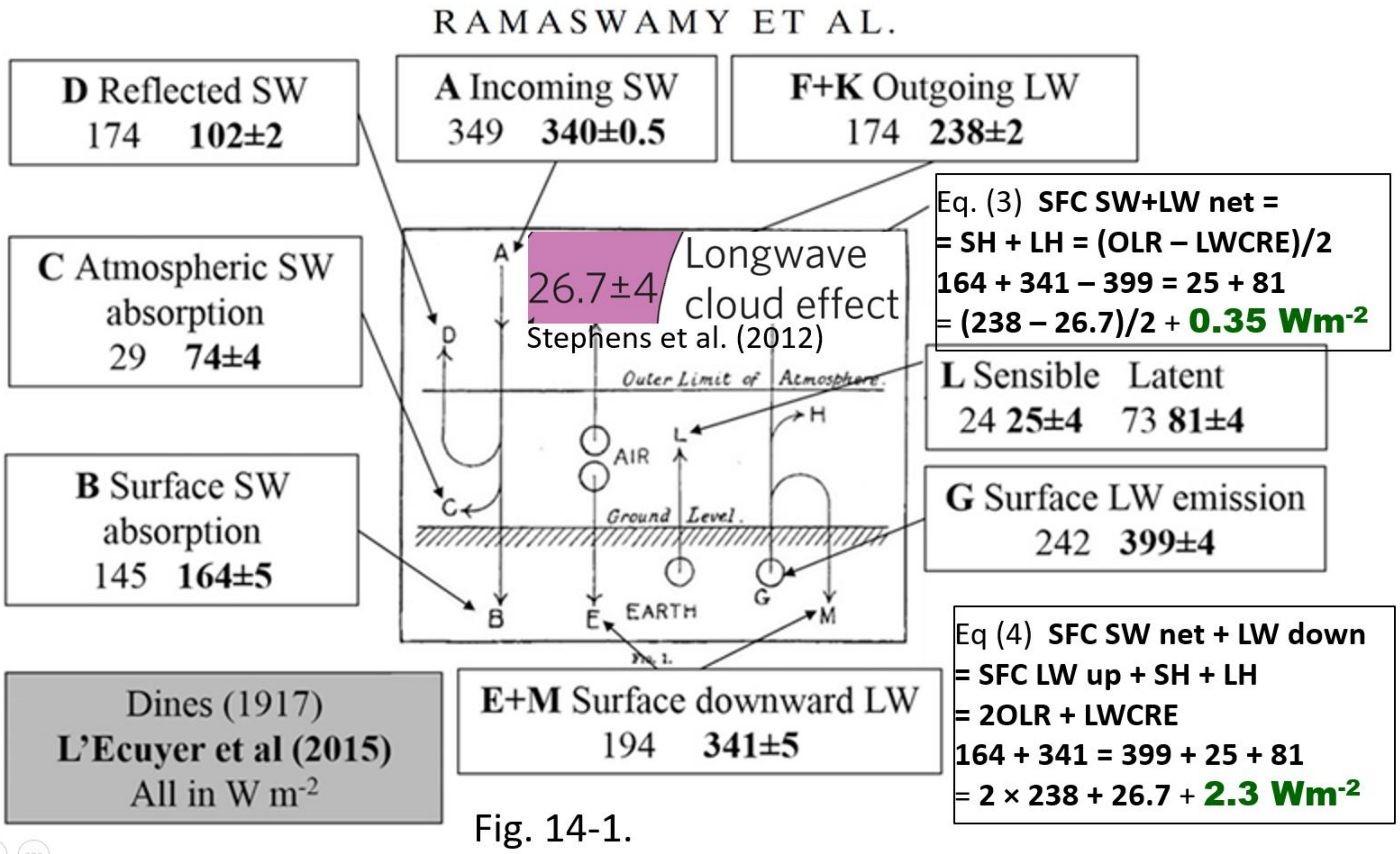


Fig. 14-1.

Fig. 9 Ramaswamy et al. (AMS, 2019) compare historical estimate of global energy budget to that of L'Ecuyer et al. (2015) (bold font). Longwave cloud effect is projected from Stephens et al. (2012) as 26,7 Wm^{-2}. Eq. (3) is valid by a difference of 0.35 Wm^{-2}, Eq. (4) by 2.3 Wm^{-2}.

An independent estimate: Clear-sky Greenhouse Effect at GFDL DL

LWCRE (TRE) = **1** = 26.68 ± 0.01 Wm^{-2}

G (clear, geo) = **15** – **10** = **5** = 133.40 ± 0.05 Wm^{-2}

G (GFDL AM4) = 133.4 ± 0.6 Wm^{-2}

Quantifying the Drivers of the Clear Sky Greenhouse Effect, 2000–2016

Shiv Priyam Raghuraman, David Paynter, V. Ramaswamy (JGR 2019)

Table 2
Global Mean and Time Mean G Comparison Between Observational, Reanalysis, and Modeling Data Sets Over March 2000 to August 2016

Quantity	ERBE	CE 4.1 "c"	CE 4.1 "t"	ERA-Interim	GFDL AM4
G_{Oceans}	146 ± 7	131.3 ± 0.5	134.1 ± 0.5	134.8 ± 0.6	135.0 ± 0.5
G	–	129.7 ± 0.6	132.4 ± 0.6	133.1 ± 0.7	133.4 ± 0.6

A Case Study: Surface Solar Radiation (all-sky)

- **Zagoni** TRE integer position: **7** units = 186.76 Wm^{-2} (**1** unit = 26.68 Wm^{-2})
- **Stackhouse** et al. GEWEX SRB (EGU 2023) 186 Wm^{-2}
- **Stackhouse**, Cox, Mikovitz, Zhang (EGU 2020) 187.8, 185.8, 185.4, 187.1:

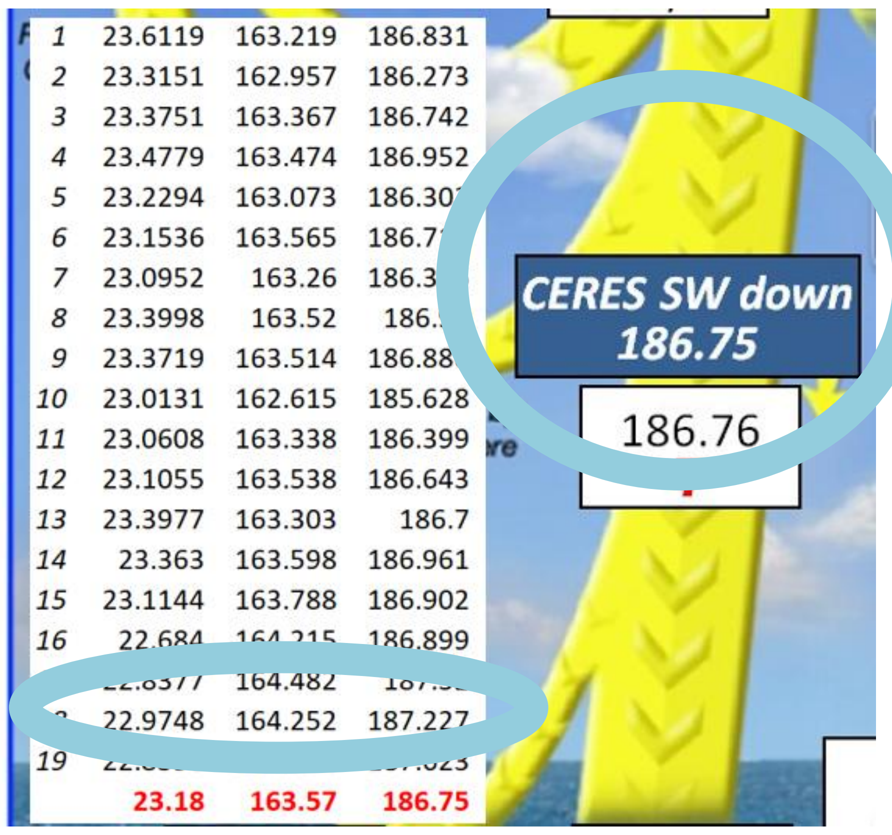
SRB (R4-IP) SW Global Annual Averages Fluxes for 2001-2007

Flux Component	Rel 3.0 (2001-2007)	Rel 4_IP (NEW algorithm, NEW inputs nnHIRS, HXS V01)	CERES Syn1Deg (Ed. 4A)	CERES EBAF (Ed 4.0)
Surface total down	187.8	185.8	185.4	187.1

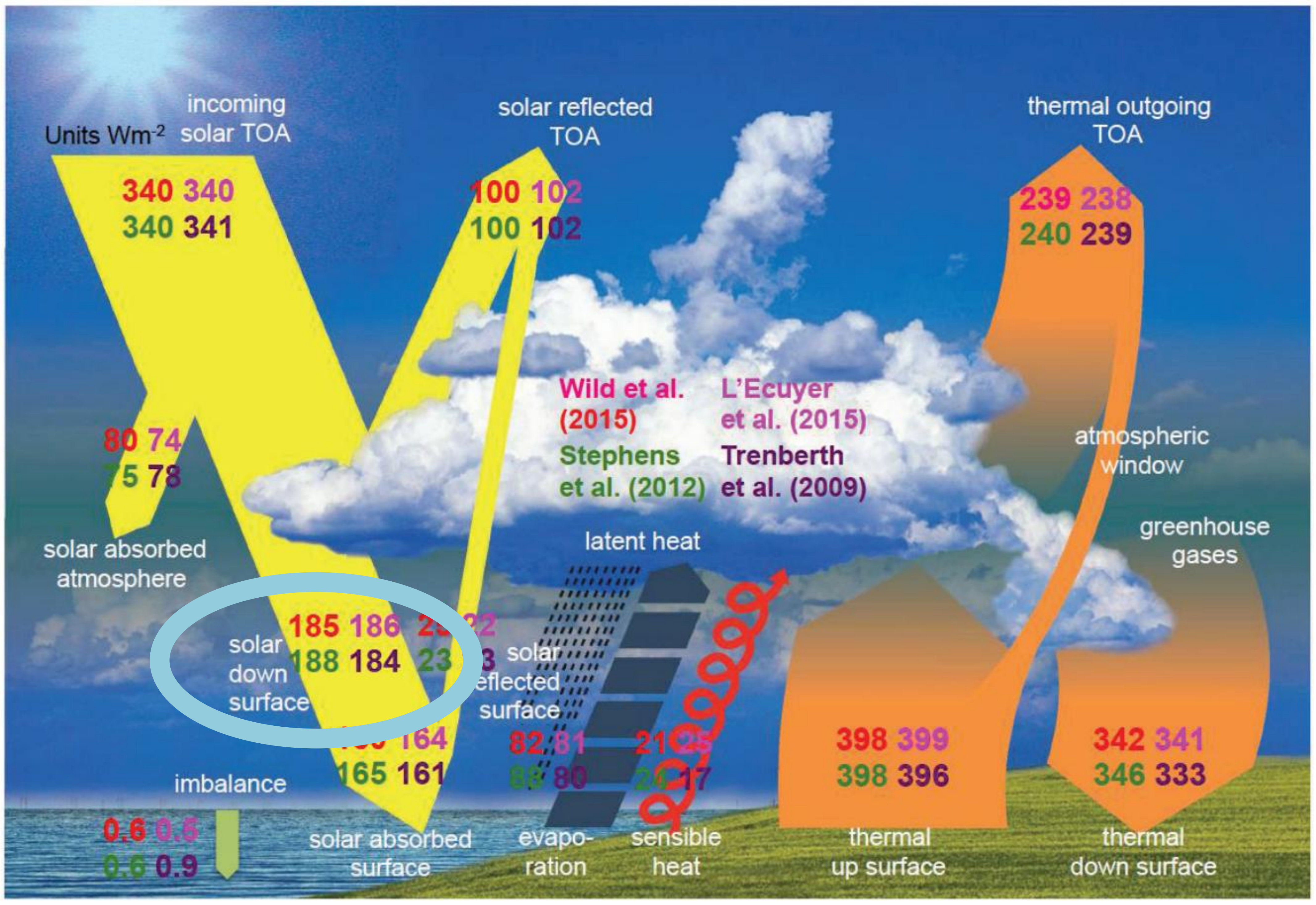
- **Trentmann and** Pfeifroth (EGU2023-2563):
  Global surface irradiance (2000 to 2017): 187 W/m²
- **Kato** et al. (2018) 187.1 (CERES EBAF Ed4); 186.6 (Ed2.8):

TABLE 5. Global annual mean irradiances (W m^{-2}) computed with Ed2.8 and Ed4 EBAF products from July 2005 through June 2015.

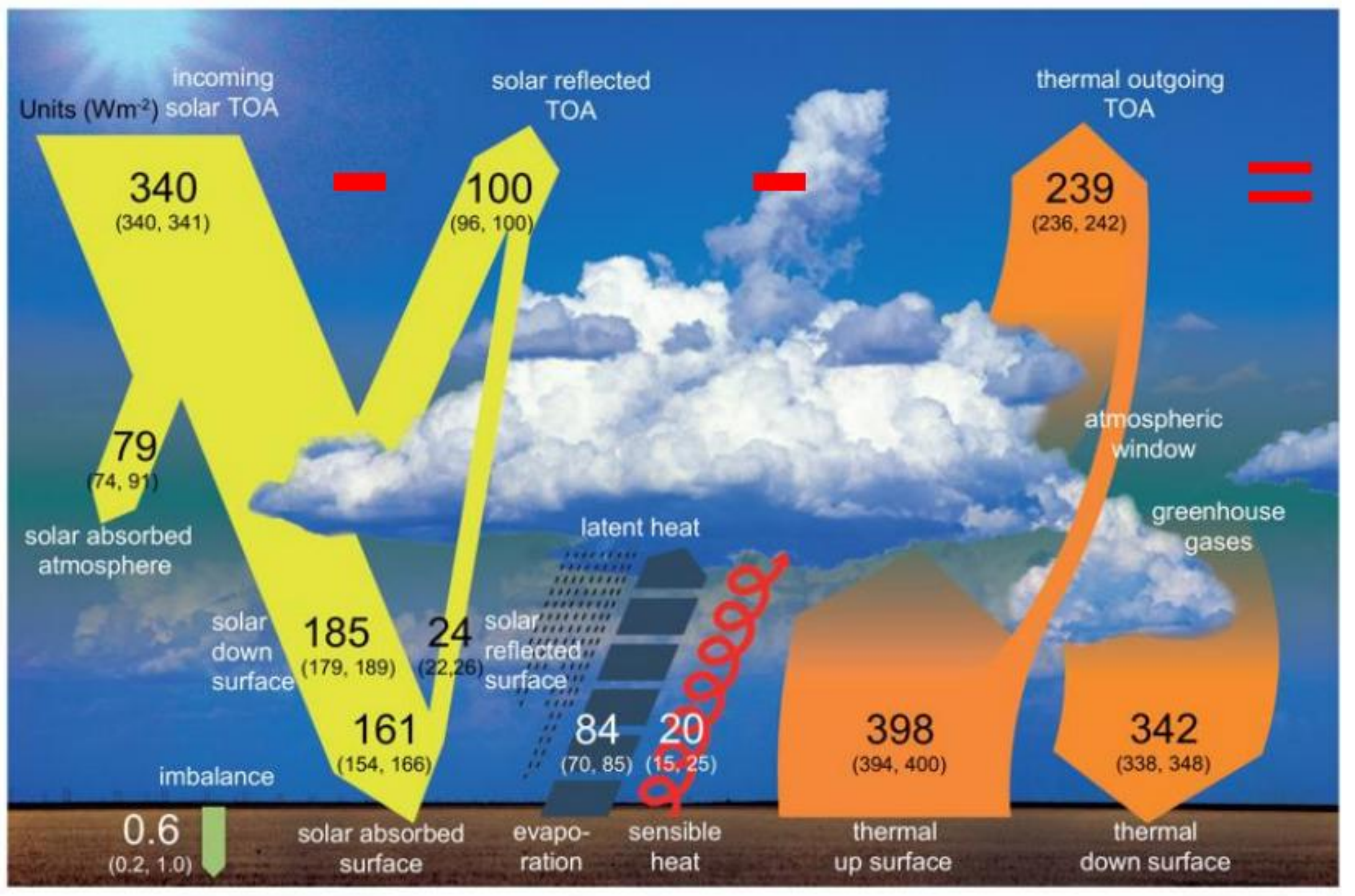
	Ed4	Ed2.8	Ed4 − Ed2.8
All-sky (Jul 2005–Jun 2015)			
TOA SW insolation	340.0	339.8	0.17
TOA SW up	99.1	99.6	−0.5
TOA LW up	240.0	239.5	0.5
SW down	187.1	186.6	0.53



- **Wild** (2017, AIP):



- **Hakuba** (2024 CERES STM), while understanding Earth's Energy Budget for Libera, refers to Wild. Solar down surface 185 Wm^{-2}.
  (The greenhouse factor is g(all-sky) = 0.3995)



Updated from IPCC AR5 / Wild et al. 2013, 2015 Climate Dynamics

Theoretical Reference Estimate for the all-sky greenhouse factor is g(TRE) = (**15** – **9**)/**15** = 0.4; while g(IPCC-Wild-2013) = (398 – 239)/398 = 0.3995.

- **Loeb** (2014, NASA LaRC, CERES): Solar down surface = 186 Wm^{-2}

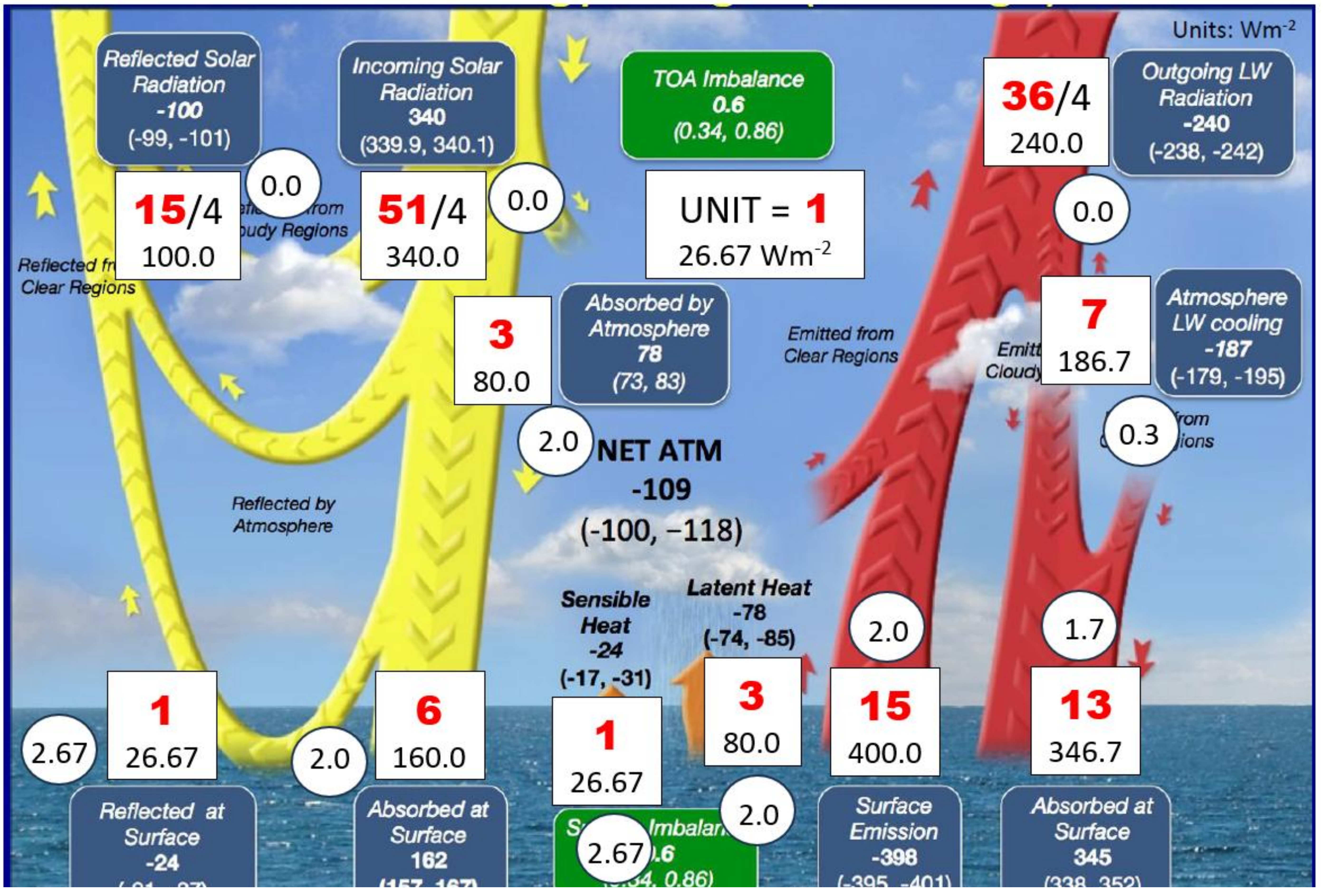
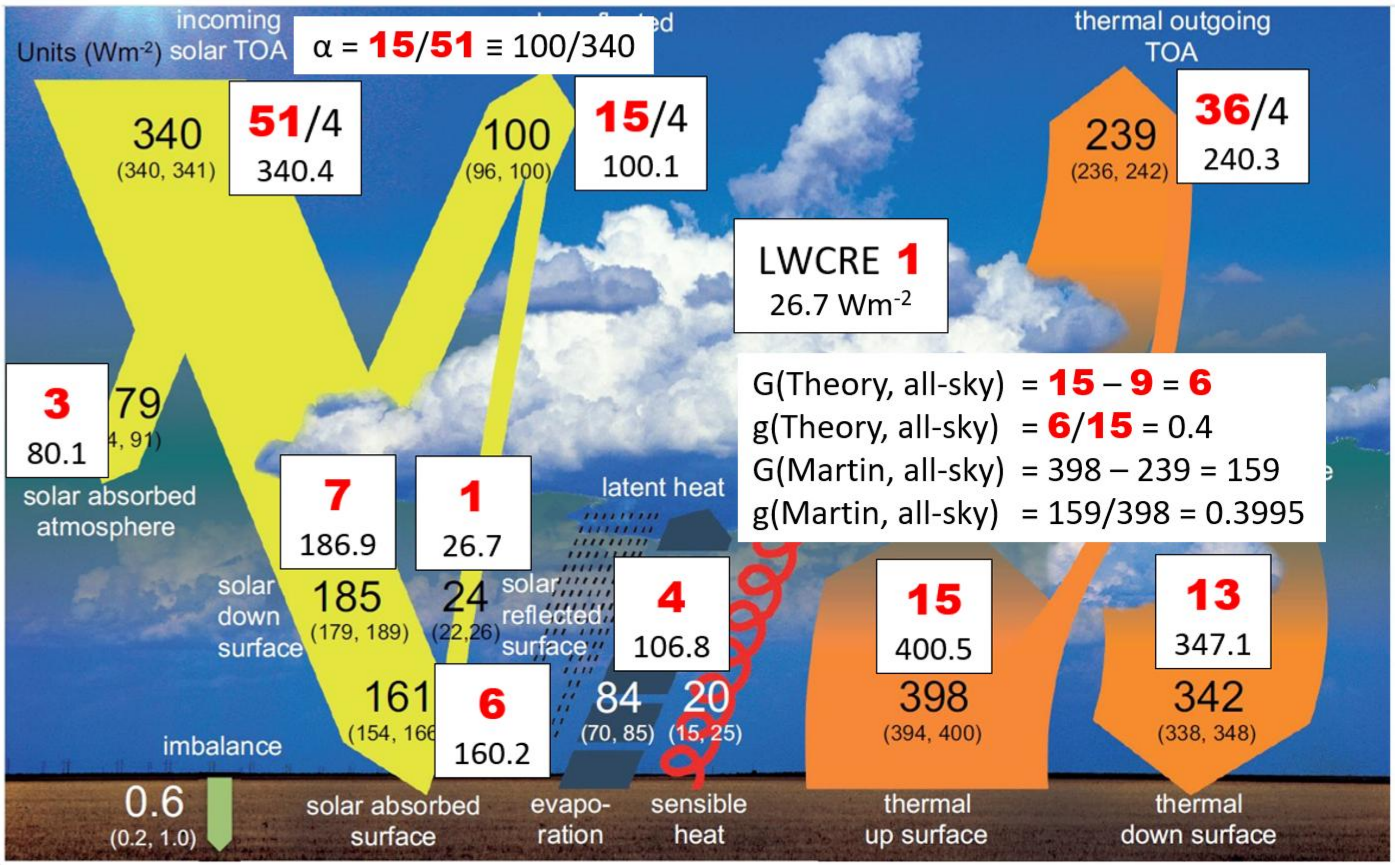


Fig. 10 Theoretical reference estimate projected on CERES EBAF Earth's Energy Budget Diagram (Loeb, 2014). Differences in circles. Each of the data is within the noted range of uncertainty. Differences at TOA = 0.0 Wm^{-2}. Largest difference at SFC = 2.67 Wm^{-2}.

Attribution of global warming: greenhouse effect?

All-sky greenhouse factor, g(TRE) = (**15** – **9**)/**15** = 0.4

All-sky greenhouse factor, g(IPCC) = (398 – 239)/398 = 0.3995



Clear-sky greenhouse factor, g(TRE) = (**15** – **10**)/**15** = 1/3

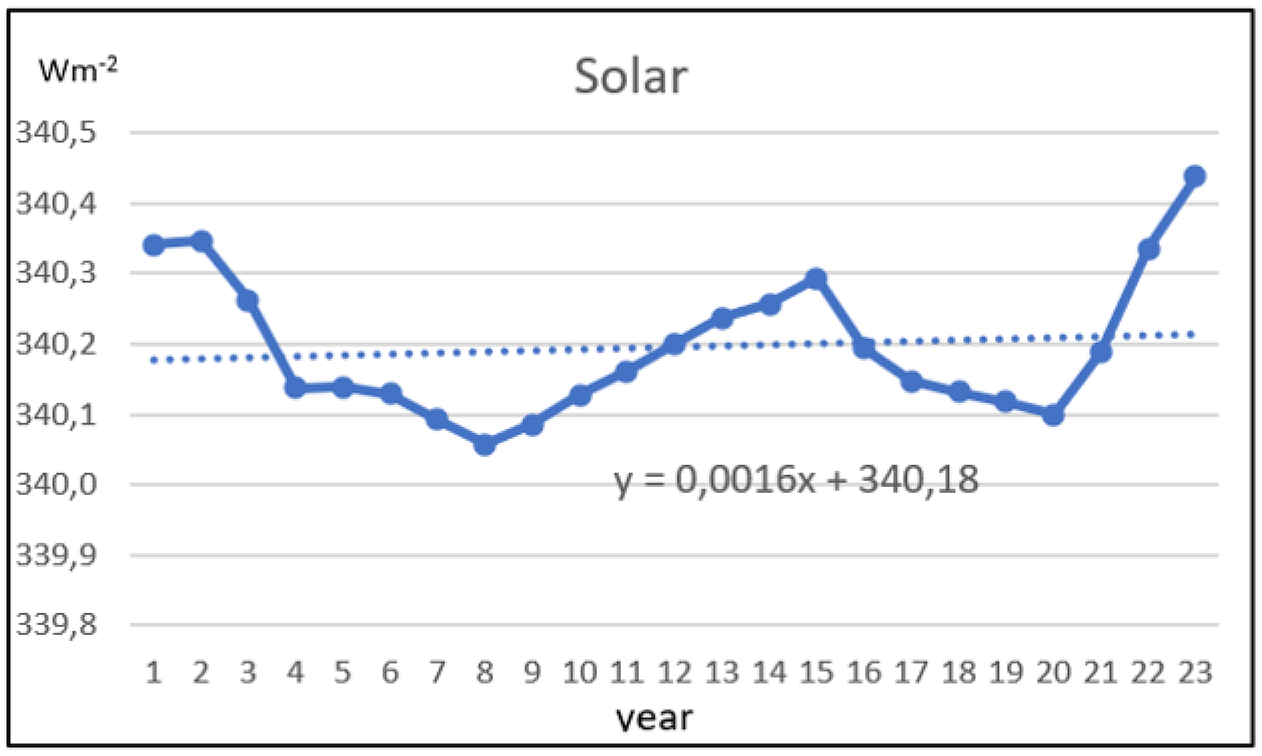
Clear-sky greenhouse factor, g(CERES) = (398.85 – 265.98)/398.85 = 0.333

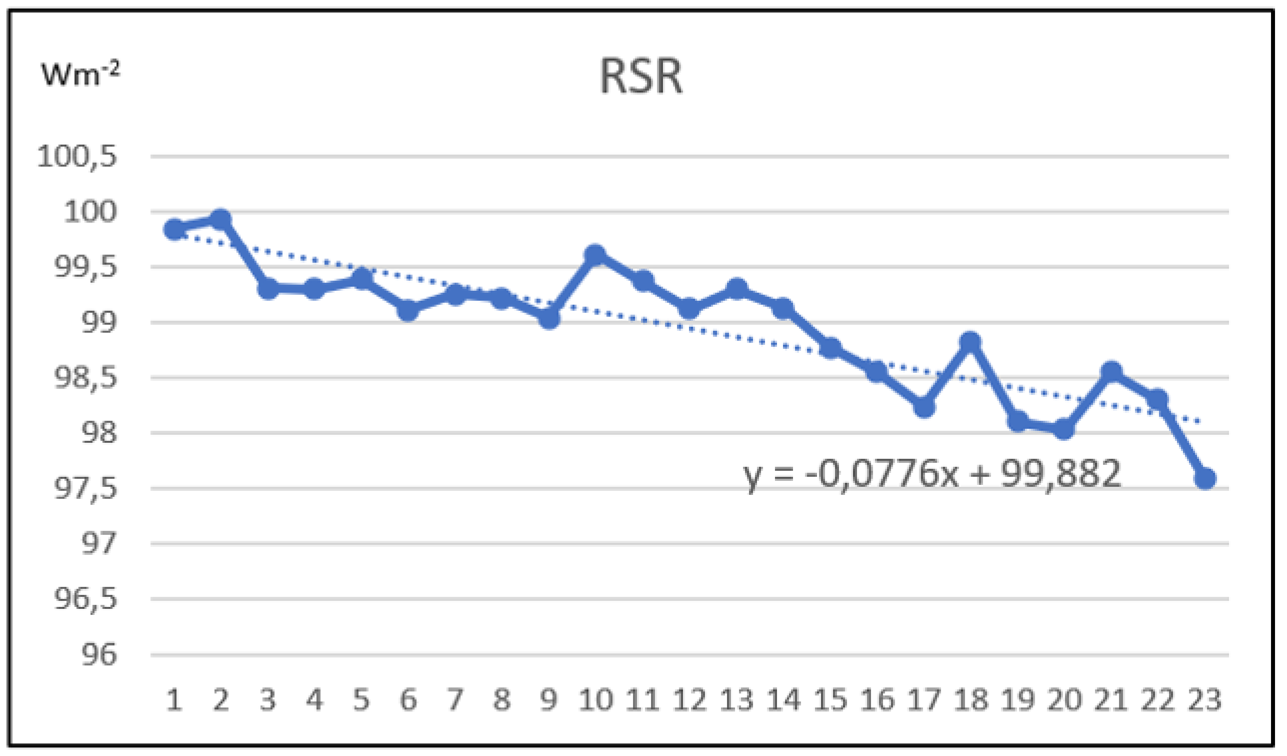
CERES EBAF Ed4.2, 24-yr mean (Aug 2000 – July 2024)

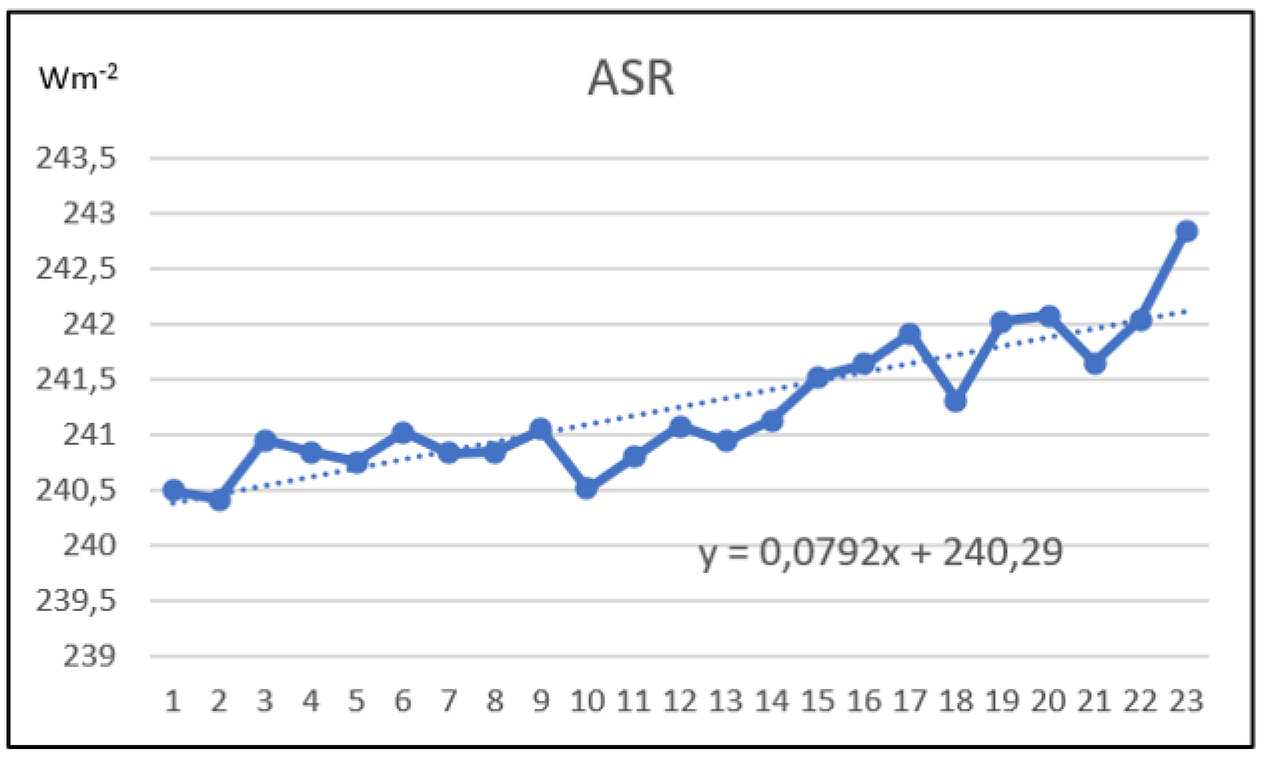
Clear-sky	N	Theory	CERES
Surface LW up	**15**	400.20	398.85
TOA LW up	**10**	266.80	265.98
G	**5**	133.40	132.87
g	5/15	1/3	0.333
All-sky	**N**	**Theory**	**CERES**
Surface LW up	**15**	400.20	398.69
TOA LW up	**9**	240.12	240.41
G	**6**	160.08	158.28
g	6/15	0.4	0.397

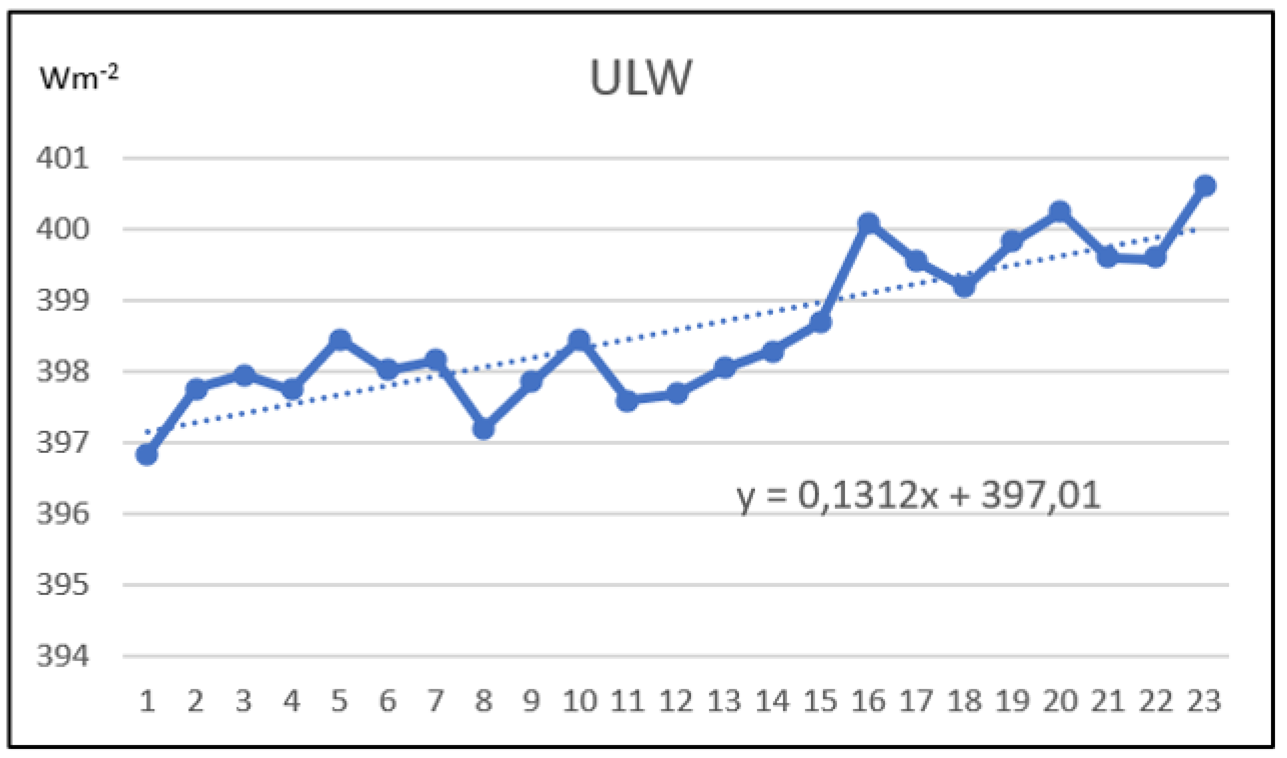
What Are the Drivers of Global Warming?

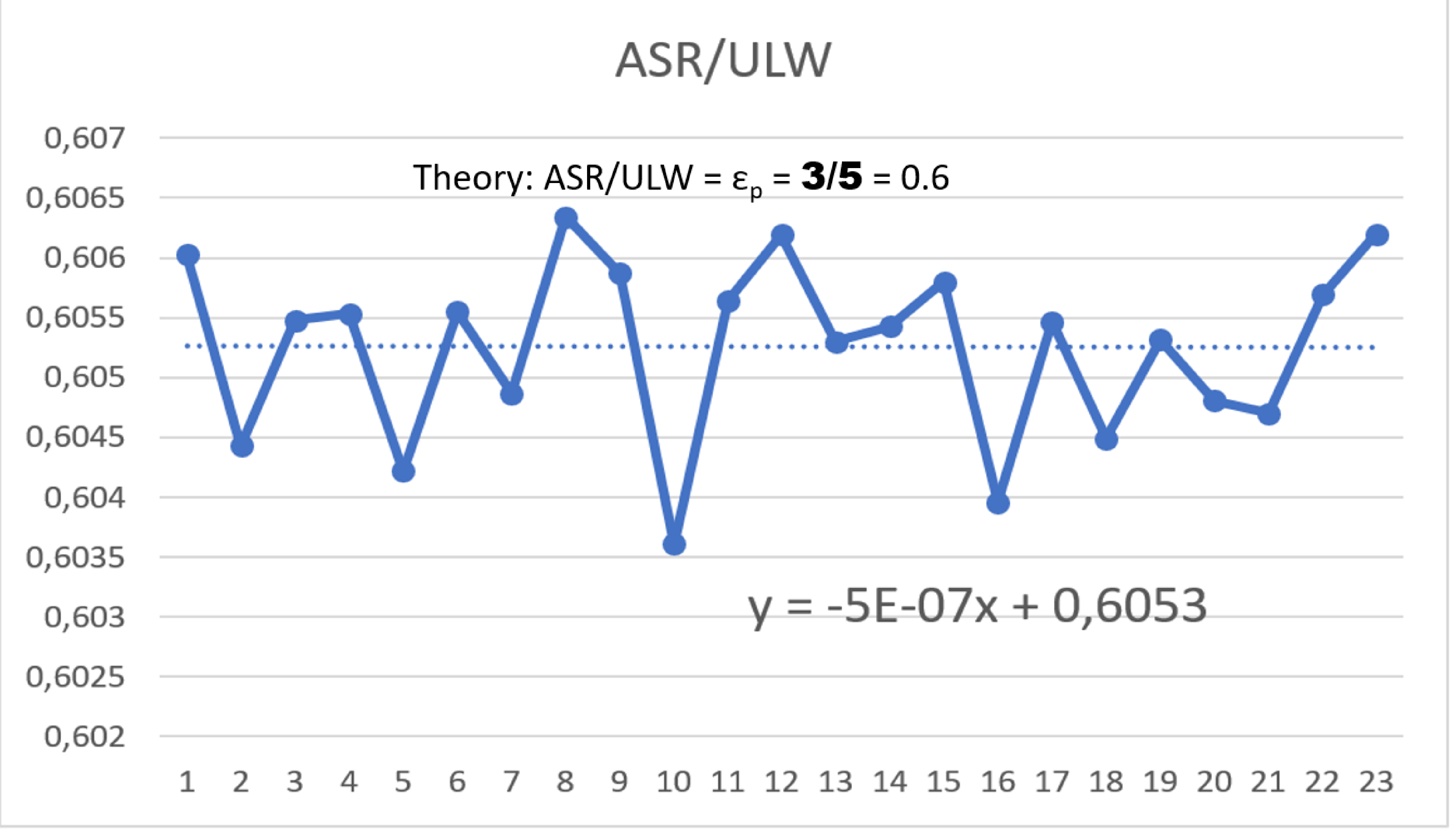
Data: CERES EBAF Ed4.2 V2, release date 2 Jan 2024 (Oct 2000 – Sept 2023)



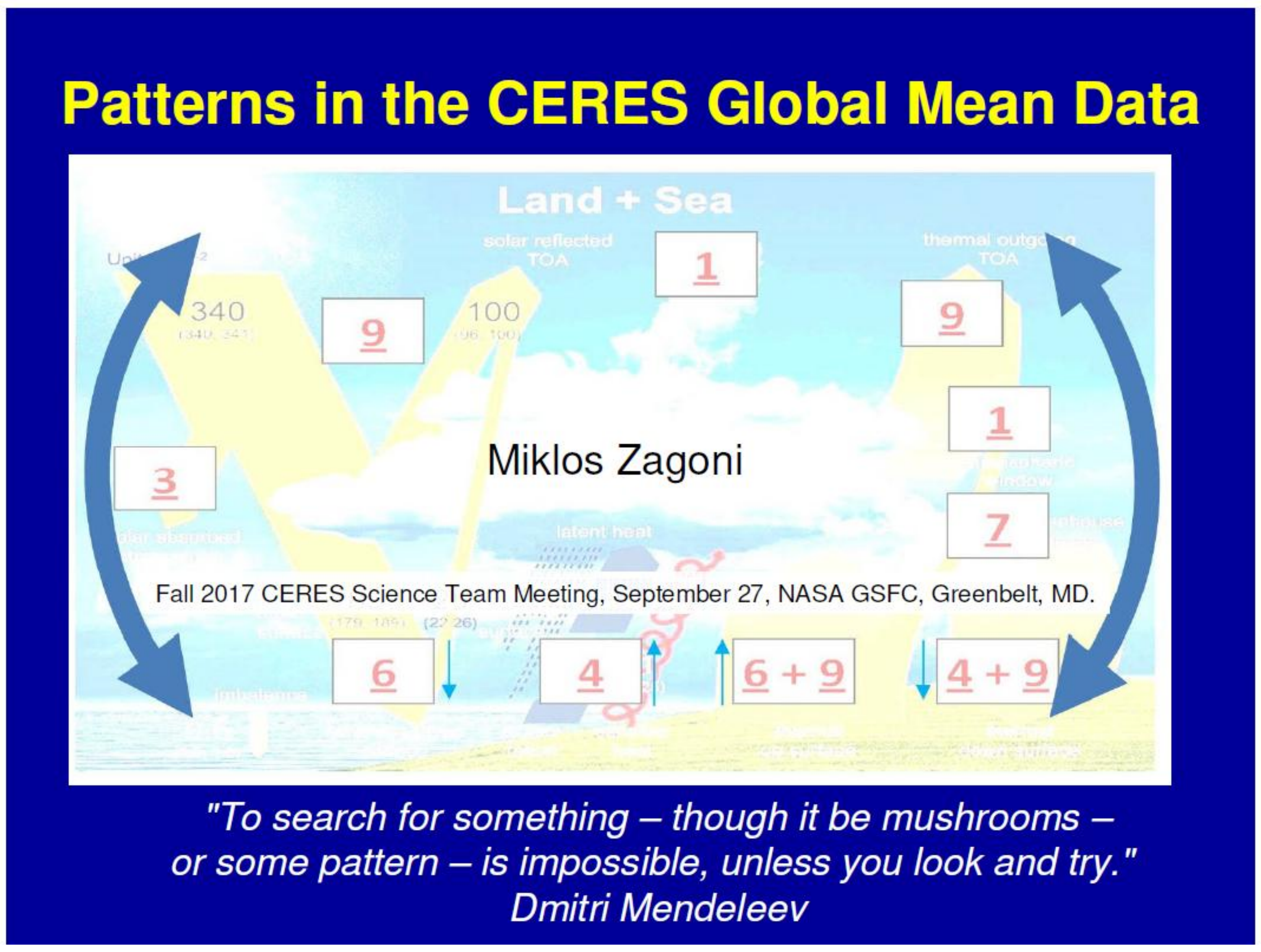








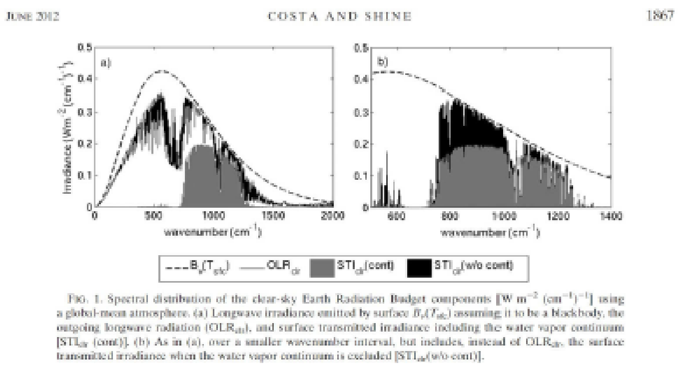
2017: Theoretical Reference Estimate introduced to the science community
NASA CERES Science Team Meeting, Goddard Space Flight Center, Washington



Pattern 3. Clear-sky integer ratios

Costa and Shine (2012) Line-By-Line

- ULW = 386 Wm^{-2}
- OLR = 259 Wm^{-2}
- ATM = 194 Wm^{-2}
- G = 127 Wm^{-2}
- STI = 65 Wm^{-2}



	STI	G	ATM	OLR	ULW	2OLR
CS12 =	65	127	194	259	386	518
Pattern	65 /	130 /	195 /	260 /	390 /	520
Ratios	**1** /	**2** /	**3** /	**4** /	**6** /	**8**
Diff	0	3	1	1	4	2 (Wm^{-2})

Direct surface–TOA coupling puts the atmospheric processes into parenthesis.

All-sky and clear-sky integer positions and their values in Wm^{-2}
(From the 2017 NASA CERES STM presentation)

EdMZ all-sky integer ratios
F = N × UNIT
UNIT = OLR(all-sky)/9

All-sky Flux	EdMZ	N
TOA SW In	340.04	
TOA SW Up	99.60	
TOA LW Up	240.14	9
SFC SW In	160.09	6
SFC LW Down	346.87	13
SFC (SW in + LW in)	506.96	19
SFC LW Up	400.23	15
SFC Net	106.73	4
G	160.09	6
SFC LWCRE	26.68	1
2 × TOA LW Up + LWCRE	506.96	19

EdMZ clear-sky and all-sky integer ratios
F = N × UNIT, F = N × UNIT
UNIT = OLR(clear-sky)/4

Clear-sky Flux	EdMZ	N	N
TOA SW In	340.04		
TOA LW Up	266.82	4	10
ATM emitted Up	200.11	3	30/4
STI	66.7	1	10/4
SFC (SW + LW) In	533.65	8	20
SFC LW Up	400.23	6	15
SFC Net	133.42	2	5
G	133.42	2	5

Model data set: EdMZ
All-sky pattern positions

All-sky	Ed2.8	Ed4.0	EdMZ
TOA SW In	339.87	340.04	340.04
TOA SW Up	99.62	99.23	99.60
TOA LW Up	239.60	240.14	240.14
SFC SW In	162.34	163.67	160.09
SFC LW Down	345.15	344.97	346.87
SFC (SW in + LW in)	507.49	508.64	506.96
SFC LW Up	398.27	398.34	400.23
SFC Net	109.22	110.30	106.73
G	158.67	158.20	160.09
SFC LWCRE	28.88	30.90	26.68

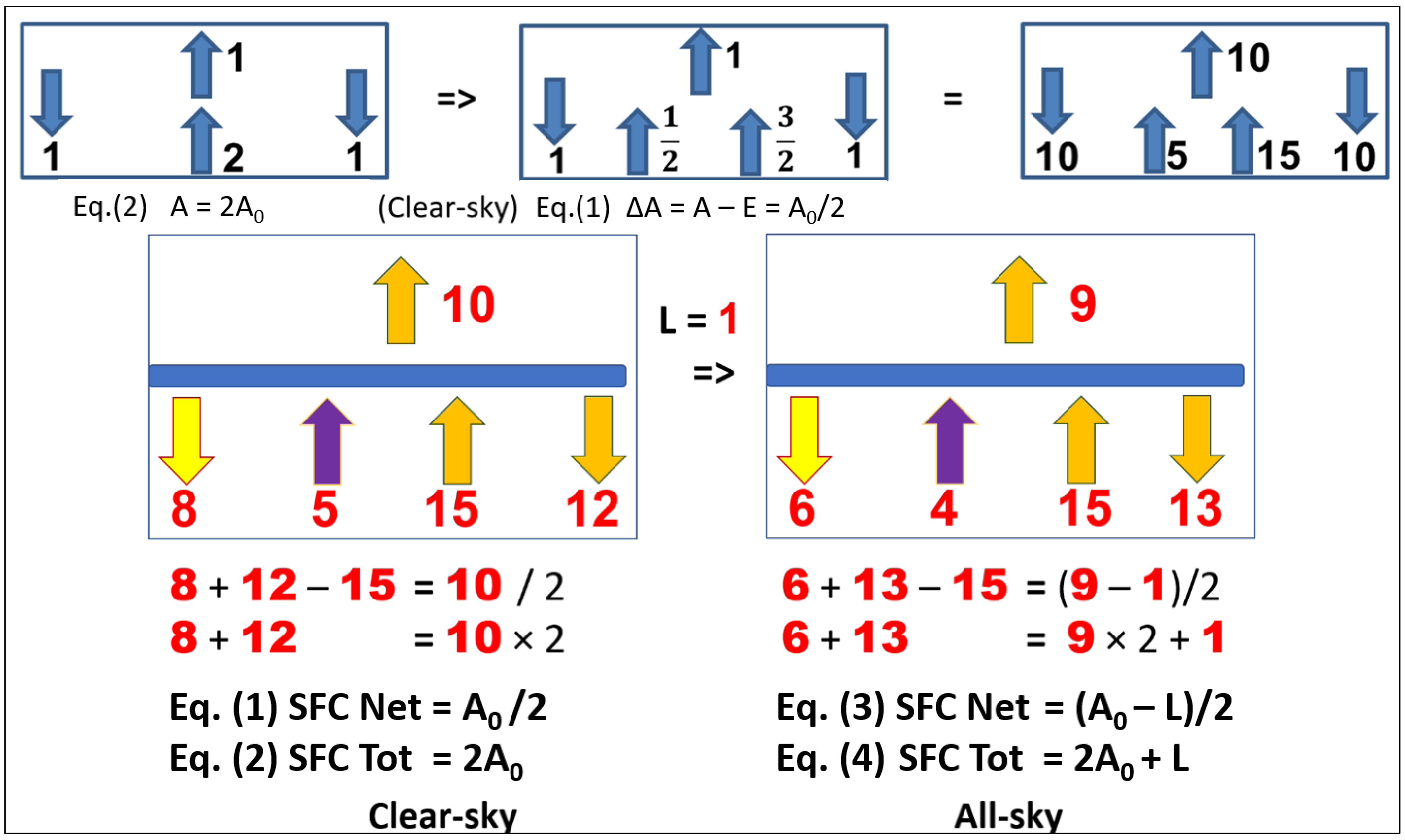
Model data set: EdMZ
Clear-sky pattern positions

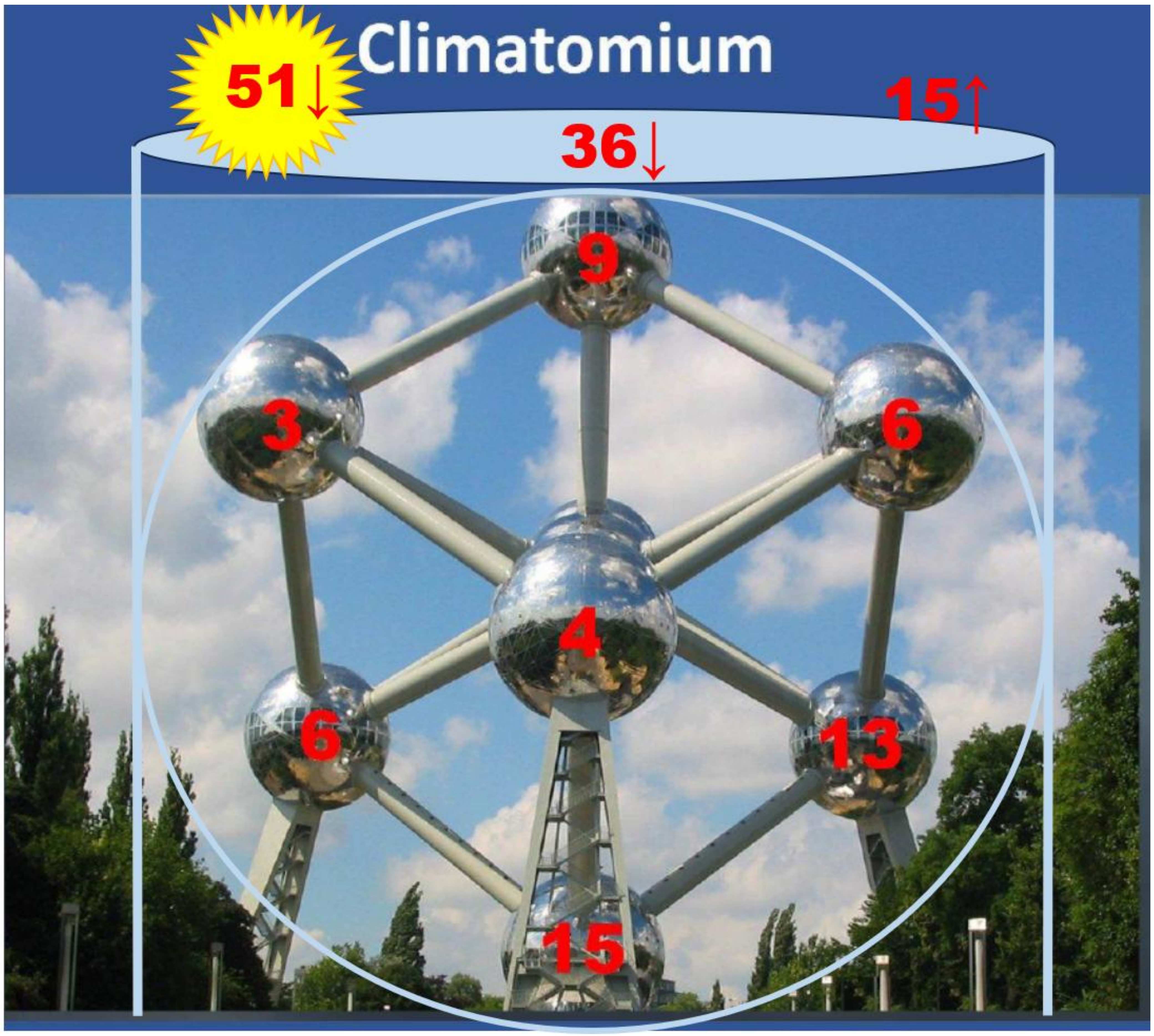
Clear-sky	Ed2.8	Ed4.0	EdMZ
TOA SW in	339.87	340.04	340.04
TOA LW up	265.59	268.13	266.82
SFC SW in	214.32	213.91	213.47
SFC LW down	316.27	314.07	320.18
SFC (SW + LW) in	530.59	527.98	533.65
SFC LW up	398.40	397.59	400.23
SFC Net	132.19	130.39	133.42
G	132.81	129.46	133.42
TOA LWCRE	25.99	27.99	26.68

CERES Science Team Meeting presentations are summarized at https://earthenergybudget.com

Homepage: https://earthenergyflows.com Contact: miklos.zagoni@earthenergyflows.com

Geometric Summary





https://www.earthenergyflows.com/Zagoni-EGU2024-Trenberths-Greenhouse-Geometry_Full-v03-480.mp4 (Length 2:2:28) + Extras

 https://earthenergyflows.com miklos.zagoni@earthenergyflows.com Page last modified: 20 February 2025 15:30