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Integer Ratios in Earth's Global Mean Energy Flow System: Observation and Explanation

--Manuscript Draft--

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Abstract:	Integer ratios can be recognized between the components of Earth's global mean energy flow system. If the unit flux is taken to be equal to the longwave cloud radiative effect (LWCRE), each flux components, all-sky and clear-sky, shortwave and longwave, at the top-of-atmosphere (TOA) and at the surface, even the components of the non-radiative energy flux (latent heat and sensible heat) are close to a small integer multiple of the unit flux, typically within the stated range of uncertainty for the given flux component. We show the integer ratio structure on several global energy budget estimates based on observed data from different sources (CERES satellite missions, the GEWEX energy and water cycle studies, or IPCC estimates from climate models and direct surface observations). In the effort of trying to find a plausible explanation for this recognition, we present four radiative transfer constraint equations, yielding a part of the observed integer structure. The first equation is a direct consequence of Schwarzschild's original two-stream relationships; the second is a specific case of them. This pair of equations is for clear-sky conditions; the second pair is an evident all-sky version of the first. A straightforward physical interpretation of the equations is suggested. The set of these four equations can be solved, and the solution is a set of small integers, being equivalent to the found ratios in the observation-based datasets.
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Dear Dr. Wang,

I am Miklos Zagoni, physicist, retired university professor from Eotvos Lorand University, Budapest, Hungary. I am submitting a manuscript for consideration to Journal of the Atmospheric Sciences, titled "Integer Ratios in Earth's Global Mean Energy Flow System: Observation and Explanation".

This manuscript is a resubmission of my original paper, JAS-D-23-200, which was rejected (instead of requiring a major revision) by one reviewer. I think the review did not falsify any of my essential statements, as I explained in detail in the attached "Response to the Reviewer", therefore I reconstructed my paper by inserting a Historical introduction to better reveal the motivations of this research. I hope this reorganized and expanded explanation will be more acceptable.

I think my manuscript fits well to your Journal's scope as it contains basic research about the physics of our atmosphere, with several quantitative and deductive aspects of the radiation transfer model implemented by our atmosphere. I do hope the Editors and the readers of this journal will find my results interesting and useful.

This work is the opposite of a sudden recognition: it is a result of more than ten years of research. It is my original work and is not under consideration for publication elsewhere.

Thank you.

Dr. Miklos Zagoni

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Budapest, January 22, 2024

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Dear Editors,

As my previous submission was rejected, this new submission is not a major revision but a profound reconstruction of JAS-D-23-0200. Thus, although my response to the former reviewer is irrelevant, I still give a detailed response to the previous review text, to help the Editors judge my approach.

The Chief Editor of the Bulletin of AMS (BAMS) proposed me to submit my manuscript to your Journal of the Atmospheric Sciences. My goal is a deeper understanding of Earth's global mean energy budget.

"Understanding" means different things in different scientific cultures. In the Platonian philosophy, it means to grab the "idea" of something. In the Philosophy of Tao, it means to have the correct Yin and Yang dynamics of the thing. I think, in this sense "Yin" is the observed data of Earth' global mean energy flow system, "Yan" is a theoretical structure, for example, the solution of certain (Schwarzschild-type) radiative transfer equations; and if the two are the same, or at least very similar, we may think we have an "understanding" of the thing.

Max Planck described this dynamics wonderfully: „Because the constant interaction between experimental and theoretical research, which is always inspiration and control, will be also in the future the safest, the only guarantee of the prosperous progress of physical science."

Of course, my paper is not about philosophy; these considerations are only to help to get a view on it. My paper is about recognized numerical structures – small integer relationships – that can be observed in all the published reliable global mean energy flow estimates, from NASA CERES satellite observations, to GEWEX global energy and water cycle studies, to the IPCC energy budgets based on climate models and direct surface observations. This is one part of my work. The other part is to offer some kind of explanation, or interpretation, that is, an "understanding" of this recognition.

The given versions of Schwarzschild's original two-stream relationships are perfectly applicable, and have a solution proved to be identical to the observed numerical structures.

But the Reviewer did not realize the observed structure; instead, she or he declares that my basic Schwarzschild-relationship (repeated in several textbooks exactly in the same form), "**cannot be valid**".

Only God is able to tell what „can" or „cannot" be valid. We, humans, with our limited experience, might only say that we think this or that seems to be valid. I show several reasons: the derivation of the equations may be given from first principles (as the English astronomer E.A. Milne wrote in his Handbook of Astrophysics in 1930); they are verified in textbooks (like Dennis Hartmann's Global Physical Climatology, 1994, repeated in 2nd Edition in 2016) with a difference of 0.31 Wm^{-2}; and, finally, they are satisfied, with an extremely high accuracy on observed datasets.

I recommend these considerations to the interest of the Readers of this submission.

Thank you.

Now my detailed response to the reviewer.

Author’s (A) Response to the Reviewer (R)

R: „The author attempts to work out the partition relationships among global mean energy fluxes at the surface and TOA on the basis of two pairs of equations inferred from the radiative transfer principles for gray-atmosphere first developed by Schwarzschild (1906).”

A: No, I do not attempt to work out partition relationships among global mean fluxes. I attempt to control (verify) those equations on published global mean energy fluxes.

R: “One pair is for surface (sensible + latent) heat fluxes and clear-sky/total-sky OLR and the other pair for the net energy absorbed by the surface and clear-sky/total-sky OLR.”

A: No, the equations connect surface fluxes to TOA LW fluxes, namely, one pair connects the surface net radiation (sensible + latent heat fluxes) to TOA LW fluxes (OLR in the clear-sky and OLR and LWCRE in the all-sky); the other pair connects the total energy absorbed by the surface to OLR in the clear-sky and to OLR and LWCRE in the all-sky.

R: “I found the second pair of relations is no more than the well-known energy balance equations at the surface and TOA”

A: No, such an energy balance equation was never assumed as an energy balance equation at the surface and TOA. If it was, how is it possible the 2 Wm^{-2} difference in the IPCC energy budget? Does it contradict the well-known energy balance? No, they describe a specific (unique) state, and its control (verification) on observed global mean data is one of the main goals of the paper;

R: “and the first pair appears invalid, suggesting the author’s misuse (or misinterpretation) of the radiative transfer principles for gray-atmosphere developed by Schwarzschild (1906)”.

A: If the first pair is invalid, then the second pair cannot be the well-known energy balance equations since they are direct (single-step) consequence of the first pair (being simply the all-sky versions of the clear-sky pair).

R: “Also the interpretation for the approximations evoked for total-sky energy relationships is inconsistent with the physical characteristics of energy balance equations for gray atmosphere.”

A: If the total-sky energy relationships are inconsistent, then how is it possible that they are valid with a difference of 0.1 Wm^{-2} on the global energy balance of Stephens et al. (2023), published in BAMS, based on 30 years of GEWEX observations?

R: “Moreover, the so called “arithmetic solution for the four equations” has been done in standard textbooks and therefore, there is nothing new there.

A: How is it possible that this solution of the four equations (where the first pair is said to be invalid and inconsistent with the physical characteristics) has been done in standard textbooks? If the first pair is invalid and the second pair is trivial, then how is it possible that they have the same absolute bias of -2.57 Wm^{-2} and +2.57 Wm^{-2}, with the mean bias of the four equations together of 0.0007 Wm^{-2} (see my manuscript)?

About the “Comments”.

R: “I. The interpretation of Eq. I (the equation above L59) is not consistent with the radiative transfer principles for gray-atmosphere. For example, Houghton’s Eq. 2.13 (1977), which was cited in the manuscript as theoretical supporting evidence for his Eq. I, is about “a temperature discontinuity at the lower boundary”, instead of the relation between the net radiative energy flux at the surface and the OLR, as the author claimed.”

A: My Eq. I (the equation above L59) is this: $A – E = A_0/2$. Houghton’s Eq. 2.13 (1977) is this: $B_g – B_0 = \Phi/2\pi$. Since B_g is the total absorbed energy at the surface (same as A); B_0 is the surface upward LW emission (same as E), and Φ is the outgoing LW energy at TOA (same as A_0), their difference is the net radiation at the surface. The two equations are identical. I do not give an interpretation; I follow Houghton. The left-hand side is a temperature discontinuity at the lower boundary (in radiative equilibrium), and the right-hand side is half of OLR. They are the same.

R: “**Eq. I cannot be valid**.”.

A: The fact is that it is satisfied by the CERES EBAF Ed2.8 data product (2017; the first 17 years of observation) with a difference of 0.59 Wm^{-2}, in a time when Earth’s energy imbalance (EEI) was regarded 0.58 Wm^{-2}. It cannot be **more** valid. In the radiative equilibrium model of Dennis Hartmann’s book (Global Physical Climatology, 1994, 2nd ed. 2016), the equality is valid with a difference of 0.31 Wm^{-2} (see lines 72-77 of my manuscript).

R: “Here are two simple reasons to support this conclusion.
(i) Consider a climate system that does not have atmosphere. In this case, the left hand side of Eq. I is exactly equal to zero and the right hand side is equal to E/2.”

A: I consider a climate system that *does have* an atmosphere. That’s why I submitted my paper to the Journal of the Atmospheric Sciences, instead of the Journal of Atmosphereless Exoplanets, or the like. Note, in that case the right-hand side is not E/2, but $A_0/2$. I never claimed – and neither did Goody, Houghton or Hartmann – that the equation is valid on planets that does not have atmosphere.

R: “(ii) According to the statement in L107-109, which is correct, left hand side of Eq. I is equal to the sum of surface latent and sensible heat fluxes (i.e., F_{SH+LH}). Should Eq. I be valid, OLR has to be equal to zero in the case that $F_{SH+LH} = 0$, which is unphysical.”

A: Again, the case when $F_{SH+LH} = 0$ is the case of the Moon, or similar to the Venus or Mars, which I never considered. On Earth, there is enough open water surface in the oceans to release enough latent heat flux, and a suitable atmosphere to generate enough convection (sensible heat flux) to satisfy the condition. The simple all-sky version of this equation is satisfied with a difference of 0.1 Wm^{-2} in the GEWEX 30 years energy budget (Stephens et al. 2023, BAMS), see my lines below L179, and my Fig. 1.

R: „Because two of the “*four simple versions of this equation*” considered by the author, i.e., Eqs. (1) and (3), or Eqs. (1a) and (3a) are based on Eq. I, the so-called “validation” results obtained from Eqs. (1) and (3) or Eqs. (1a) and (3a) are simply misinterpretation of the global energy balance of a climate system."

A: Both Eqs (1) and (3), and Eqs. (1a) and (3a), based on Eq. (I), are valid with differences far within the stated range of uncertainty. The Reviewer should consider this fact, if she or he has a specific interpretation of these equations, at least in the spirit of Max Planck's instruction about the interaction of experimental and theoretical research: if some equations are justified by data, it is advised to take them seriously, independently of our preconceptions or expectations. That's the message of my paper.

R: "2. Applying the energy balance model of a one-layer gray-atmosphere climate system to the difference between the second and first equation the second equation of Eq. (11) of Schwarzschild (1906) yields (his or her Eq. (1))."

A: Really, this is the case when $\tau = 2$ when $\varepsilon = 1$.

R: "Therefore, Eq. II, on which the other two of the "*four simple versions of this equation*" considered by the author, i.e., Eqs. (2) and (4), or Eqs. (2a) and (4a) are based, is only a special case of a one-layer gray-atmosphere climate system, in which the atmosphere is a blackbody atmosphere. Therefore, the so-called "validation" is no more saying the global mean atmosphere behaves more like a blackbody atmosphere according to the global mean radiative energy fluxes at the surface and TOA. Therefore, there is nothing new beyond this well-known fact."

A: Earth's atmosphere acts like $\tau = 2$; this is not at all a well-known fact, this is a model, and it is an experimental result that the equation is valid with this τ very exactly on some energy balance estimates (GEWEX), but there are sizeable differences for example in the IPCC data. It cannot be assumed that the author of the IPCC distribution wouldn't take it into consideration if the equation were a well-known energy balance condition.

R: 3. If one wants to apply a one-layer gray-atmosphere to account for the difference between total-sky and clear-sky radiative energy fluxes at the surface and TOA, the difference has to be attributed to the SW effect of clouds, as the LW portion has to be the same for a blackbody atmosphere. Therefore, the extra term "L" in Eq. III and Eq. IV, which the author is called as "*the longwave cloud radiative effect*" is inconsistent with the fact the atmosphere behaves like a blackbody emitter/absorber from the perspective of the global mean energy balance. "

A: I do not *call* the extra term L as the longwave cloud radiative effect. It is *defined* that way, as the difference of clear-sky and all-sky OLR. This way, longwave radiation transfer in the atmosphere is separated from the longwave cloud radiative effect; the shortwave effect is taken into account in the *all-sky values* on both sides of the equations.

My overall remark is that the Reviewer uses an interpretative approach to equations that are justified (satisfied) with high accuracy by several observed data products, without having one single word about the fact that these equations are *valid* on these datasets.

Thank you for your attention.

Miklos Zagoni

Budapest, Hungary
January 22, 2024

Integer Ratios in Earth's Global Mean Energy Flow System: Observation and Explanation

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ABSTRACT

Integer ratios can be recognized between the components of Earth's global mean energy flow system. If the unit flux is taken to be equal to the longwave cloud radiative effect (LWCRE), each flux components, all-sky and clear-sky, shortwave and longwave, at the top-of-atmosphere (TOA) and at the surface, even the components of the non-radiative energy flux (latent heat and sensible heat) are close to a small integer multiple of the unit flux, typically within the stated range of uncertainty for the given flux component. We show the integer ratio structure on several global energy budget estimates based on observed data from different sources (CERES satellite missions, the GEWEX energy and water cycle studies, or IPCC estimates from climate models and direct surface observations). In the effort of trying to find a plausible explanation for this recognition, we present four radiative transfer constraint equations, yielding a part of the observed integer structure. The first equation is a direct consequence of Schwarzschild's original two-stream relationships; the second is a specific case of them. This pair of equations is for clear-sky conditions; the second pair is an evident all-sky version of the first. A straightforward physical interpretation of the equations is suggested. The set of these four equations can be solved, and the solution is a set of small integers, being equivalent to the found ratios in the observation-based datasets.

SIGNIFICANCE STATEMENT

Sunlight is the primary energy source of the climate system. Some part of it reflected on the atmosphere and the clouds, some from the surface; the rest is absorbed and re-emitted by the atmosphere and the surface. The resulted energy flow system has shortwave and longwave components and non-radiative fluxes as well, both in cloudless and in average-cloudy conditions, published in global mean energy budgets nowadays with an accuracy of some watt-per-square-meters. The purpose of this work is to point out that definite integer ratios exist between the components, valid within the stated range of uncertainty. We present the integer ratio system, and offer a theoretical explanation based on the solution of known radiative transfer equations.

1. Historical introduction

The first reliable estimate of the Earth annual global mean energy flow system, presented in the landmark study of Kiehl and Trenberth (1997), was based primarily on the ERBE data. In the next year, Wild et al. (1998) showed a disposition of radiative energy in the global climate system from general circulation models and direct surface observations, exhibiting remarkably differences; the largest is in all-sky downward longwave radiation (DLR), 344 Wm^{-2}, compared to 324 Wm^{-2} in Kiehl and Trenberth (1997). This 20 Wm^{-2} discrepancy remained unresolved for more than a decade, when Wild (2012) offered a ‘facelift’ for the picture of the global energy balance, reinforcing to increase DLR by 20 Wm^{-2} and suggesting a +10 Wm^{-2} change in non-radiative fluxes (up to a value of 105 Wm^{-2}) and a 10 Wm^{-2} decrease in solar radiation absorbed by the surface (down to 158 Wm^{-2}) which, together, yield a 7 Wm^{-2} increase in surface upward longwave (ULW) radiation (397 Wm^{-2}). With these changes, the mean values in the global energy flow system have occupied their position within typically ±3 Wm^{-2} as to we know them today. The results from the first decade of CERES observations was summarized in the same year in an updated diagram by Stephens et al. (2012, Figure B1).

Studying this depiction in detail, an unexpected recognition can be made. In the longwave part, each flux component is an integer multiple of the longwave cloud effect (given by the value of 26.7±4 Wm^{-2}), with differences much smaller than the indicated uncertainty; see Figure 1.

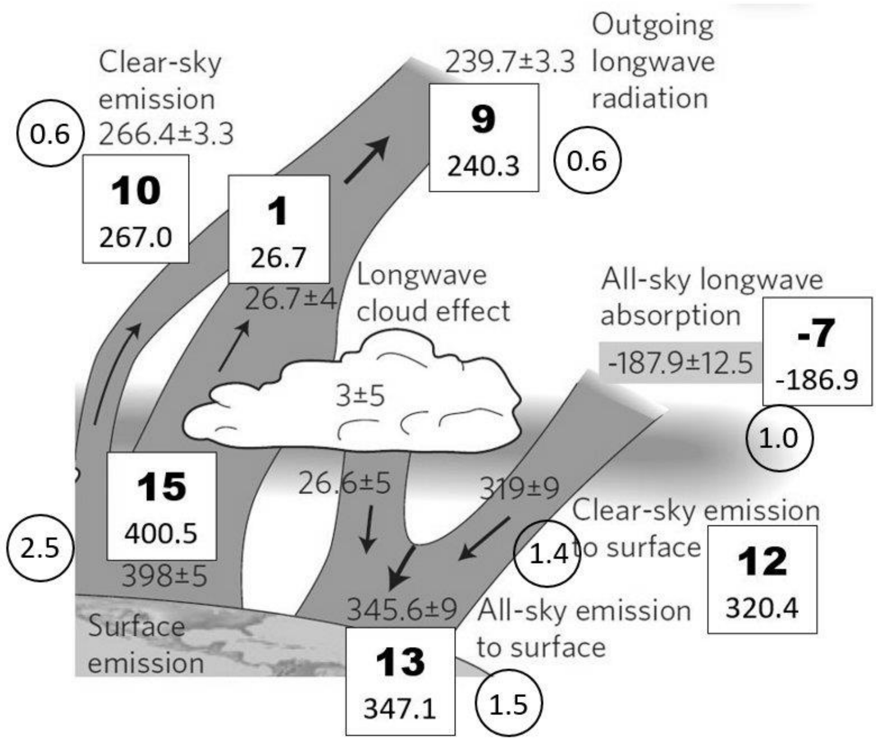

Figure 1 Longwave part of the global mean energy budget of Stephens et al. (2012), with our additions in textboxes (integer position in bold typeface and integer position multiplied by the unit flux in Wm^{-2}) and circles (the differences in Wm^{-2})

There, in added text boxes, we indicated the integer number (one LWCRE = **1**) in black bold typeface and the computed value (the integer position multiplied by the unit flux) in Wm^{-2}, also their difference in circles. The largest difference is 2.5 Wm^{-2} (in surface emission). It is obvious that the clear-sky and all-sky LW emissions (both to space and to surface) differ by one longwave cloud radiative effect (this comes from the definition of LWCRE). But that the values themselves are very close to integer multiples of LWCRE is really unexpected, counterintuitive and even implausible. How is that possible? By chance? The accuracy of the fit makes the mere coincidence improbable; but further investigation is needed.

Next year, in the IPCC Working Group I (The Physical Science Basis) 5th Assessment Report (Hartmann et al. 2013), the global mean energy budget (Figure 2.11, adapted from Wild et al. 2013), the integer system could also be found, with two further additional components. First, the shortwave components (solar absorbed atmosphere, solar down surface and solar reflected surface) occupy values very close to integer multiple positions; this way, the non-radiative (turbulent) flux component (the sum of evaporation and sensible heat) also finds a place in the integer system. Second, solar reflected at TOA is very accurately an integer if regarded on the cross-section disk to incoming solar radiation (that is, before division by 4 for spherical weighting); the difference is 0.1 Wm^{-2}, see Figure 2.

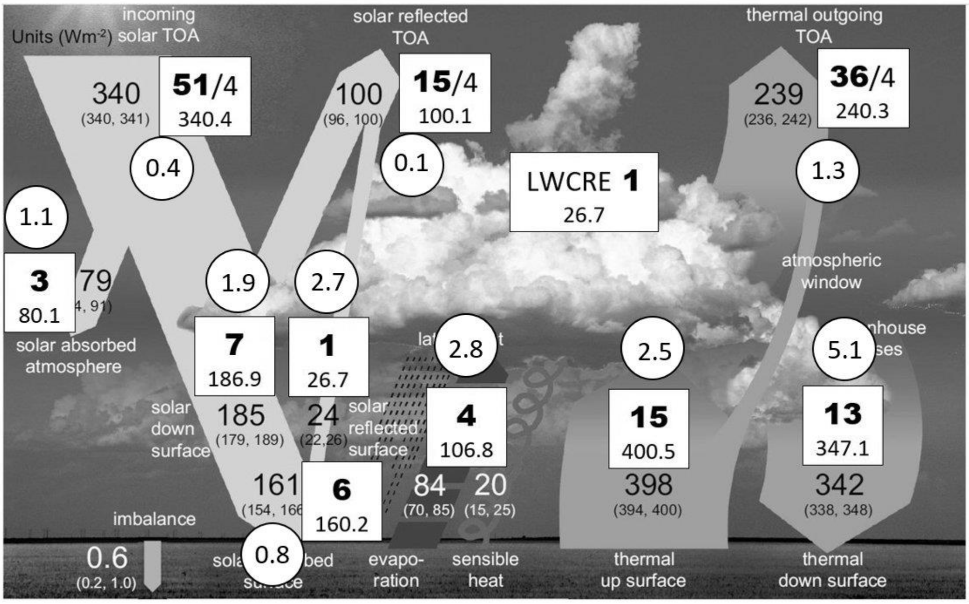

Figure 2 The global mean energy budget of the IPCC WGI AR5 (2013, Fig. 2.11) (Hartmann et al. 2013), with our additions in textboxes and circles (as above)

The largest difference is in thermal down surface, 5.1 Wm^{-2}, still within the stated uncertainty range of (338, 348). The probability of a mere coincidence has decreased further.

One year later an energy budget was presented, based on NASA CERES EBAF radiative data (Loeb 2014). The speciality of this distribution is that each data in the diagram has an integer position within the stated range of uncertainty. Here we refined the unit flux as 26.67 Wm^{-2}. The TOA fluxes have their place on the cross-section disk with a difference of zero. The sensible heat and latent heat components of the convective flux also occupy integer positions. The largest difference at the surface is 2.67 Wm^{-2}; see Figures 3a and 3b.

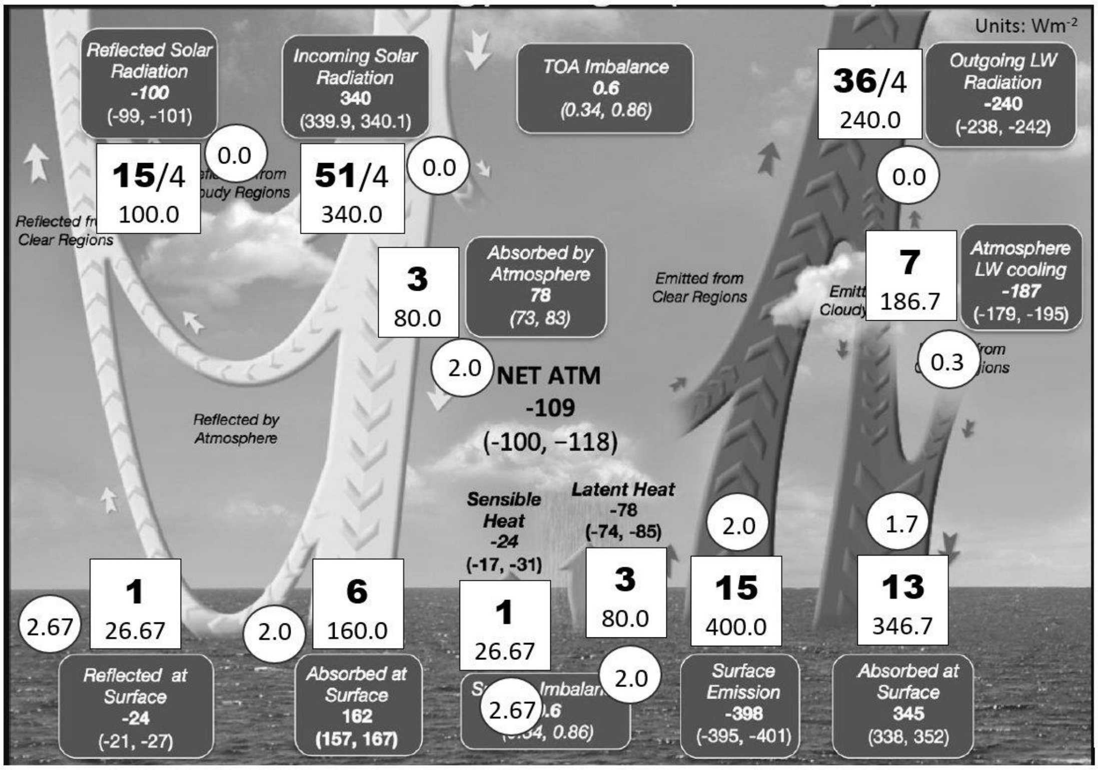

Figure 3a The global mean energy budget by Loeb (2014), based on CERES data, with our additions in textboxes and circles (as above). Unit = 26.67 Wm^{-2}. Each integer multiple value is within the stated range of uncertainty; the largest difference is 2.67 Wm^{-2}

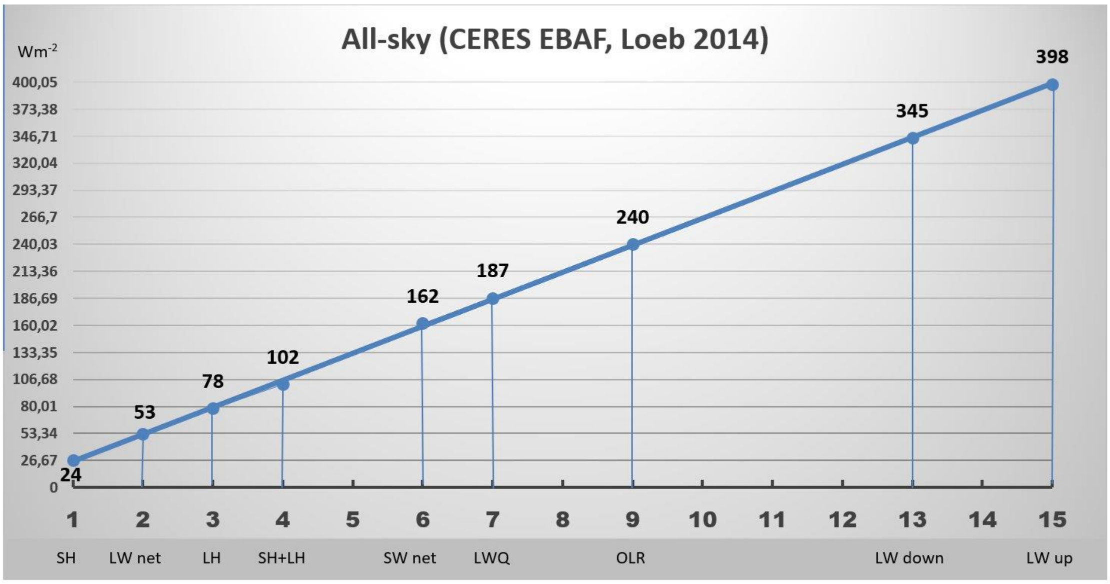

Figure 3b CERES EBAF data (Loeb 2014) against integer multiple positions. Unit = 26.67 Wm^{-2}. LW net and SW net are for the surface, as well as LW down and LW up. LWQ = Atmosphere LW Cooling

Now looking at the most recent GEWEX global energy budget (Stephens et al. 2023), with a unit flux of 26.68 Wm^{-2}, the largest difference at the TOA is 0.62 Wm^{-2} (OLR); at the surface is 3.38 Wm^{-2} in reflection (see Figure 4). Notice the remarkable accuracy of TOA incoming solar radiation (0.03 Wm^{-2} difference), and in the components of the turbulent flux (sensible heat and evaporation).

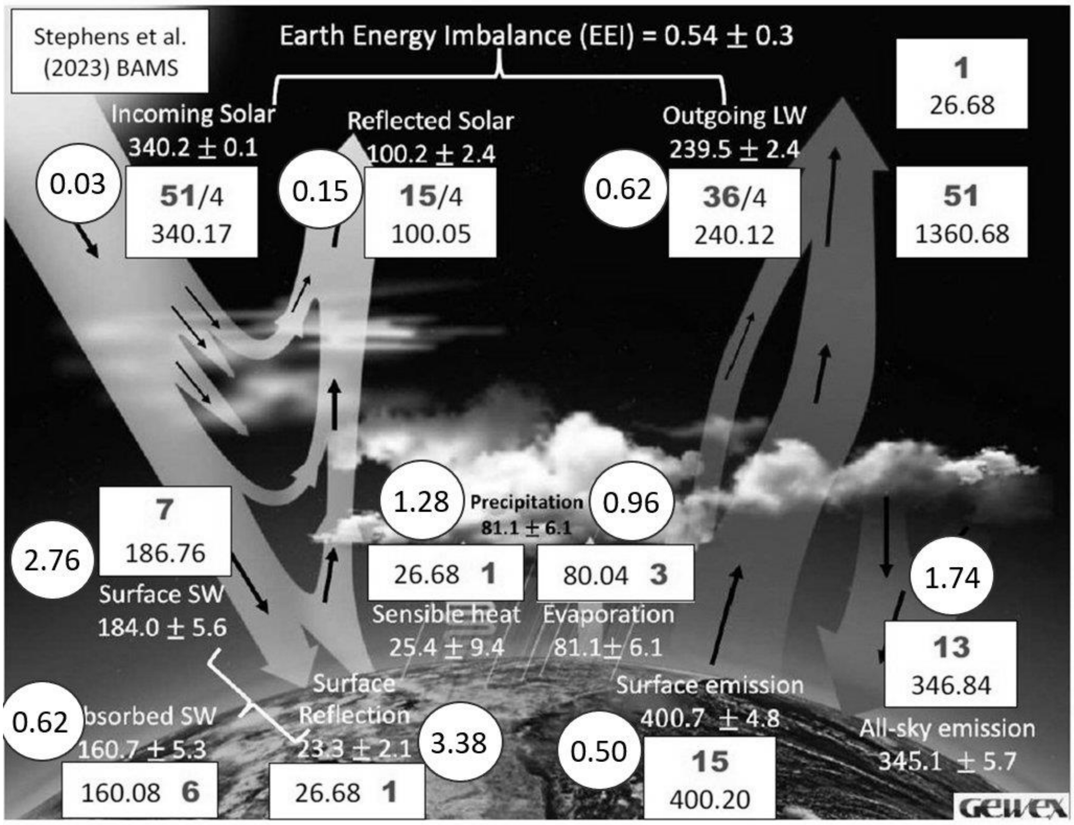

Figure 4 The global mean energy budget based on 30 years of GEWEX data (Stephens et al. 2023, Fig. SB3), with our additions in textboxes and circles (as above). The unit flux is 26.68 Wm^{-2}. The largest difference is 3.38 Wm^{-2} (Surface Reflection)

Since these diagrams depict only all-sky data, we tried the integer system on CERES clear-sky data as well, using the most recent available product, EBAF Edition 4.2, Version 2, release date January 2, 2024, full 23 years of observation, from April 2000 through March 2023, see Table 1. Total solar irradiance is taken to be TSI = 51 units = 1360.68 ± 0.5 Wm^{-2}; notice that the most accurate value is 1360.8 ± 0.5 Wm^{-2} (Kopp and Lean 2011).

Table 1 Clear-sky and all-sky N positions, EBAF Edition 4.2 Version 2 data (global monthly means for 23 full years, time period April 2000 — March 2023, and their difference. Unit flux is 26.68 Wm^{-2}

TSI = **51**	**Clear-sky**	**N**	**N**×unit	Ed4.2	Diff
TOA	SW in	**51**/4	340.17	340.19	0.02
	SW up	**8**/4	53.36	53.80	0.44
	LW up	**40**/4	266.80	265.95	-0.85
	Net	**3**/4	20.01	20.44	0.43
Surface	SW Net	**8**	213.44	211.35	-2.09
	LW down	**12**	320.16	317.92	-2.24
	LW up	**15**	400.20	398.64	-1.56
	LW Net	**-3**	-80.04	-80.72	-0.68
	TOT Net	**5**	133.40	130.64	-2.76
	All-sky	**N**	**N**×unit	Ed4.2	Diff
TOA	LW up	**36**/4	240.12	240.35	0.23
	SW up	**15**/4	100.05	99.05	-1.00
Surface	SW Net	**6**	160.08	163.69	3.61
	LW down	**13**	346.84	346.15	-0.69
	LW up	**15**	400.20	398.48	-1.72
	LW Net	**-2**	-53.36	-52.32	-1.04
	TOT Net	**4**	106.73	111.36	4.63

The largest difference is in Surface Total Net, 2.76 Wm^{-2} in the clear-sky and 4.63 Wm^{-2} in the all-sky. Note that with the most recent CERES Edition4 instrument calibration improvements, the SSF1deg_Edition4A net imbalance is ~+5 W m^{-2} (CERES_SSF1deg-Hour/Day/Month_Ed4A Data Quality Summary Version 2, updated 8/4/2023). It can be stated that each flux component, both clear-sky and all-sky, at TOA and at the surface, shortwave and longwave, is consistent with observations within the stated range of observation / calibration uncertainty.

The primary purpose of this paper has now been fulfilled: to present the integer system. Nonetheless, this structure requires theoretical explanation. We find that four versions of Schwarzschild’s (1906, Eq. 11) two-stream radiative transfer relationships are applicable to the global mean energy flow system of Earth, including some of the flux components of Table I. The first relationship describes the SW+LW net radiation at the surface; the second the total SW+LW radiation at the surface; both under clear-sky conditions; the third and fourth equations are their all-sky versions.

2. Schwarzschild's equation of radiative transfer

Schwarzschild (1906, Eq. 11) presented the grey, plane-parallel approximation to the problem of radiation transfer in an absorbing and emitting medium (originally the Sun's atmosphere), where a constant flux is coming from the 'far interior' (lower boundary) and leaves the system at the top-of-atmosphere (upper boundary). The atmosphere is assumed to be in radiative equilibrium and in local thermodynamic equilibrium, and in this equation the optical depth is assumed to be 'gray' (that is, one integrated energy flow, 'flux', is being considered). With these conditions, the model seems to be well applicable on the atmosphere Earth-like planets, with an outer energy source coming at the upper boundary and a black surface as the lower boundary.

The equation consists of three terms: the emission of a layer E, the upwelling beam A and the downwelling beam B are expressed as a function of the constant incoming and emerging flux A_0 and the optical depth τ:

$$E = A_0(1 + \tau)/2, \quad A = A_0(2 + \tau)/2, \quad B = A_0\tau/2 \qquad \text{Schwarzschild (1906, Eq.11)}$$

Emden (1913) realized that there is a discontinuity in the Planck-function at the surface, expressed by the difference of the second and first terms, $\Delta A = A - E$, implying a discontinuity between the temperature of the surface and the temperature of the lowest atmospheric layer in radiative equilibrium; but he noted in the same sentence that this discontinuity was greatly diminished by the evaporation of water and convection (this way, discovering radiative-convective equilibrium). Milne (1930) mentions that these relationships may be derived from first principles. We choose this relationship as the first equation of our model.

$$A - E = A_0/2 \qquad \text{(I)}$$

Eq. (I) is shown in the same form for planetary applications in standard atmospheric textbooks like Goody (1964, Eq. 2.115), Houghton (1977, Eq. 2.13) or Goody and Yung (1989, Eq. 2.146), and graphically represented for example in Chamberlain (1978, Fig. 1.4) or Hartmann (1994, Fig. 3.11). It evidently applies only for climate systems having an atmosphere; for the Moon and for atmosphereless planets the equality is trivially invalid. Manabe and Strickler (1964, Fig. 4) and Manabe and Wetherald (1967, Fig 5.) reproduce the size of discontinuity (quoted and re-calculated by Hartmann 1994, 2016, Fig. 3.16) under all-sky conditions, but do not utilize the equation as a constraint on its magnitude.

Hartmann (1994, pp. 61-63, Figs. 3.10-3.11) describes a two-layer radiative equilibrium model where the equation is not presented explicitly but its validity can be shows on the given data with a difference of 0.31 Wm^{-2}: the temperature of the surface is T_S = 335 K, the temperature of the air adjacent to the surface is T_{SA} = 320 K and the effective emission temperature at TOA is T_e = 255 K, hence $\sigma(T_S^4 – T_{SA}^4 – T_e^4/2) = – 0.31\ Wm^{-2}$.

For planetary applications, A is the total shortwave (SW) plus longwave (LW) energy absorbed by the surface, being equivalent (in equilibrium) with the total energy emitted by the surface (upward longwave radiation plus convection, the latter is the sum of latent heat, LH, and sensible heat, SH); E expresses the surface upward longwave radiation, and A_0 the outgoing LW radiation (OLR) at the top-of the atmosphere (TOA). The equation connects the surface net SW + LW radiation (SH + LH) to half of the outgoing longwave flux at TOA, independently of the optical depth. It is a requirement that the surface of the planet should have the opportunity to release enough latent heat to fulfill the equality, which requirement is satisfied by the open evaporating surfaces of Earth’s oceans. Since in the original logic (deduction for the Sun’s atmosphere) no clouds were considered, we regard this a clear-sky equation.

Stephens et al. (1994) use the same model, Eq. (1a) and (1b) describing the transfer equations in radiative equilibrium, with a solution in Eq. (5a) and (5b) for the upward and downward hemispheric fluxes:

$$F\uparrow(\tau_S) = \sigma T_S^4 = F_\infty (2 + \tau_S)/2 \qquad \text{(Stephens et. al 1994, 5a)}$$

$$F\downarrow(\tau_S) = F_g \quad = F_\infty \tau_S/2 \qquad \text{(Stephens et. al 1994, 5b)}$$

These relationships, with $F_0(\tau_S) = \sigma T_0^4 = F_\infty(1 + \tau_S)/2$, are equivalent with the three terms in Schwarzschild’s (1906, Eq. 11); hence $\Delta\sigma T_S^4 = \sigma T_S^4 – \sigma T_0^4 = F_\infty /2$, as in Eq. (I).

The second term in Schwarzschild’s Eq. (11) describes the total energy absorbed and emitted at the surface as a function of OLR and the optical depth. We tried the equation with several optical depth values. τ = 0 defines the upper boundary; τ = 1 gives the fluxes where OLR is generated; we found that the choice of τ = 2 at the surface satisfies the equations in all datasets. Therefore, the second equation of our model will be, still for clear-sky conditions:

$$A = 2A_0 \qquad \text{(II)}$$

A geometric representation of Eq. (II) is shown in textbooks of Hartmann (1994, Fig. 2.3), or Marshall and Plumb (2008, Fig. 2.7), where the greenhouse effect is introduced by the simplest radiative equilibrium model, consisting of a surface and a single shortwave-transparent, infrared-opaque atmospheric layer. In this model the LW radiation emitted by the surface is absorbed by the atmosphere completely and emitted upward and downward equally; hence the energy emitted by the surface is twice the emerging LW flux at TOA.

Notice that the choice of $\tau = 2$ defines the first term in Schwarzschild (1906, Eq. 11) as $E = 3A_0/2$, thus Eqs. (1) and (2) together determine a ratio of $\Delta A : A_0 : E : A = 1 : 2 : 3 : 4$; same as in Table 1 for the corresponding flux components.

As the all-sky versions of these two relationships are created by separating atmospheric radiation transfer from the longwave cloud radiative effect (LWCRE). Our third equation therefore will be the same as Eq. (I), with all-sky values in the left-hand side, all-sky A_0 on the right-hand side and L stands for LWCRE:

$$A - E = (A_0 - L)/2 \tag{III}$$

The fourth equation is created from Eq. (II), again with all-sky value in A and A_0, and adding the longwave cloud effect:

$$A = 2A_0 + L \tag{IV}$$

To compare to observed data, we write these equations in CERES notation. Let be Surface SW net = Surface (SW down – SW up); Surface LW net = Surface (LW down – LW up) and TOA LW = OLR, then

$$\text{Surface (SW net + LW net) (clear-sky)} = \text{OLR(clear-sky)} / 2 \tag{1}$$

$$\text{Surface (SW net + LW down) (clear-sky)} = 2\text{OLR(clear-sky)} \tag{2}$$

$$\text{Surface (SW net + LW net) (all-sky)} = [\text{OLR(all-sky)} - \text{LWCRE}] / 2 \tag{3}$$

$$\text{Surface (SW net + LW down) (all-sky)} = 2\text{OLR(all-sky)} + \text{LWCRE} \tag{4}$$

If there is equilibrium at the surface, the absorbed and emitted energy are equal, and the sensible heat (SH) plus latent heat (LH) flux components are equal to the net radiation at the surface, thus the equations look like:

$$\text{Surface (SH + LH) (clear-sky)} = \text{OLR(clear-sky)} / 2 \tag{1a}$$

$$\text{Surface (LW up + SH + LH) (clear-sky)} = 2\text{OLR(clear-sky)} \tag{2a}$$

Surface (SH + LH) (all-sky) = [OLR(all-sky) – LWCRE] / 2 (3a)

Surface (LW up + SH +LH) (all-sky) = 2OLR(all-sky) + LWCRE (4a)

We validate the equations on several published global mean energy flow distributions (Section 3) and show that this set of equations has a numerical solution where a part of observed integer ratio system appears (Section 4).

3. Validation of the four equations

a. Stevens and Schwartz (2012)

Stevens and Schwartz (2012, see their Table 1) give an estimate of Earth's global mean energy flow system based on observation and simulations, with the following all-sky data: Latent heat flux = 86, Sensible heat flux = 20, OLR = 239; and calling LWCRE from CERES as 26.5 Wm^{-2} the all-sky net equations (3a) and (4a) look like:

Sensible heat + latent heat = (Outgoing LW – LWCRE)/2 (3a)

20 + 86 = (239 – 26.5)/2 – 0.25 Wm^{-2} Stevens and Schwartz (2012)

Sensible heat + latent heat + Surface upward LW = 2 × Outgoing LW + LWCRE (4a)

20 + 86 + 397 = 2 × 239 + 26.5 –1.5 Wm^{-2} Stevens and Schwartz (2012)

The differences are 0.25 Wm^{-2} and 1.5 Wm^{-2}, resp.

b. NASA Energy and Water-cycle Study (L'Ecuyer et al. 2015)

This study aims to apply balance constraints on energy and water cycles since in contemporary flux datasets surface net radiation exceeds the corresponding turbulent heat fluxes by 13-24 Wm^{-2}. Based on regional assessments of the components of the hydrological cycle (evaporation, precipitation and runoff), their best estimate of the net radiation at the surface (and therefore the sum of the sensible and latent heat flux) globally is 106 Wm^{-2}, which can be regarded as the most accurate estimate. The assessment is constrained only to all-sky conditions. Since longwave cloud effect (LWCRE) is not indicated, we took it from the L'Ecuyer et al. (2019) [27.1 Wm^{-2} at TOA and 26.3 Wm^{-2} at the surface] as the mean value of 26.7 Wm^{-2} (same as in Stephens et al. 2012):

$$\text{SFC (SW net + LW net)} = \text{SH} + \text{LH} = [\text{OLR} - \text{LWCRE}]/2 \quad \text{(3a)}$$

$$164 - 58 = 25 + 81 = (238 - 26.7)/2 + 0.35 \quad \text{L'Ecuyer et al.}$$

$$\text{SFC (SW net + LW down)} = \text{ULW} + \text{SH} + \text{LH} = 2\text{OLR} + \text{LWCRE} \quad \text{(4a)}$$

$$164 + 341 = 399 + 25 + 81 = 2 \times 238 + 26.7 + 2.3 \quad \text{L'Ecuyer et al.}$$

The difference in the net all-sky equation (3a) is only 0.35 Wm^{-2}, less than the indicated Net Absorption at the surface, 0.45 ± 0.4 Wm^{-2}. The noted uncertainty in the sensible heat flux (25 Wm^{-2}) and latent heat flux (evaporation) (81 Wm^{-2}) are ± 4 Wm^{-2}. The mean bias of the two all-sky equation is 1.33 Wm^{-2}.

c. Stephens and L'Ecuyer (2015)

Stephens and L'Ecuyer (2015) in an update to L'Ecuyer et al. (2015) provided a second optimization where the TOA fluxes are more tightly constrained to CERES EBAF fluxes. OLR and downward longwave radiation (DLR) have been increased, and, as a result, turbulent fluxes become 108 Wm^{-2} (with SH = 26 Wm^{-2}, evaporation = 82 Wm^{-2}); with the accuracy of Eq. (3a) as 1.35 Wm^{-2} and of Eq. (4a) as 0.3 Wm^{-2}; the mean bias of Eqs. (3a) and (4a) has decreased to 0.82 Wm^{-2}.

$$\text{SFC (SW net + LW net)} = \text{SH} + \text{LH} = (\text{OLR} - \text{LWCRE})/2 \quad \text{(3a)}$$

$$163 - 55 = 26 + 82 = (240 - 26.7)/2 + 1.35 \quad \text{St \& L'E}$$

$$\text{SFC (SW net + LW down)} = \text{ULW} + \text{SH} + \text{LH} = 2\text{OLR} + \text{LWCRE} \quad \text{(4a)}$$

$$163 + 344 = 399 + 26 + 82 = 2 \times 240 + 26.7 + 0.3 \quad \text{St \& L'E}$$

d. Hartmann (2016)

Hartmann (2016) has the following values (data from its Fig. 2.4, LWCRE = 26 Wm^{-2} from its Table 3.2):

$$\text{Thermals} + \text{Latent heat} = (\text{OLR} - \text{LWCRE}) / 2 \quad \text{(3a)}$$

$$20 + 88 = (239 - 26)/2 + 1.5 \quad \text{Hartmann}$$

$$\text{Thermals} + \text{Latent heat} + \text{IR emission from surface} = 2\text{OLR} + \text{LWCRE} \qquad (4a)$$

$$20 + 88 + 396 = 2 \times 239 + 26 \qquad \text{Hartmann}$$

Eq. (3a) is valid with a difference of 1.5 Wm^{-2}; Eq. (4a) has zero difference.

e. CERES EBAF Edition 2.8

At the time this study has started, the best satellite-based data product was CERES EBAF Edition 2.8, spanning over 16 years (from March 2000 to February 2016). The accuracy of Eq. (1) is 0.60 Wm^{-2}, with the immediate consequence of connecting surface net radiation unequivocally to half of the outgoing TOA LW radiation in the clear-sky. The estimated heat uptake of Earth in that time was the same, 0.58±0.38 Wm^{-2} (Loeb et al. 2012). The same accuracy of Eq. (2), 0.59 Wm^{-2} is a strong verification of the choice of $\tau = 2$. Bias of the all-sky equations (2.4 Wm^{-2} and 2.3 Wm^{-2}) is half the magnitude of the CERES instrument calibration uncertainty of 4.2 Wm^{-2} or of the standard CERES TOA net flux of 6.5 Wm^{-2} (Loeb et al. 2009):

$$\text{SFC (SW net + LW net) (clear)} = \text{OLR (clear)}/\,2 \qquad (1)$$

$$214.32 + 316.26 - 398.40 = 265.59/2 - 0.60 \qquad \text{Ed2.8}$$

$$\text{SFC (SW net + LW down) (clear)} = 2\text{OLR (clear)} \qquad (2)$$

$$214.32 + 316.26 = 2 \times 265.59 - 0.59 \qquad \text{Ed2.8}$$

$$\text{SFC (SW net + LW net) (all)} = (\text{OLR(all)} - \text{LWCRE})\,/\,2 \qquad (3)$$

$$162.34 + 345.15 - 398.27 = (239.59 - 25.99)/2 + 2.42 \qquad \text{Ed2.8}$$

$$\text{SFC (SW net + LW down) (all)} = 2\text{OLR(all)} + \text{LWCRE} \qquad (4)$$

$$162.34 + 345.15 = 2 \times 239.59 + 25.99 + 2.30 \qquad \text{Ed2.8}$$

f. CERES EBAF Edition 4.1

Using this data product on the full available time range (22 running years from April 2000 through March 2022; 264 monthly means), the mean bias of the four equations together is 0.0007 Wm^{-2}, indiscernible from zero in the netCDF file.

SFC (SW down – SW up + LW down – LW up) = OLR (clear)/2 (1)

240.8680 – 29.0724 + 317.4049 – 398.5211 = 266.0122 /2 – 2.3267 Ed4.1

SFC (SW down – SW up + LW down) = 2OLR (clear) (2)

240.8680 – 29.0724 + 317.4049 = 2 × 266.0122 –2.8238 Ed4.1

SFC (SW down – SW up + LW down – LW up) = [OLR (all) – LWCRE] /2 (3)

186.8544 – 23.1629 + 345.0108 – 398.7550 = (240.2450 – 25.7671)/2 +2.7083 Ed4.1

SFC (SW down – SW up + LW down) = 2OLR (all) + LWCRE (4)

186.8544 – 23.1629 + 345.0108 = 2 × 240.2450 + 25.7671 + 2.4450 Ed4.1

Mean bias: 0.0007 Wm^{-2}.

g. GEWEX (Stephens et al. 2023)

Sensible heat + Evaporation = (OLR – LWCRE) / 2 (3a)

25.4 + 81.1 = (239.5 – 26.7)/2 + 0.1 GEWEX

Surface (SW net + LW down) = 2OLR + LWCRE (4)

160.7 + 345.1 = 2 × 239.5 + 26.7 + 0.1 GEWEX

This quantification is based on 30 years of up-to-date GEWEX data products (Stephens et al. 2023 Fig. SB3, reproduced in our Fig. 4). Since neither clear-sky data nor cloud effects are indicated, we take the LWCRE again from the study of L'Ecuyer et al. (2019) with a mean value of 26.7 Wm^{-2} as above. Both of the all-sky equations are valid with a difference of 0.1 Wm^{-2}.

4. Solution to the equations

a. The integer system

The geometric representation of $A = 2A_0$ is given in Fig. 5 (based on Hartmann 1994, Fig. 2.3).

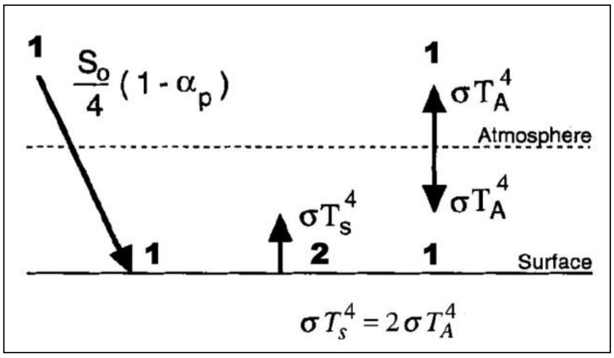

Figure 5 The simplest greenhouse model representing Eq. (2), $A = 2A_0$, in radiative equilibrium. The ratios are indicated. After Hartmann (1994, Fig. 2.3)

Improving the model by allowing convective fluxes at the surface according to Eq. (1), $A - E = \Delta A = A_0/2$ and $E = 3A_0/2$, the model will become as shown in Fig. 6:

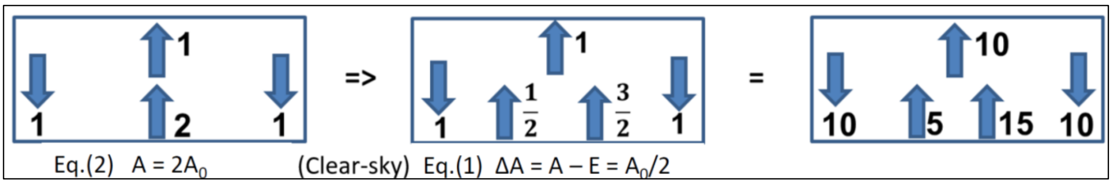

Figure 6 Left, the geometry according to Eq. (2); Middle: including the net radiation at the surface according to Eq. (1) and (2). The right panel is equivalent to the middle, after multiplying the unit by ten

In radiative equilibrium, $\Delta A = A_0/2$ is discontinuity at the surface. In radiative-convective equilibrium, $\Delta A = A_0/2$ is the convective flux. Since the unit is not yet specified, as a preparation, we multiply it by ten.

Including the all-sky Eqs. (3) and (4) and choosing the longwave cloud radiative effect as the unit: LWCRE = **1** (in the diagrams shown in red bold typeface), we have a geometric solution for the four equations, see Fig. 7.

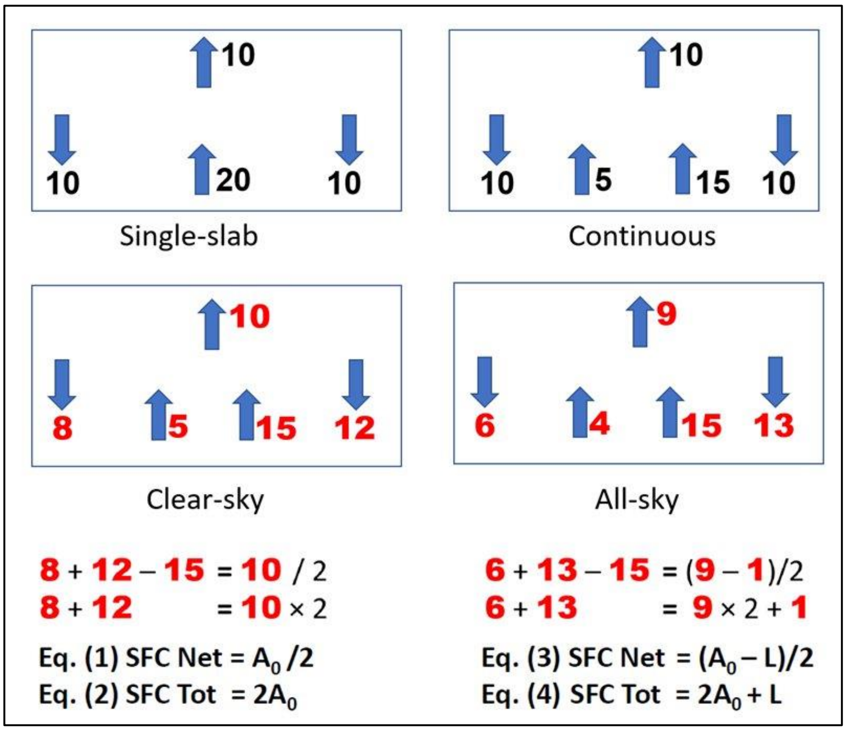

Figure 7 Idealized geometric equilibrium representation of the four equations with integers. No reference to atmospheric gaseous composition was made. Surface fluxes are unequivocally connected to the TOA fluxes. Numbers shown in red bold typeface are integer multiples of L = LWCRE = **1**

5. Conclusions

Integer ratios and specific arithmetic relationships may be recognized in observation-based global mean energy flow distributions published in the last decade, with the unit flux being equal to the longwave cloud radiative effect. The integer multiple system appears both for TOA, in-atmosphere and surface fluxes, for shortwave and longwave radiations, even for the non-radiative flux components, both under clear-sky and all-sky conditions. The observed flux values are close to the integer positions typically within the stated range of uncertainty. This is a common feature of global mean flux estimates coming from different sources (satellite-based observations, climate models or hydrological cycle assessments).

Our primary task in this study was to call attention to the integer structure and present it on all the available data sources. We also attempted to give a theoretical explanation, which was found in Schwarzschild’s (1906) original two-stream equations for radiation transfer. A part of the global mean flux components was derived from four specific versions of the equations: a pair for the net radiation and another for the total absorbed (and emitted) energy at the surface, in for clear-sky and all-sky conditions. The net equations are independent of the optical thickness; the total energy equations are taken at $\tau = 2$. Each equation connects energy at the surface to TOA LW radiation. On the completed, archived satellite-based CERES EBAF Edition 4.1 data product (full 22 years from April 2000 through March 2022) the difference of the individual equations is $\pm$ 3 Wm^{-2}, less than the stated range of CERES data uncertainty; the mean bias of the four equations is 0.0007 Wm^{-2}.

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