

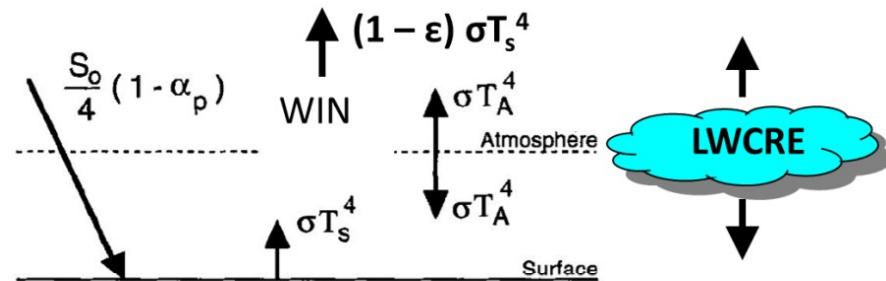
Section 4. Equation (5) and Clouds

- Our Eqs (1) and (3) were validated with high accuracy for clear-skies.
- The original equations of Schwarzschild were given for the Sun's atmosphere, where "clear-sky" is evidently meaningless.
- The textbooks for planetary applications, like Goody, or Houghton, use Φ_s on the right-hand side, noting that it equals both the incoming solar and the outgoing terrestrial radiation.
- This latter in Earth's conditions holds in the all-sky.
- But we used these equations for the clear-sky, where a large imbalance exists between the absorbed solar and the emitted thermal energy.
- It is not evident why the equations are valid with OLR, instead of ASR?



More about the single-slab

- In the single-slab model, $ASR = OLR$, and $\pi B_g = 2ASR = 2OLR$
- But since WIN is lost, and only LWCRE is gained back,
- Surface LW+SW gross absorption cannot be equal to $2ASR$, but only **$2ASR - WIN + LWCRE = 2OLR$**
- That's why in the clear-sky gross equation we have $\pi B_g = \mathbf{2OLR}$, instead of $2ASR$.
- In the all-sky, the 'leak' is closed by LWCRE:
- $ASR(\text{all}) = OLR(\text{all})$ and $WIN(\text{all}) = LWCRE$.
- **But how much is WIN(clear) in theory?**



Eq. (5) $2ASR = 2OLR + WIN - LWCRE$

- How much is $WIN(\text{clear})$? As we have seen, Costa and Shine (2012) found $WIN(\text{clear}) = 65 \text{ Wm}^{-2}$ for their model- $OLR = 259 \text{ Wm}^{-2}$.
- Assuming proportionality, for our $OLR = 266.80 \text{ Wm}^{-2}$ the corresponding $WIN(\text{clear})$ is 66.96 Wm^{-2} .
- The **N**-position in the integer system is 66.70 Wm^{-2} .
- That is, $WIN(\text{clear}) = OLR(\text{clear})/4 = \mathbf{10}/4$ units.



Eq. (5) $2ASR = 2OLR + WIN - LWCRE$

All-sky: ASR all = OLR all = **9**; WIN all = LWCRE = **1**

Clear-sky: OLR clear = **10**; WIN clear = **10/4 = 2.5** =>

Clear-sky: ASR clear = **10.75**; **Clear-sky TOA net = 3/4**

This was for the spherical surface of Earth.

For the intercepting cross-section disk (x4):

TSI = **51**, **Clear-sky:** ASR = **43**, OLR = **40**, WIN = **10**,

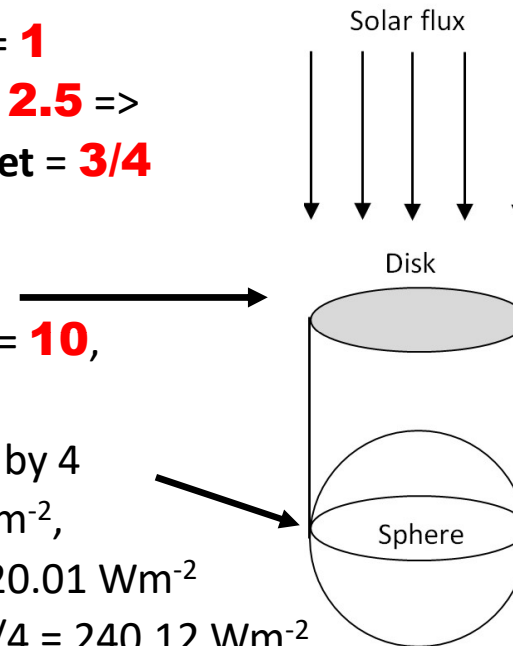
Reflected (RSR) = **8**, **Clear-sky TOA net = 3**.

Substitute TSI = **51** = 1360.68 Wm^{-2} and divide by 4

RSR = **2** = 53.36 Wm^{-2} , ASR = **43/4** = 286.81 Wm^{-2} ,

OLR = **10** = 266.80 Wm^{-2} ; Clear-sky TOA net = 20.01 Wm^{-2}

All-sky: TSI = **51**, RSR = **15/4**, ASR = OLR = **36/4** = 240.12 Wm^{-2}



It seems that the energetic role of clouds in the LW is to close the open atmospheric window. The radiative energy being lost in the window is gained back by the greenhouse effect of clouds. This interplay is expressed by Eq. (5).



Effective cloud area fraction

- As we know that $WIN(\text{all}) = LWCRE = 1$ and $WIN(\text{clear}) = 10/4$, we can construct a single IR-opaque effective cloud area fraction:
- $WIN(\text{all}) = (1 - \beta_{\text{eff}}) \times WIN(\text{clear}) \Rightarrow$
- $\beta_{\text{eff}} = 3/5 = 0.60$.
- Not unrealistic, since $\beta_{\text{obs}} = 0.67$.



Total LWCRE

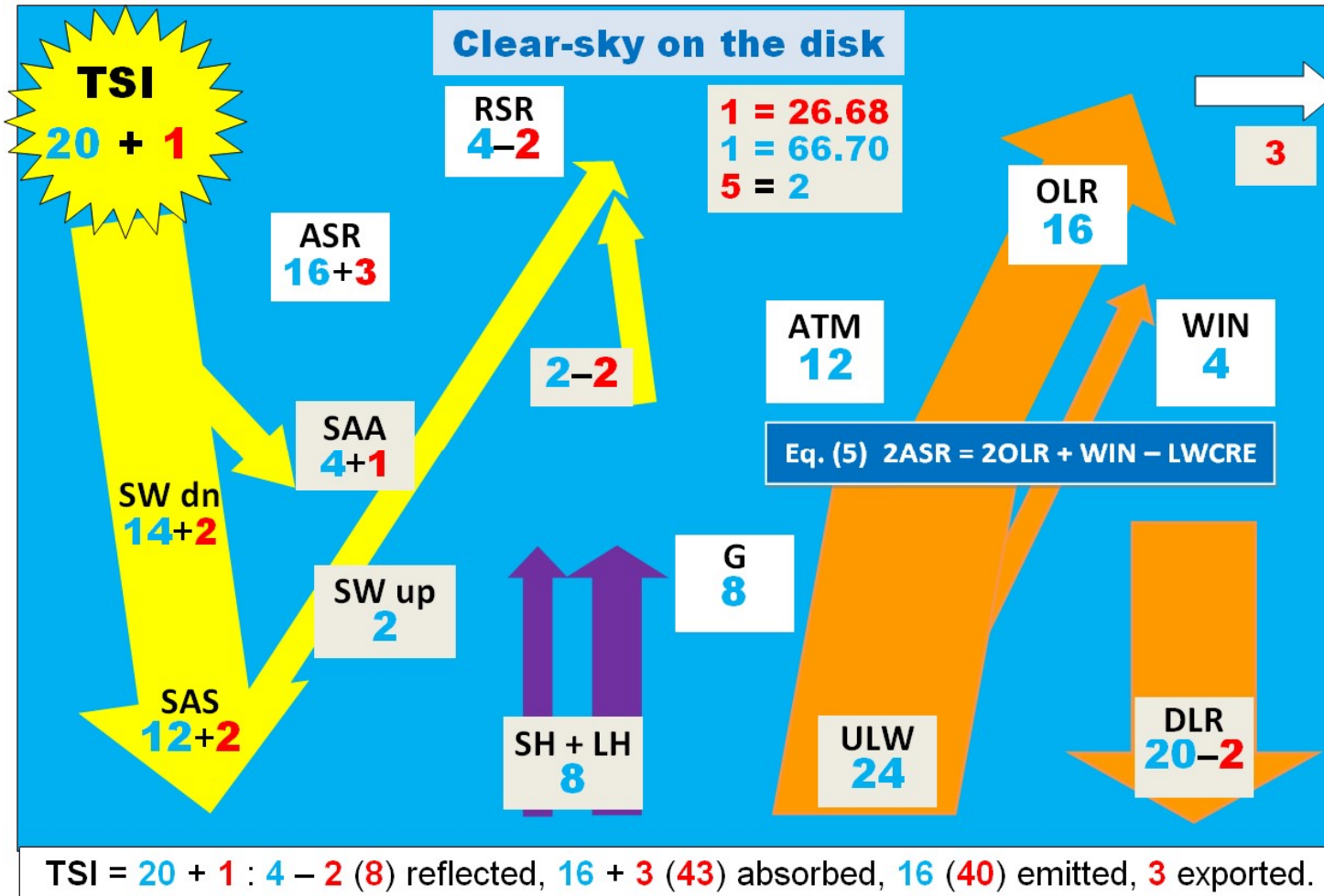
- This way, it is easy to define the “total” longwave cloud radiative effect as
- $LWCRE_T = LWCRE / \beta_{\text{eff}} = 26.68/0.6 = 44.466667 \text{ Wm}^{-2}$
- Evidently,
- $LWCRE_T = OLR(\text{clear}) - OLR(\text{cloudy})$
- Therefore $OLR(\text{cloudy}) = 222.33 \text{ Wm}^{-2}$.
- Further, $DLR(\text{cloudy}) = DLR(\text{clear}) + LWCRE_T = 364.63 \text{ Wm}^{-2}$, and
- $G(\text{cloudy}) = G(\text{clear}) + LWCRE_T = ULW - OLR(\text{cloudy}) = 177.867 \text{ Wm}^{-2}$.
- Now we are ready to construct an idealized global mean cloudy energy flow system with $\beta_{\text{eff}} = 0.6$

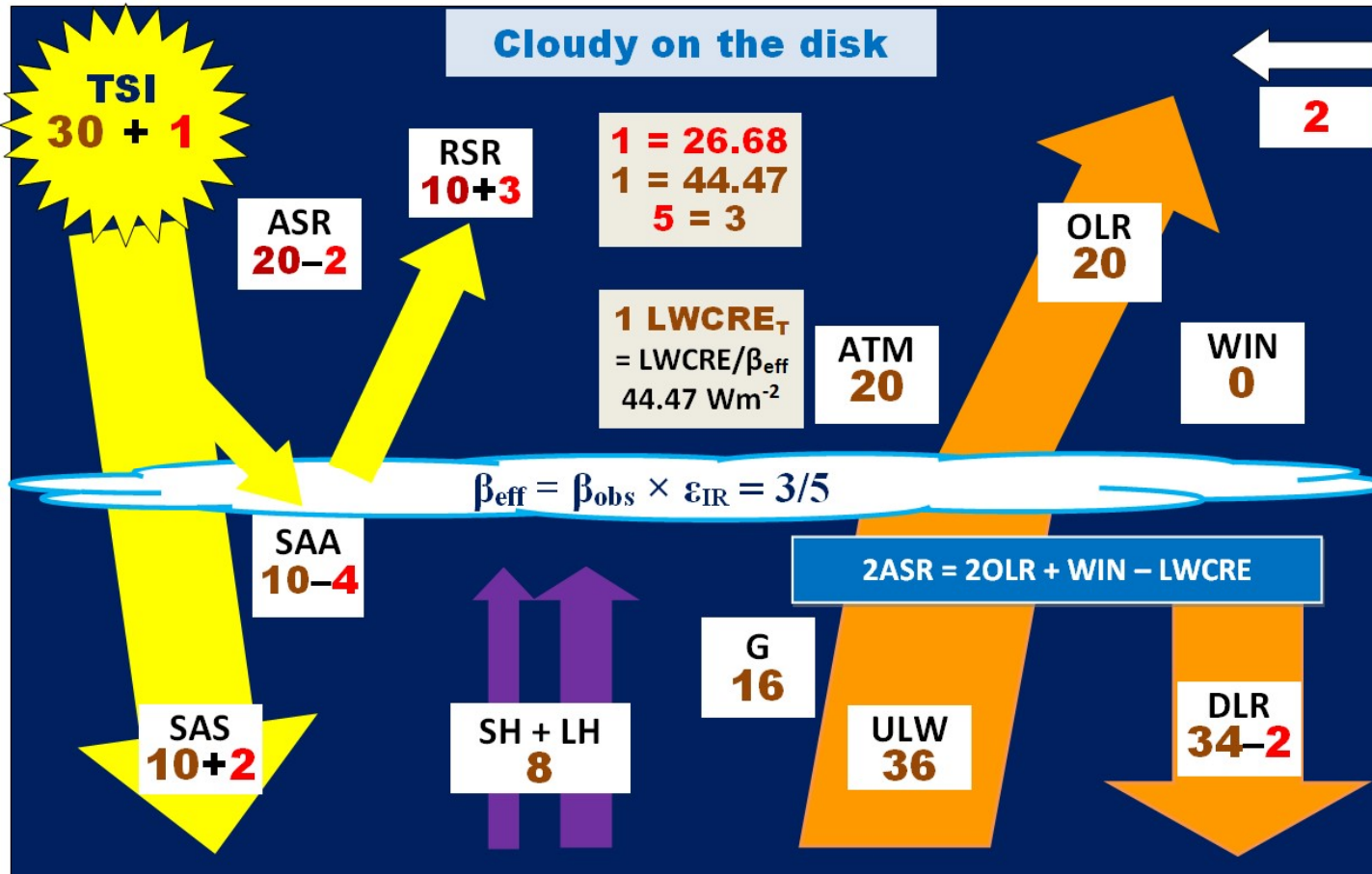


Clouds: The real single-slab model

- The general weighting equation, connecting the fluxes is
- $F(\text{all}) = \beta_{\text{eff}} \times F(\text{cloudy}) + (1 - \beta_{\text{eff}}) \times F(\text{clear})$.
- We know the all-sky and the clear-sky fluxes from the equations, we use them to construct the cloudy fluxes. *E.g.*,
- $\text{DLR}(\text{cloudy}) = 364.63 \text{ Wm}^{-2} = \text{DLR}(\text{clear}) + \text{LWCRE}_{\text{T}}$, as ought.
- Since there is a **3**/4 unit = 20.01 Wm^{-2} surplus in the clear-sky part, this energy is transported to the cloudy area, weighted by the clear/cloudy area ratio of 2/3.
- That is, an energy of **1**/2 unit = 13.34 Wm^{-2} is to be added.







1 cloudy unit =
 $\text{LWCRE}_T =$
 $44.466667 \text{ Wm}^{-2}$

1 all-sky unit =
 $\text{LWCRE} =$
 26.68 Wm^{-2}

$\text{TSI} = 1360.68 \text{ Wm}^{-2}$
 $30 + 1 = 50 + 1$

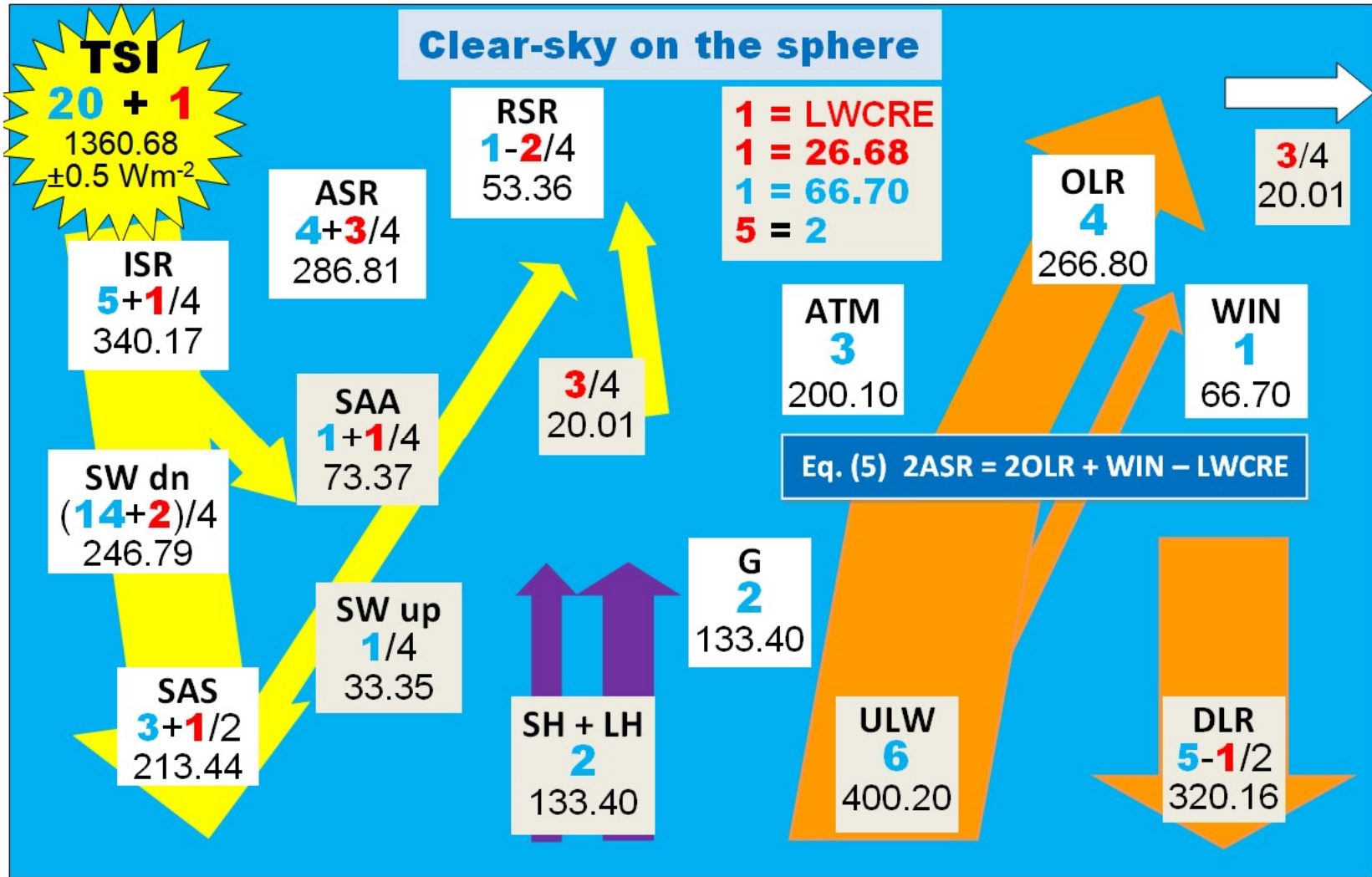
Equations:

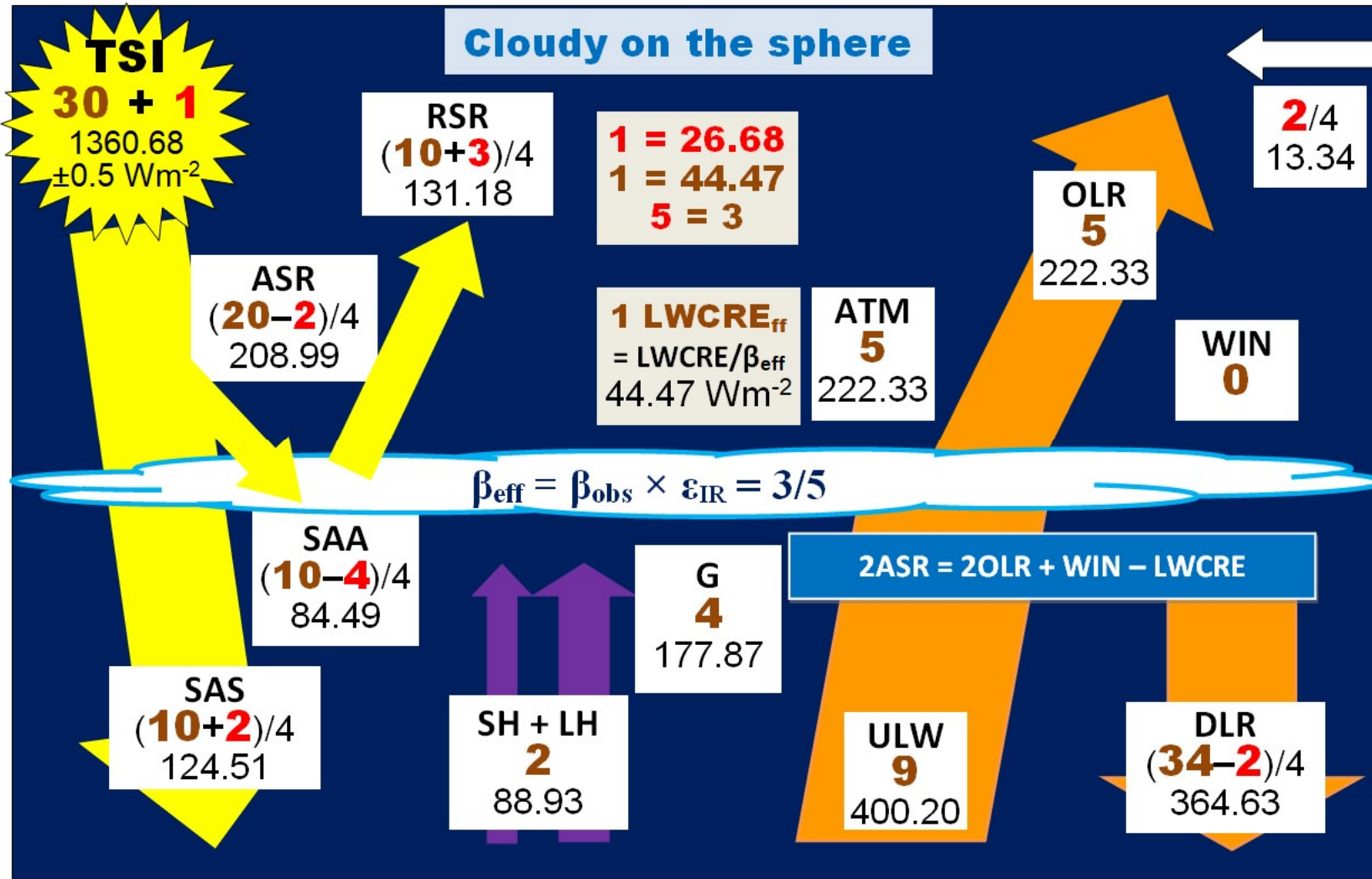
SFC net =
 $(\text{OLR} - \text{LWCRE}_T) / 2$

SFC gross =
 $2\text{OLR} + \text{LWCRE}_T$

TSI = 30 + 1; 10 + 3 reflected, 20 - 2 absorbed, 20 emitted, 2 imported.



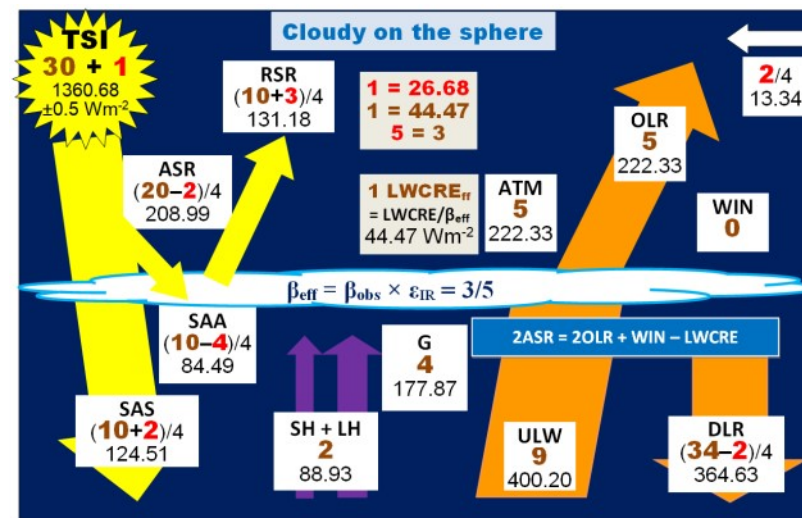
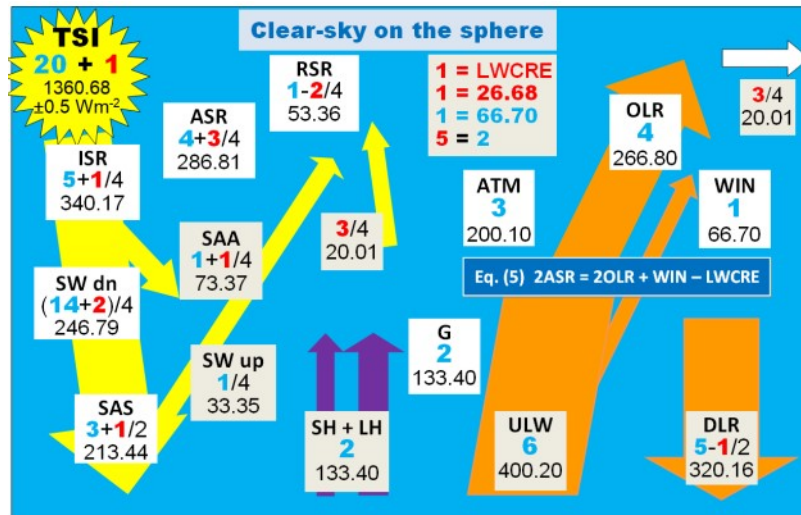




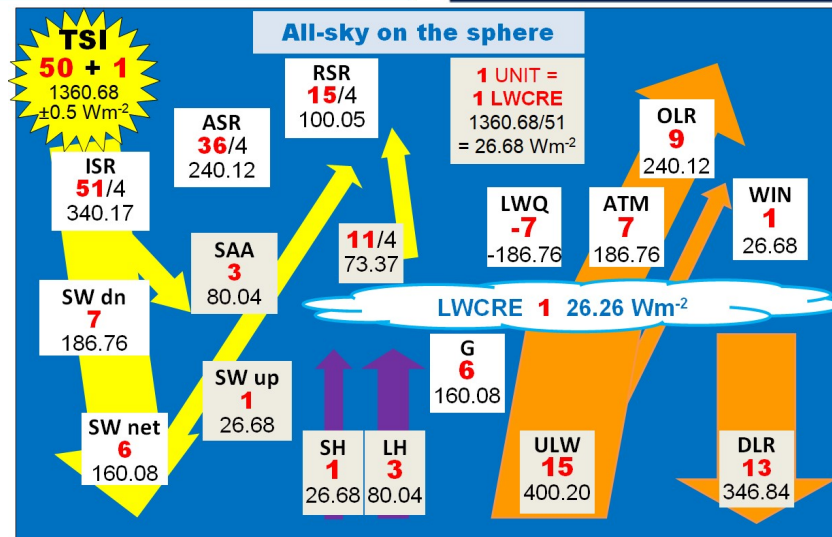
Cloudy on the sphere

- The numbers in the cloudy equations look like this:
- SFC net = **2** = $(\text{OLR} - \text{LWCRE}_T)/2 = (\mathbf{5} - \mathbf{1})/2$
- SFC gross = **9** + **2** = $2\text{OLR} + \text{LWCRE}_T = 2 \times \mathbf{5} + \mathbf{1}$
- Their form is the “all-sky” form, including LWCRE_T in the place of LWCRE.
- The all-sky system is now can be created as the weighted sum of the clear-sky and cloudy fluxes:





OLR(all) =
 $4 \times 2/5 + 5 \times 3/5 = 9$
 ULW =
 $6 \times 2/5 + 9 \times 3/5 = 15$
 SH+LH(all) =
 $2 \times 2/5 + 2 \times 3/5 = 4$
 G(all) =
 $2 \times 2/5 + 4 \times 3/5 = 6$



TSI = 1360.68 Wm^{-2}
 =
 $20 + 1$
 =
 $30 + 1$
 =
 $50 + 1$



Eq. (5) Cloudy numerical check

- $2ASR(\text{cloudy}) = 2OLR(\text{cloudy}) + WIN - LWCRE$
- $WIN = 0 \Rightarrow ASR(\text{cloudy}) = OLR(\text{cloudy}) - LWCRE/2 = 222.33 - 13.34 = 208.99 \text{ Wm}^{-2}$
- $ASR(\text{all}) = \beta_{\text{eff}} \times ASR(\text{cloudy}) + (1 - \beta_{\text{eff}}) \times ASR(\text{clear})$
- $240.12 = 0.6 \times 208.99 + 0.4 \times 286.81$
- $TOA \text{ net IMB (clear)} = ASR(\text{clear}) - OLR(\text{clear}) = 20.01 \text{ Wm}^{-2}$
- $TOA \text{ net IMB (cloudy)} = ASR(\text{cloudy}) - OLR(\text{cloudy}) = -13.34 \text{ Wm}^{-2}$
- $20.01 \times 0.4/0.6 = 13.34 \text{ Wm}^{-2}$



Equations

1. SFC net, clear = $OLR/2$
 2. SFC net, all = $(OLR - LWCRE)/2$
 3. SFC gross, clear = $2OLR$
 4. SFC gross, all = $2OLR + LWCRE$
 - 2 (cloudy) SFC net, cloudy = $(OLR - LWCRE_T)/2$
 - 4 (cloudy) SFC gross, cloudy = $2OLR + LWCRE_T$
 5. $2ASR = 2OLR + WIN - LWCRE$
- $TSI = 20 + 1 = 30 + 1 = 50 + 1$
 $LWCRE = \text{All-sky unit} = 1$
 $LWCRE_{Total} = \text{Cloudy unit} = 1$
 $WIN(\text{clear}) = \text{Clear-sky unit} = 1$
 $1 = 26.68 \text{ Wm}^{-2}$
 $1 = 44.47 \text{ Wm}^{-2}$
 $1 = 66.70 \text{ Wm}^{-2}$
- $2 = 4 / 2; \quad 3 = 6 / 2; \quad 5 = 10 / 2;$
 $8 = 4 \times 2; \quad 12 = 6 \times 2; \quad 20 = 10 \times 2$
 $2 = (5 - 1)/2; \quad 2 + 9 = 11 = 5 \times 2 + 1$
 $4 = (9 - 1)/2; \quad 4 + 15 = 19 = 9 \times 2 + 1$

Essential flux ratios

from the solution of Eqs. (1)-(4)

$$ULW (\text{clear} = \text{cloudy} = \text{all}) = 6, 9, 15$$

$$OLR \text{ clear; cloudy; all} = 4, 5, 9$$

$$G \text{ clear; cloudy; all} = 2, 4, 6$$

$$OLR \text{ clear} = 4 = 6 = 10$$

$$G \text{ clear} = 2 = 3 = 5$$

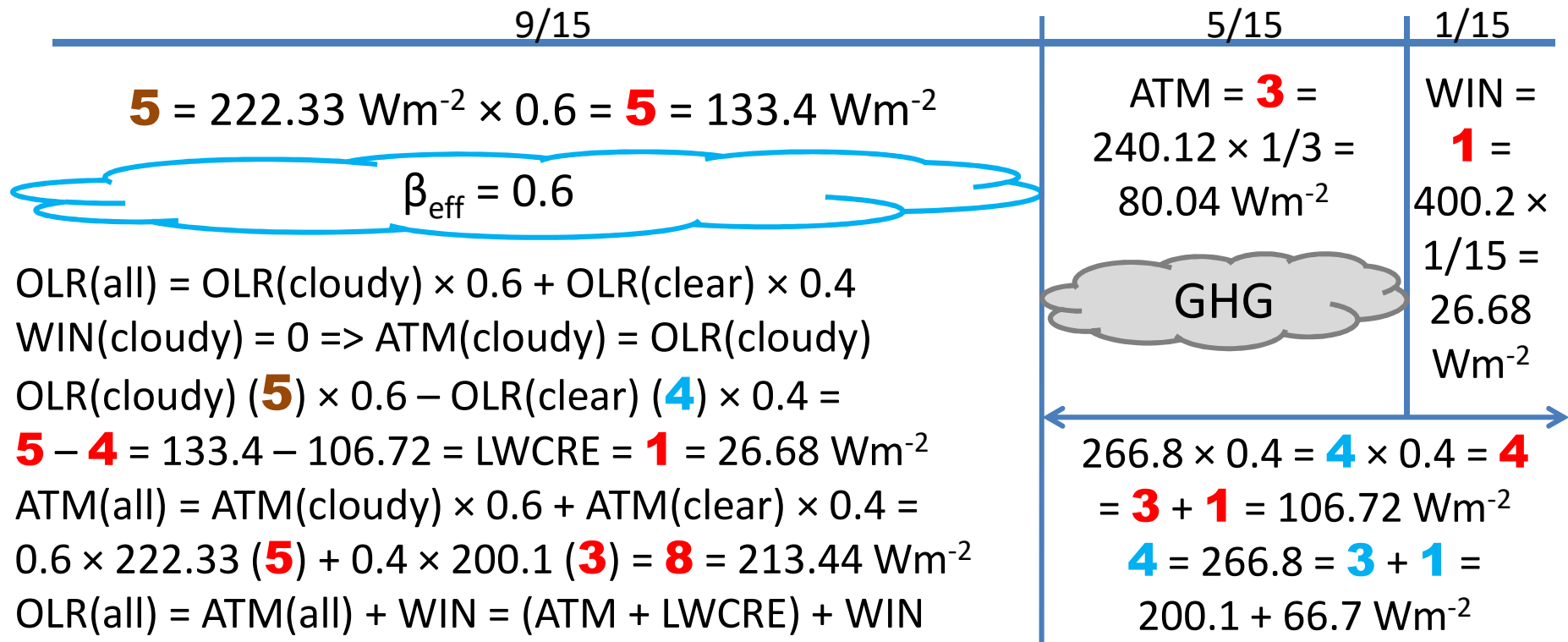
$$g (\text{all}) = (15 - 9) / 15 = 0.4$$

$$\begin{aligned}
 g(\text{clear}) &= (15 - 10) / 15 = \\
 &= (9 - 6) / 9 = \\
 &= (6 - 4) / 6 = 1/3
 \end{aligned}$$

$$g (\text{cloudy}) = (9 - 5) / 9 = 4/9$$



$$\text{OLR}(\text{all-sky}) = \mathbf{9} = 240.12 \text{ Wm}^{-2}$$



$$\text{ULW} = \mathbf{15} = 400.2 \text{ Wm}^{-2}$$

