EBAF Ed4.2.1 Data vs. Theory

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Budapest, Hungary

Spring CERES Science Team Meeting

May 13-15, 2025, NASA LaRC, Hampton, VA.

Remote Presentation

Theory

- „*Globally averaged, the surface has a net surplus of radiant energy* (…)" (ceres.larc.nasa.gov/science)
- This is an observation-based statement.
- But also a theoretical assertion.
- In the literature, there is an expectation on that.
- I review the literature and check the validity of that expectation on CERES data

Liou (2002)

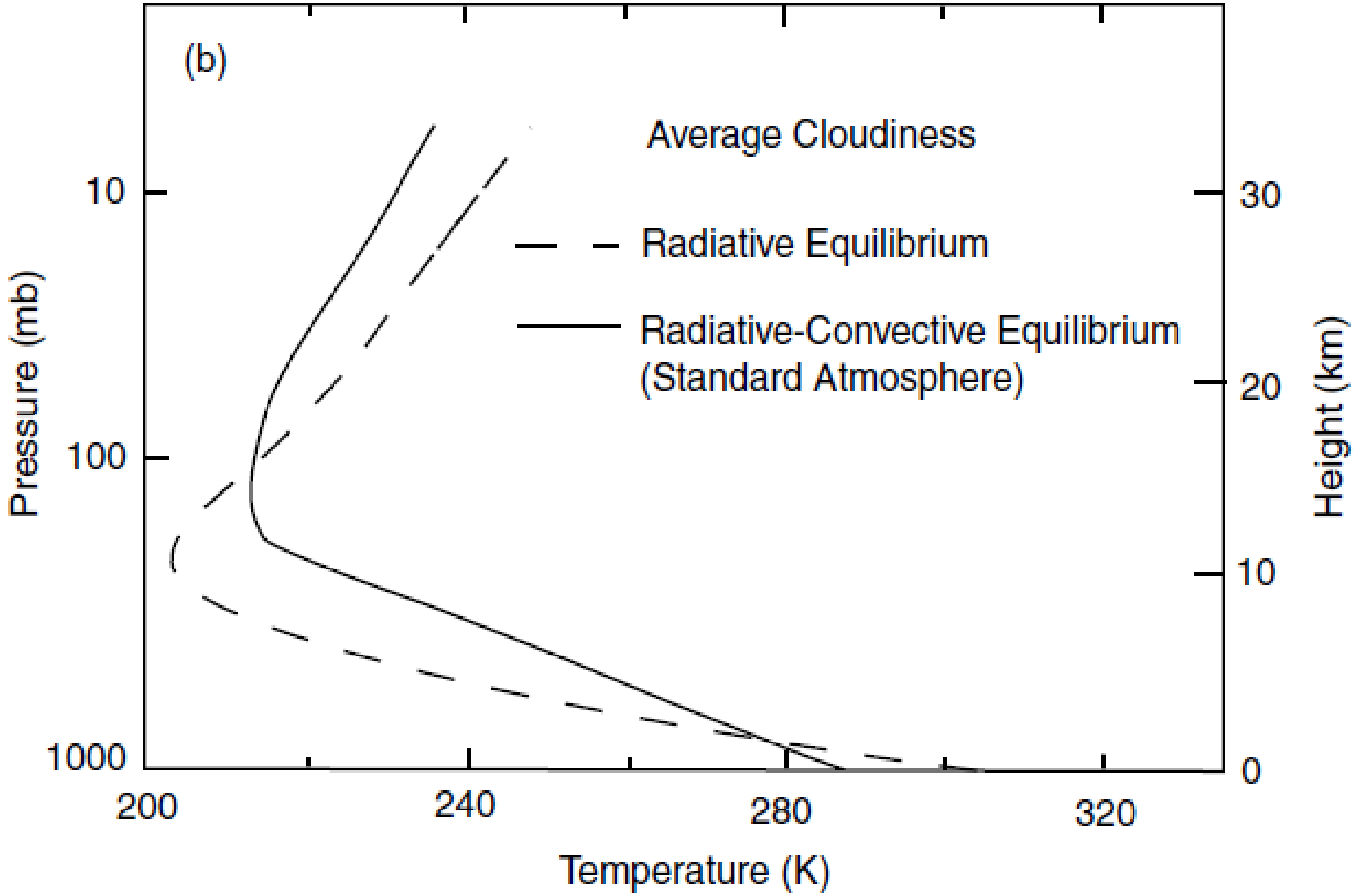


Figure 8.9 Vertical distributions of radiative and radiative–convective equilibrium temperatures in clear (a) and average cloud (b) conditions, simulated from a one-dimensional radiative–convective climate model.

Hartmann (1994)

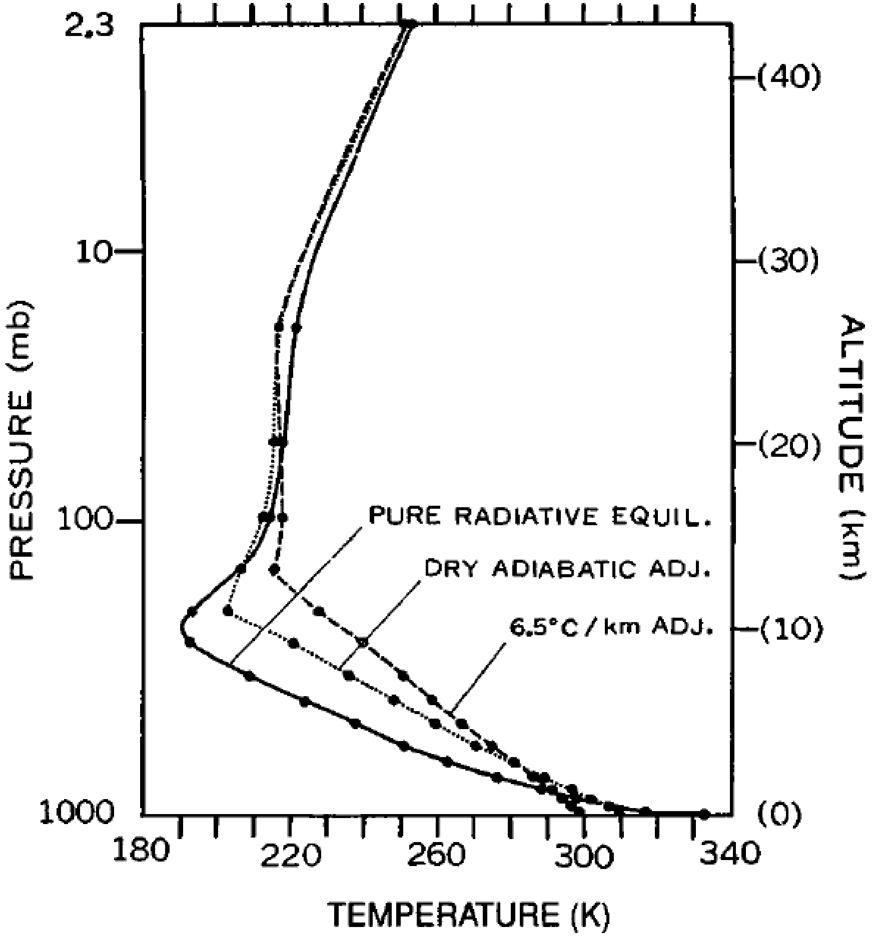


Fig. 3.16 Calculated temperature profiles for radiative equilibrium, and thermal equilibrium with lapse rates of 9.8°C km^{-1} and 6.5°C km^{-1}. [From Manabe and Strickler (1964). Reprinted with permission from the American Meteorological Society.]

Hartmann has an equation for that

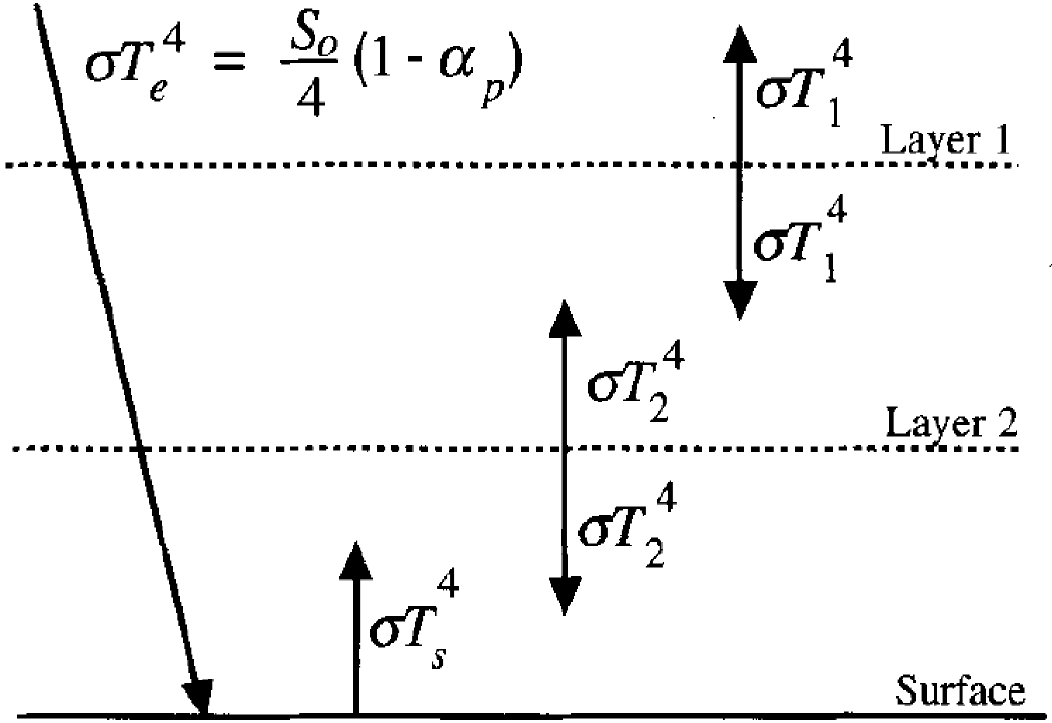


Fig. 3.10

$$\sigma T_2^4 = 2\sigma T_1^4 \quad (3.48)$$

$$\sigma T_1^4 + \sigma T_s^4 = 2\sigma T_2^4 \quad (3.49)$$

$$\frac{S_0}{4}\left(1-\alpha_p\right) + \sigma T_2^4 = \sigma T_s^4 \quad (3.50)$$

$$\varepsilon\sigma T_s^4 + \varepsilon\sigma T_2^4 = 2\varepsilon\sigma T_{SA}^4 \quad (3.54)$$

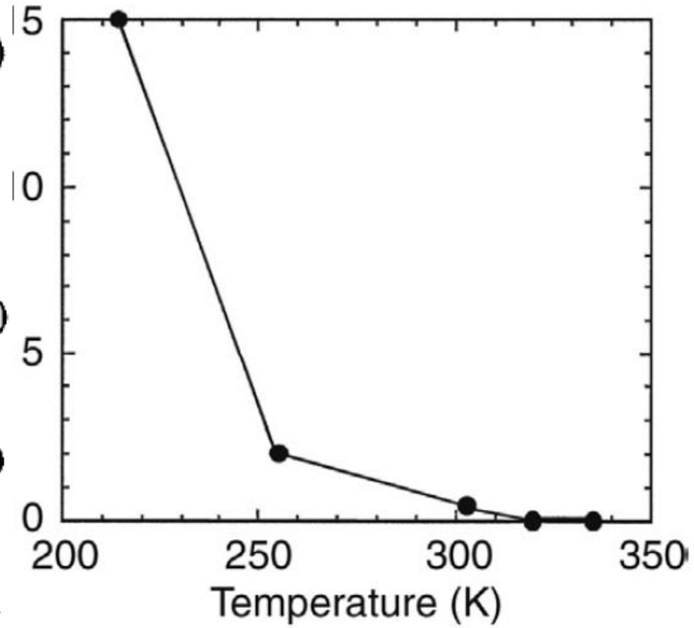


Fig. 3.11

$$\Rightarrow \boldsymbol{\sigma T_s^4 - \sigma T_{SA}^4 = \sigma T_e^4/2}$$

Emission temperature T_e = 255 K
Air adjacent to surface T_{SA} = 320 K
Surface temperature T_S = 335 K

$\boldsymbol{\sigma T_s^4}$	$-$ $\boldsymbol{\sigma T_{SA}^4}$		$= \boldsymbol{\sigma T_e^4/2}$
$5.67\,[(3.35)^4 - (3.20)^4]$			$= 5.67(2.55)^4/2$
714.11	$-$ 594.54	= 119.56	= 119.87 – **0.31**

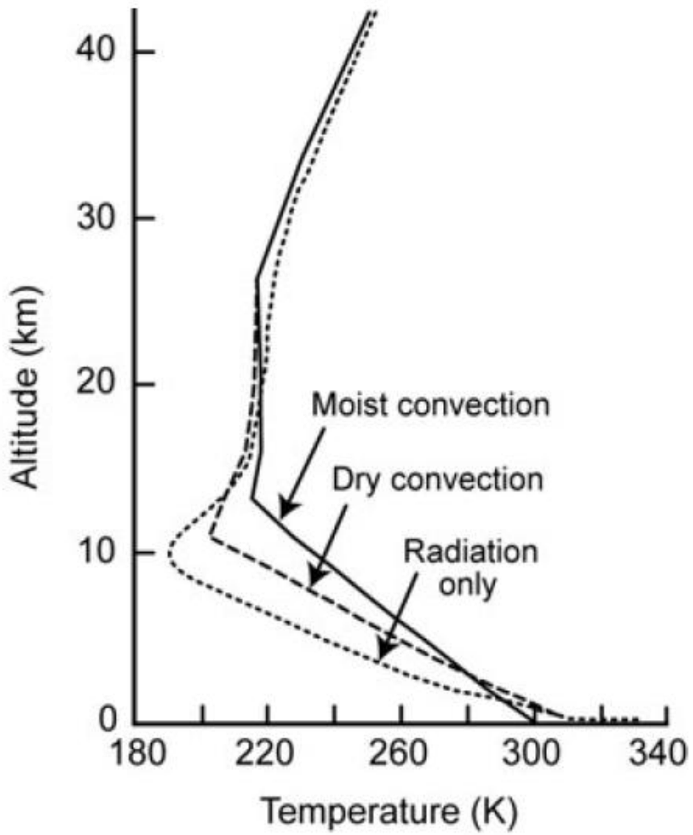


FIG. 2.11. Representative vertical temperature profiles calculated assuming (top curve) convective equilibrium in a moist atmosphere, (middle) convective equilibrium in a dry atmosphere, and (bottom) radiative equilibrium. (See Manabe and Wetherald 1967)

Vardavas and Taylor:
Radiation and Climate
Oxford (2007) Fig. 2.11

„An atmosphere in radiative equilibrium (see Fig. 2.11) produces essentially a discontinuity (of about 20 K) between the Earth's surface temperature and the near–surface atmospheric temperature."

„Thus the mean intensity can be expressed as"

$$J = I^{+} - f/2\pi.$$

Graeme L. Stephens: Radiative Transfer Notes AT 622.

Colorado State University, Fort Collins (1992-2013)

Example 6.3: Skin temperatures and temperature discontinuities

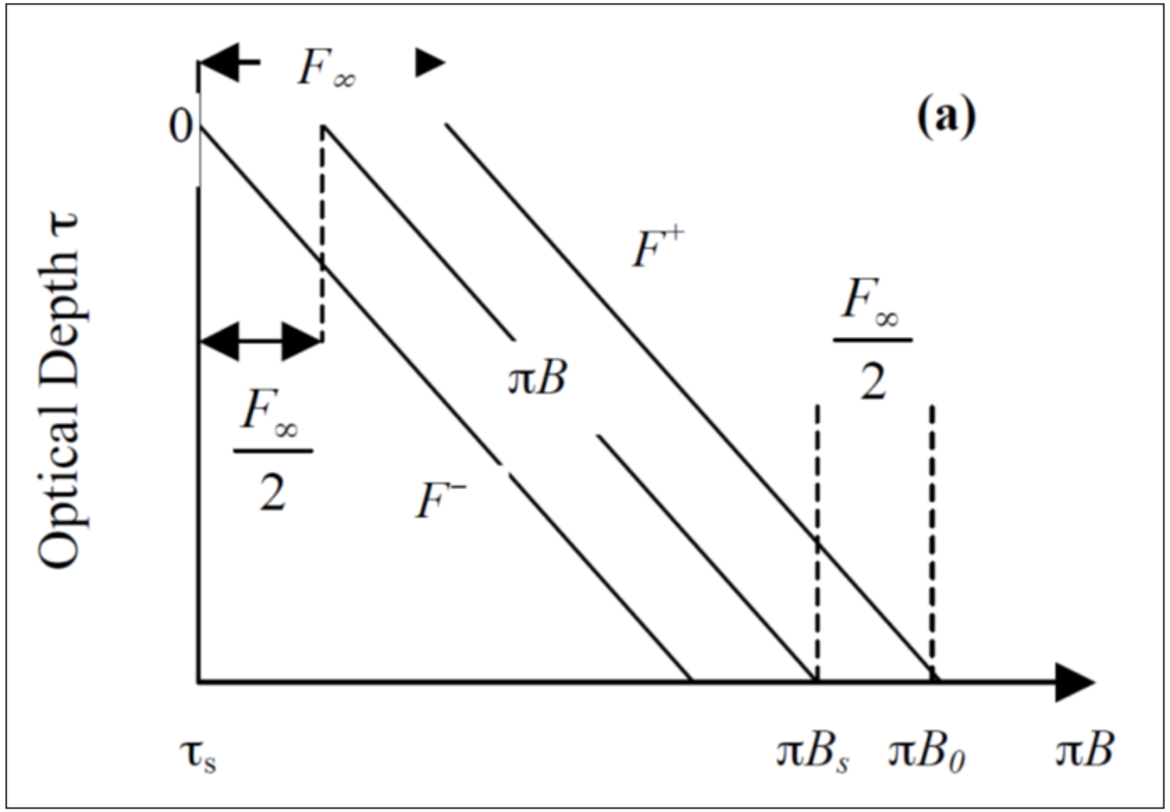
The solutions represented by Eqns. (6.10a) and (6.10b) provide rather interesting insights into the temperature profiles that are predicted by these equations. One of the results of this model is an estimate of the 'skin' temperature, which we think of as a measure of the stratospheric temperature. We obtain this using Eqn. (6.10a) with $\tilde{\tau} = 0$

$$\sigma T^4(\tilde{\tau} = 0) = \frac{F_\infty}{2}$$

and with $F_\infty \approx 235$ Wm^{-2}, it follows that this temperature is $T_{skin} = [117.5/5.68 \times 10^{-8}]^{0.25} = 213$ K.

The solutions in Eqns. (6.10a) and (6.10b) predict a discontinuity between the surface temperature T_s and the air temperature just above the ground $T(\tilde{\tau}_s)$. Differencing these equations and with $\tilde{\tau} = \tilde{\tau}_s$,

$$\sigma T_s^4 - \sigma T^4(\tilde{\tau}_s) = \frac{F_\infty}{2}.$$



"This radiative equilibrium profile is unstable w.r.t. vertical motion and is destroyed by convection"

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 100, NO. D6, PAGES 11,585–11,591, JUNE 20, 1995

Deductions from a simple climate model: Factors governing surface temperature and atmospheric thermal structure

C. P. Weaver and V. Ramanathan

Center for Clouds, Chemistry and Climate, Scripps Institution of Oceanography, La Jolla, California

Temperature discontinuity. Equations (6a) and (6b) predict a temperature jump at the surface:

$$\sigma T_g^4 - \sigma T(\tau^*)^4 = \frac{f_0}{2} \quad (7)$$

3. Modified Schwarzschild's Grey Model

3.1. Semigrey Model With One Spectral Window

The first modification is a simple extension of the Schwarzschild equations in which we add a spectral window

Schwarzschild (1906, Eq. 11)
grey, two-stream

Ueber das Gleichgewicht der Sonnenatmosphäre

Von

K. Schwarzschild.

Vorgelegt in der Sitzung vom 13. Januar 1906.

On the Equilibrium of the Sun's Atmosphere

by K. Schwarzschild

(*Presented at the meeting of the Berlin Academy of Sciences on January 13, 1906*)

$$E = \frac{A_0}{2}(1 + \bar{\tau}), \qquad A = \frac{A_0}{2}(2 + \bar{\tau}), \qquad B = \frac{A_0}{2}\bar{\tau}. \tag{11}$$

E emission of a layer, *A* upward beam, *B* downward beam, A_0 emerging flux at TOA, τ optical depth

Equation (1) $\mathbf{A - E = A_0 / 2}$

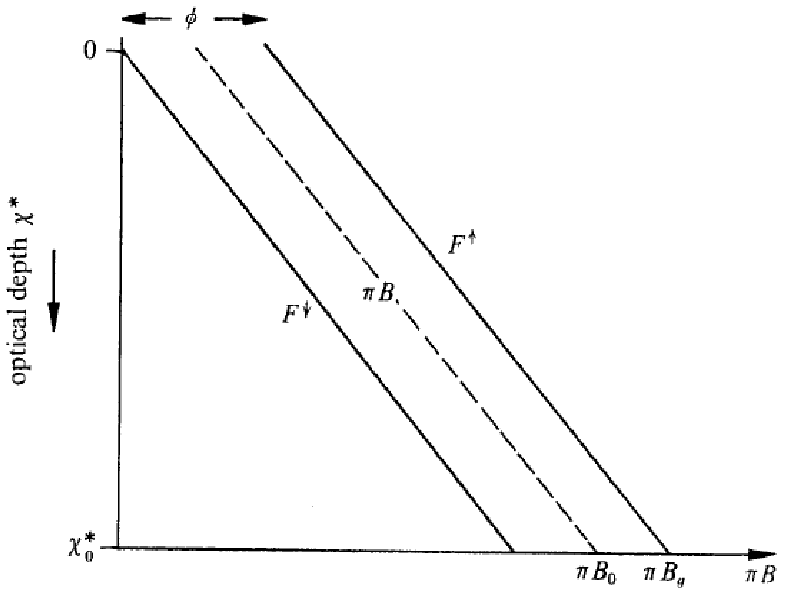
Net radiation at the surface equals half of the effective emission

John Houghton: The Physics of Atmospheres
Cambridge (1977, 1986, 2002) Eq. 2.13

At the bottom of the atmosphere where $\chi^* = \chi_0^*$, $F^\uparrow = \pi B_g$, B_g being the black-body function at the temperature of the ground. It is easy to show that there must be a temperature discontinuity at the lower boundary, the black-body function for the air close to the ground being B_0, and

$$\boxed{B_g - B_0 = \frac{\phi}{2\pi}} \qquad (2.13)$$

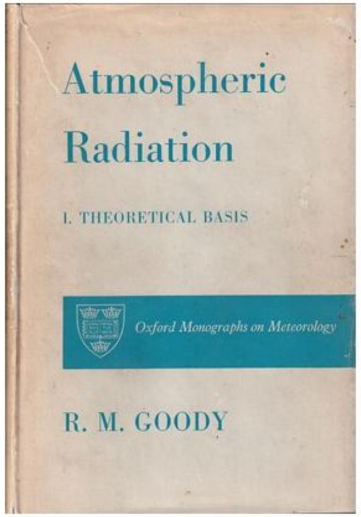
DISCONTINUITY = OLR/2
(independent of the optical depth)



such a steep lapse rate is very unstable with respect to vertical motion, and will soon be destroyed by the process of *convection*

Discontinuity = Convection = OLR/2

R. Goody: Atmospheric radiation. Theoretical basis. Oxford, 1964

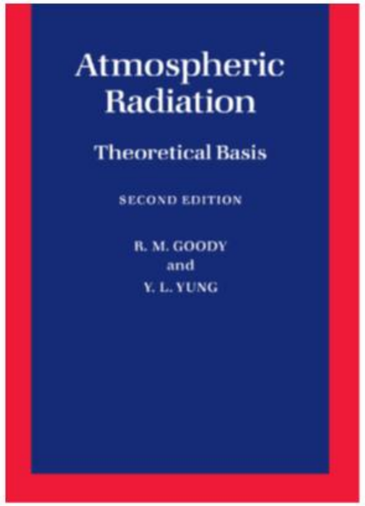


$$F_\nu/2\pi = J_\nu(0) - B^*_\nu(0) = B^*_\nu(\tau^*_\nu) - J_\nu(\tau^*_\nu). \qquad (2.115)$$

Note that if the medium is in thermodynamic equilibrium ($J_\nu = B_\nu$), equation (2.115) requires a discontinuity in B_ν (i.e. a *temperature discontinuity*) at both boundaries.

the step in B always equals $F/2\pi$

Goody – Yung: Atmospheric radiation. Theoretical basis. Oxford, 1989



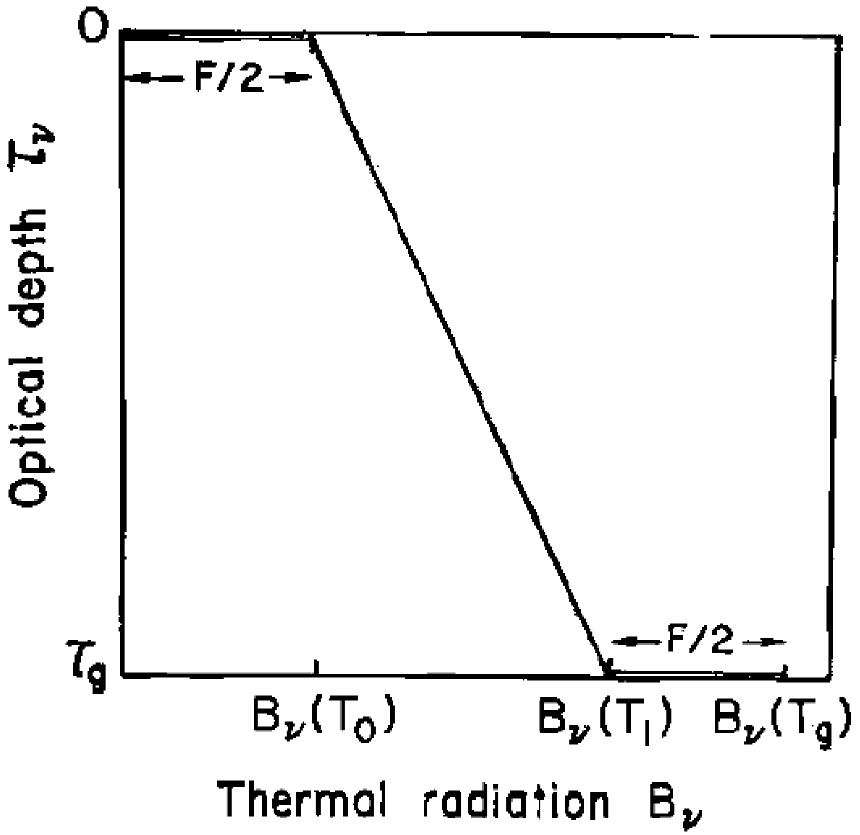
$$F/2\pi = B(0) - B^*(0) = B^*(\tau_1) - B(\tau_1). \qquad (2.146)$$

Equation (2.146) requires a discontinuity in the Planck function, implying a discontinuity of temperature, at the boundary.

2. There are discontinuities,

$$\boxed{\Delta B = \frac{F_S}{2\pi}}$$

DISCONTINUITY = OLR/2



Joseph Chamberlain
Theory of Planetary Atmospheres (1978, 1987)
Academic Press, Fig. 1.4, Eq. 1.2.29

Fig. 1.4 The MRE solution for $T(\tau)$, presented as $B_\nu(T)$ vs. τ. Note the discontinuity at the ground and the finite skin temperature at $\tau = 0$.

DISCONTINUITY = OLR/2
(independent of τ)

Hence the upward intensity at the ground is

$$I_g^+ \equiv B_\nu(T_g) = B_\nu(T_1) + \tfrac{1}{2}F_\nu \qquad (1.2.29) \qquad (1.2.29)$$

Visconti: Fundamentals of Physics and Chemistry of the Atmospheres
Springer (2001)

$$T^4 = \frac{T_e^4}{2}\left(\frac{3}{2}\tau + 1\right) \tag{3.47}$$

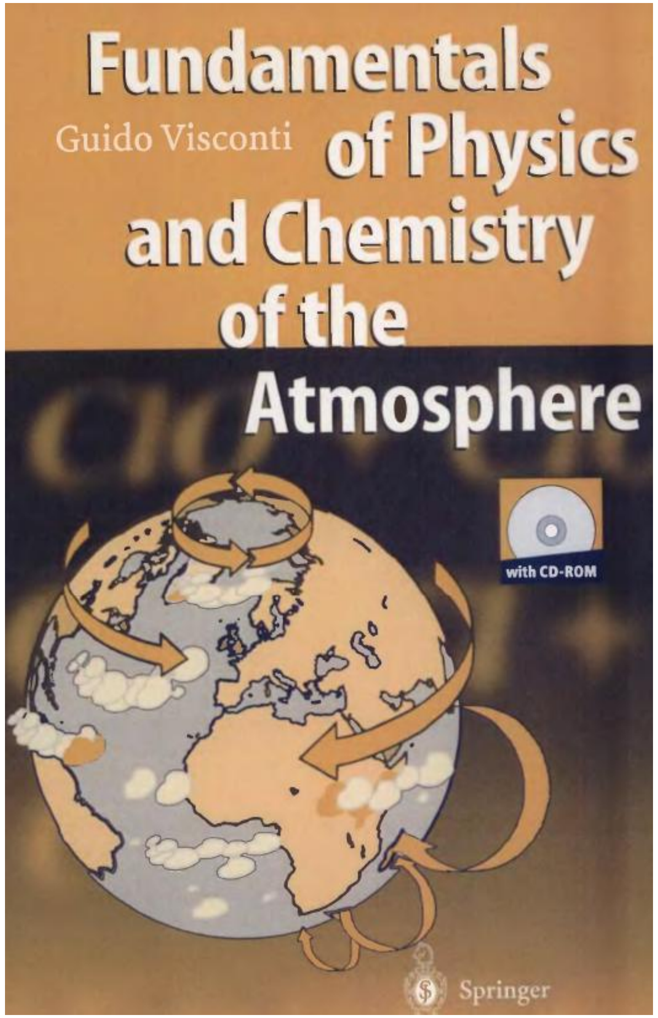
where τ has been replaced by its effective value. An interesting consequence of this solution is to discover what happens at the surface, where the optical thickness is τ^* and the temperature is T_s; we have

$$F^{\uparrow}(\tau^*) = \sigma T_s^4 = \frac{1}{2}\sigma T_e^4\left(\frac{3}{2}\tau^* + 2\right) \tag{3.48}$$

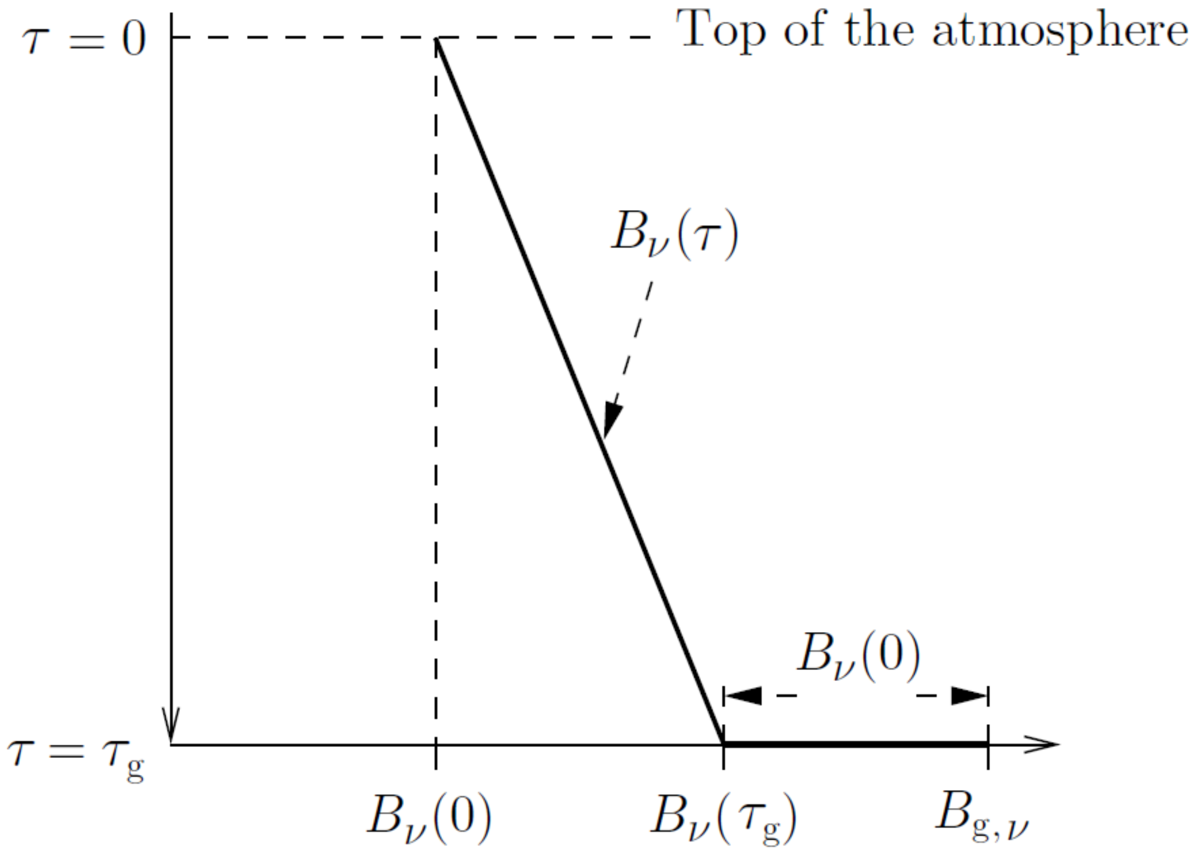
The temperature at the bottom of the atmosphere at τ^* is given by Equation (3.47), so that we have a discontinuity between the air temperature and that of the surface

$$T_s^4 - T(\tau^*) = T_e^4/2 \tag{3.49}$$

DISCONTINUITY = OLR/2



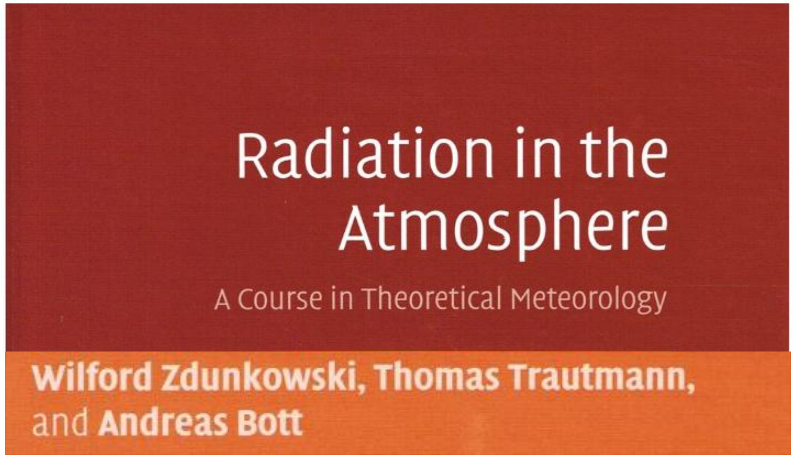
Two-stream methods for the solution of the RTE



Radiation in the Atmosphere

Zdunkowski, Trautmann and Bott

Cambridge (2008) Fig. 6.7

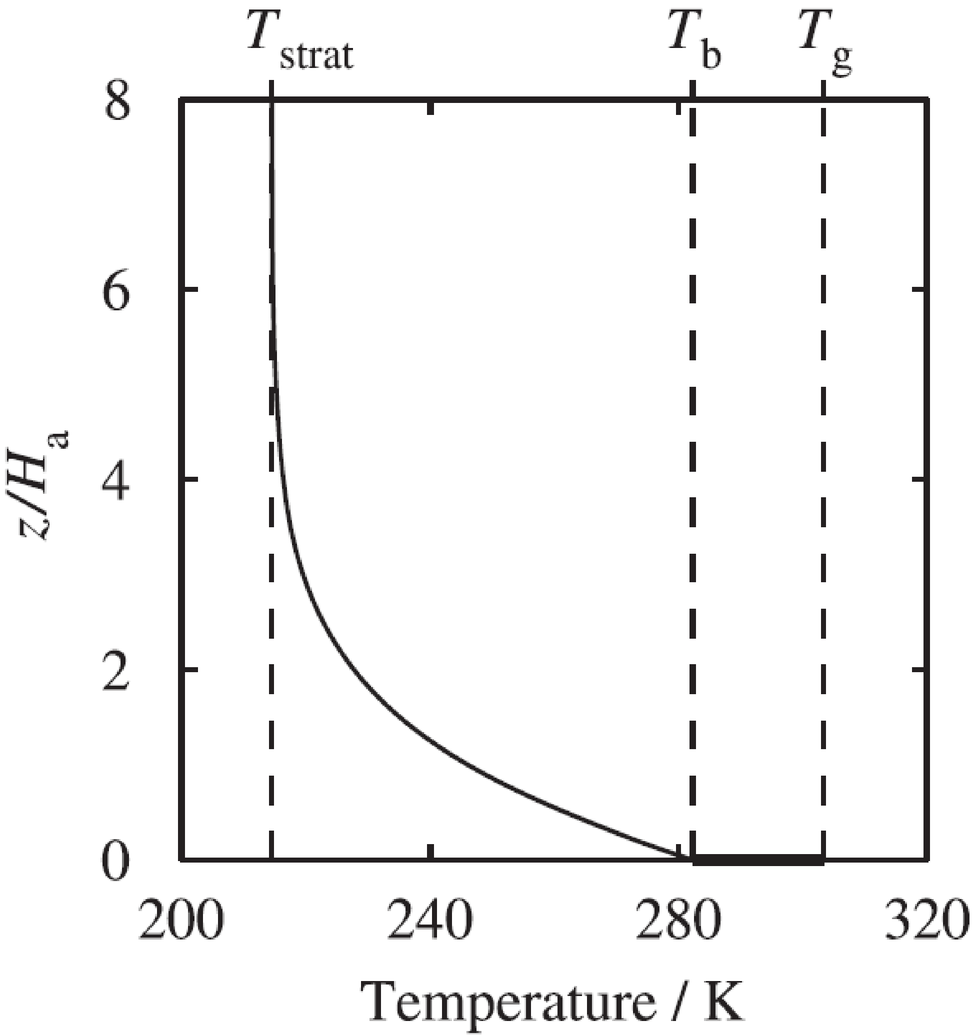


DISCONTINUITY = OLR/2

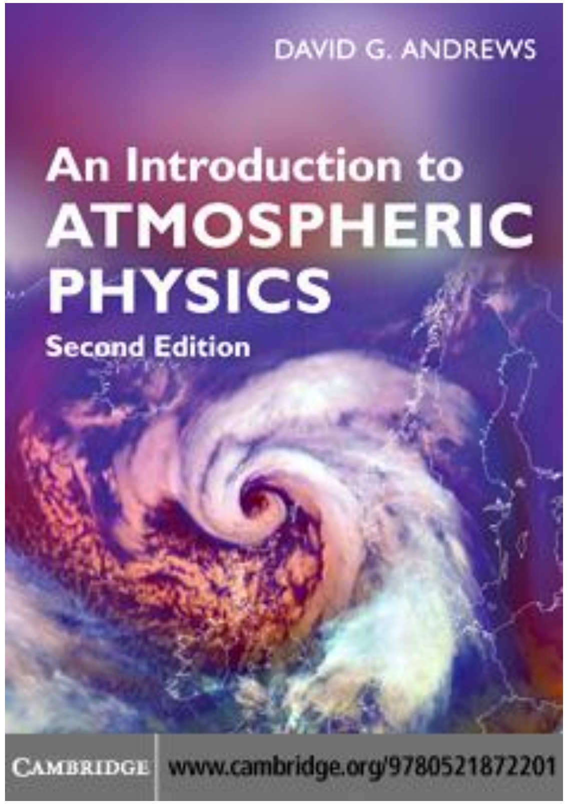
$$\boxed{B_{g,\nu} = B_\nu(\tau_g) + \frac{1}{2\pi} E_{net,\nu}}$$

observe a discontinuity of the curve expressing a temperature jump ΔT_g between the surface temperature T_g and the lowest atmospheric layer $T(\tau_g)$, i.e.

$$\Delta T_g = T_g - T(\tau_g) > 0 \qquad (6.155)$$



Andrews: An Introduction to Atmospheric Physics
Cambridge (2010), pp 85-86.



$$T_b \equiv T_e \left(\frac{1 + \chi_g^*}{2} \right)^{1/4}$$

$$T_g \equiv T_e \left(\frac{2 + \chi_g^*}{2} \right)^{1/4}$$

$$\boxed{T_g^4 - T_b^4 = T_e^4/2}$$

DISCONTINUITY = OLR/2

discontinuity between the bottom of the atmosphere and the ground.

Inclusion of convection in the model removes the temperature discontinuity

8.5 Thermal equilibrium 235

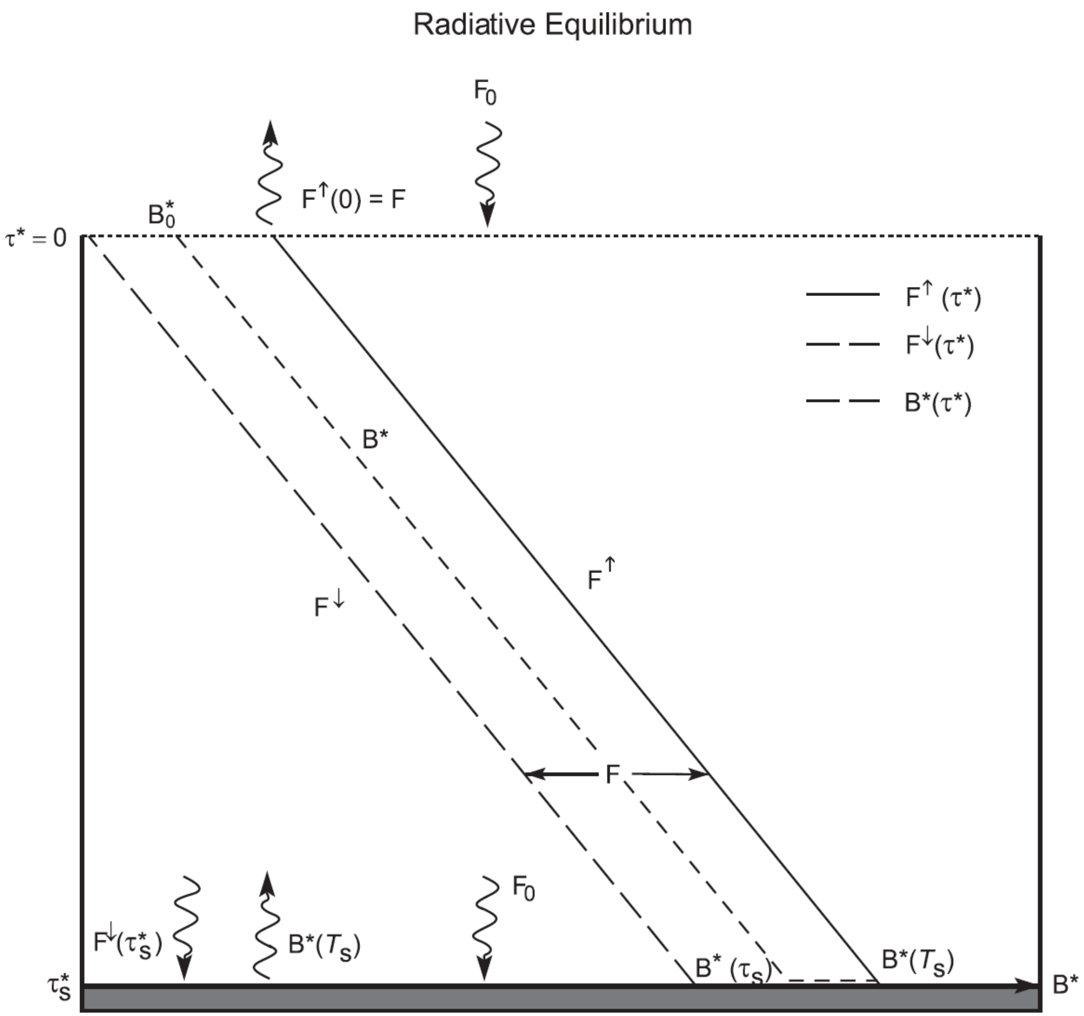
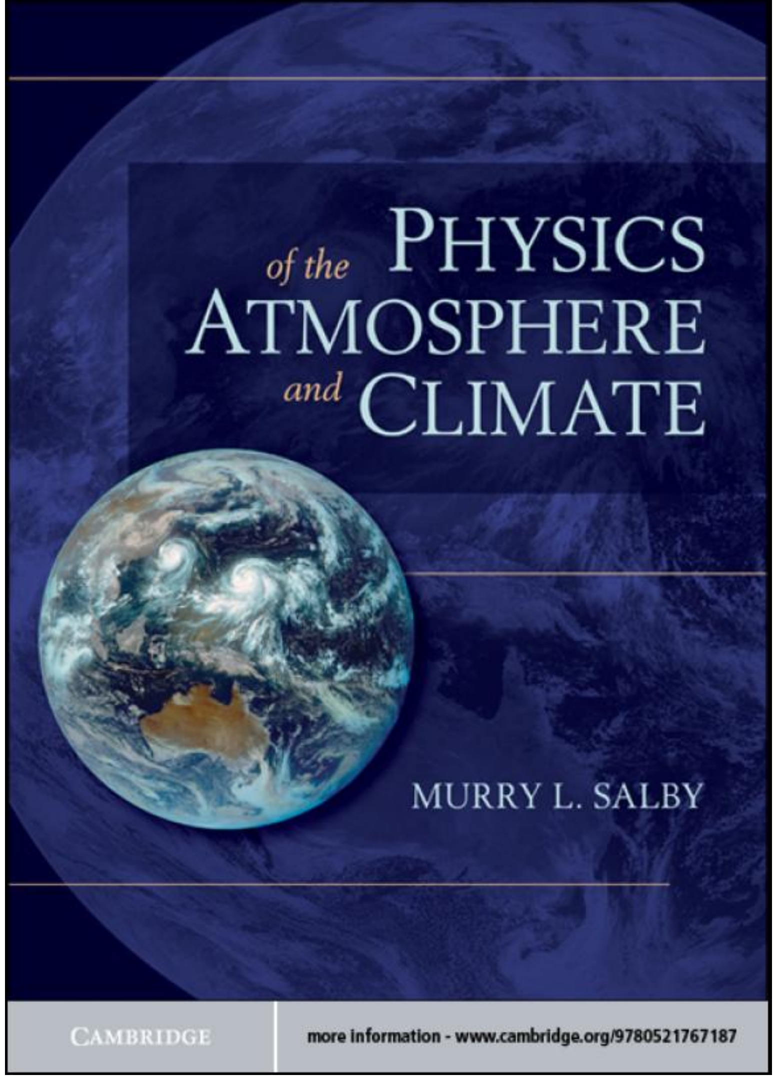


Figure 8.20 Upwelling and downwelling LW fluxes and LW emission in a gray atmosphere that is in radiative equilibrium with an incident SW flux F_0 and a black underlying surface. Note: the emission profile is discontinuous at the surface.



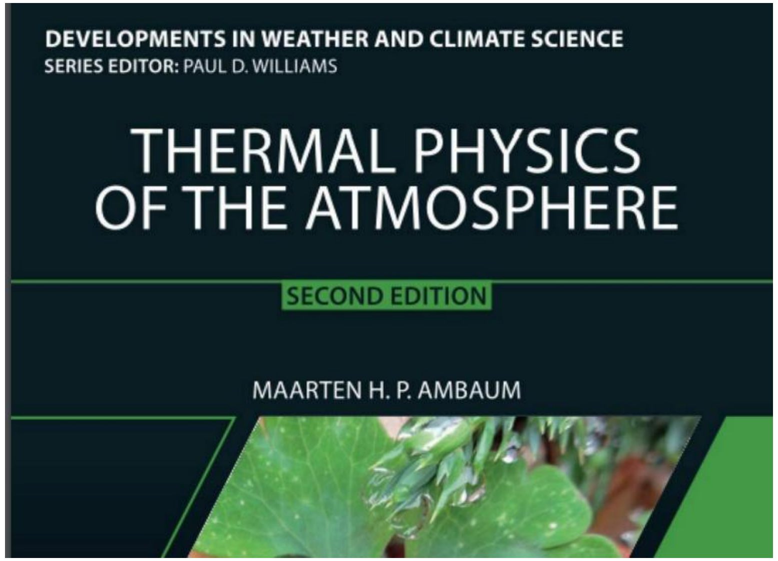
M. Salby: Physics of the Atmosphere and Climate
Cambridge, 2012

$$B^*(T_s) = B^*(\tau_s^*) + \frac{F_0}{2}. \tag{8.67}$$

According to (8.67), the temperature predicted by radiative equilibrium is discontinuous at the surface, the ground being warmer than the overlying air.

ELSEVIER

RMetS
Royal Meteorological Society



These two budget equations can be combined by eliminating $L_{\downarrow 0}$ to find

$$\blacktriangleright \qquad \sigma T_E^4 = \sigma T_S^4 + S_{n0}/2. \qquad (10.56)$$

So we find that under radiative equilibrium the surface temperature is higher than the temperature of the adjoining atmosphere. This temperature discontinuity is unstable in practice and there will be turbulent heat exchange which will remove the temperature discontinuity. This instability of the radiative state is a driver of surface layer turbulence: the radiation will force the lower part of the boundary layer, the surface layer, towards instability and this tendency is compensated by turbulent fluxes near the surface.

Kevin E. Trenberth (2022):

The Changing Flow of Energy Through the Climate System

Sidebar 3.2: How Does a Greenhouse Effect Work?

(page 30: Earth's Energy Balance)

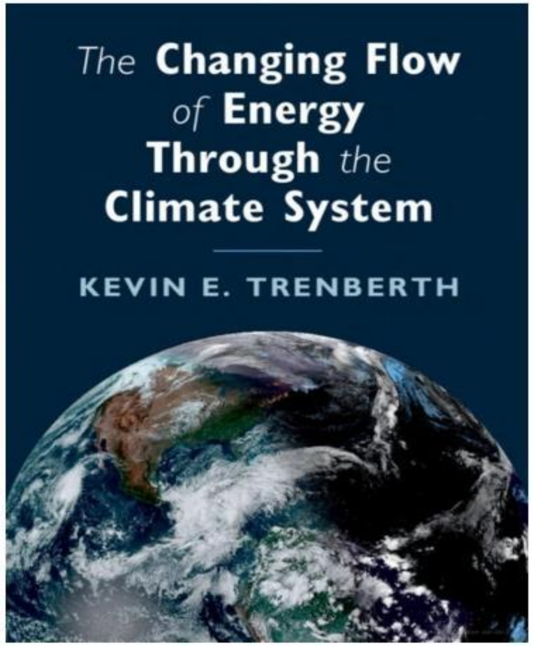
The answer is:

Imagine two plates in space

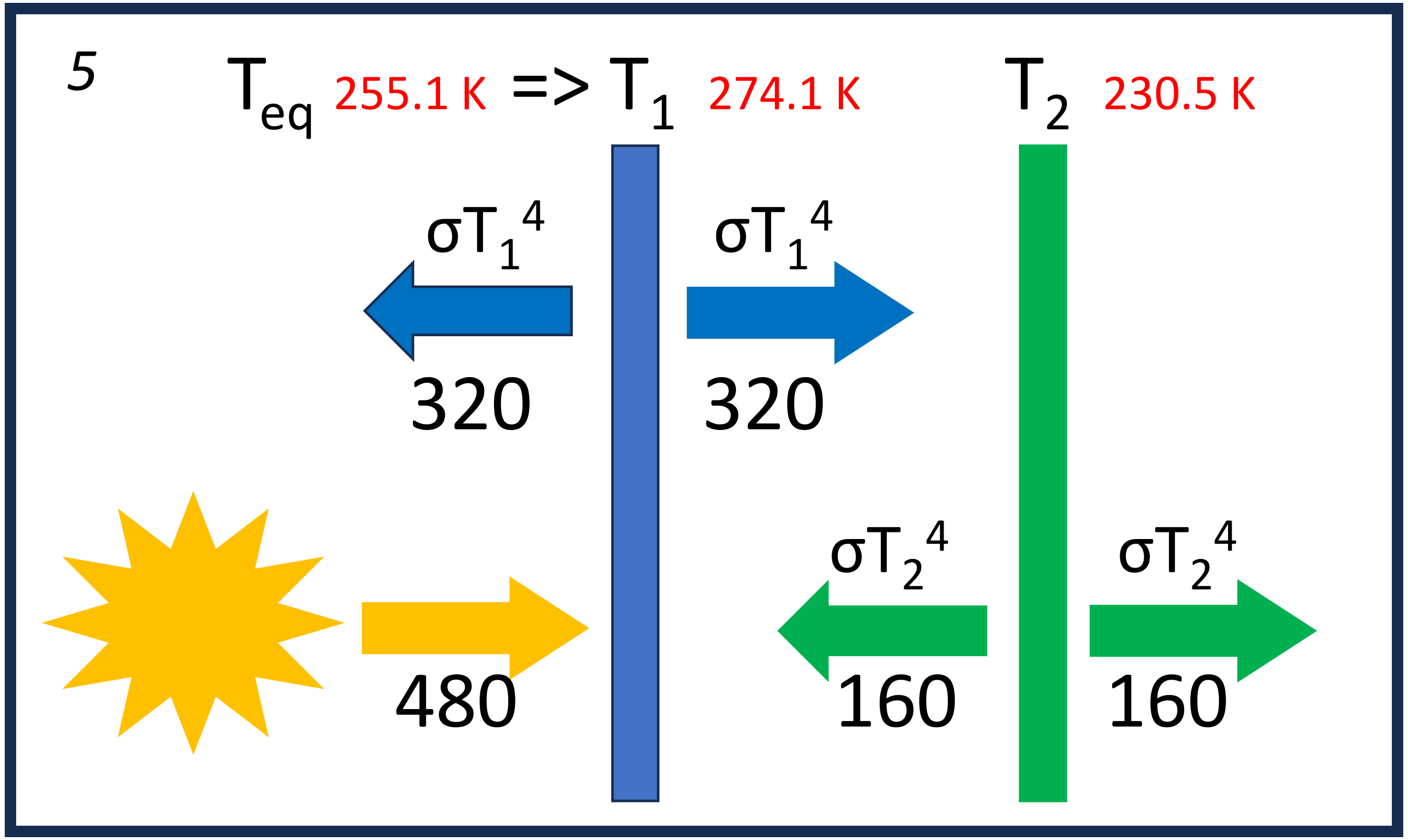
Incoming solar radiation: 240 Wm^{-2}

The plate is two-sided: 480 Wm^{-2}

Five panels, four equations.



The final state



$$G : F_2 : F_{eq} : F_1 : F_{sol} = 1 : 2 : 3 : 4 : 6$$

Trenberth's four equations

(T1) $480\ \mathrm{Wm^{-2}} = 2\sigma T_{eq}^4$

(T2) $480\ \mathrm{Wm^{-2}} = \sigma T_1^4 + \sigma T_2^4$

(T3) $\sigma T_1^4 = 2\sigma T_2^4$

(T4) $480\ \mathrm{Wm^{-2}} = 1.5\ \sigma T_1^4$

$F_{sol} = 480\ \mathrm{Wm^{-2}}$

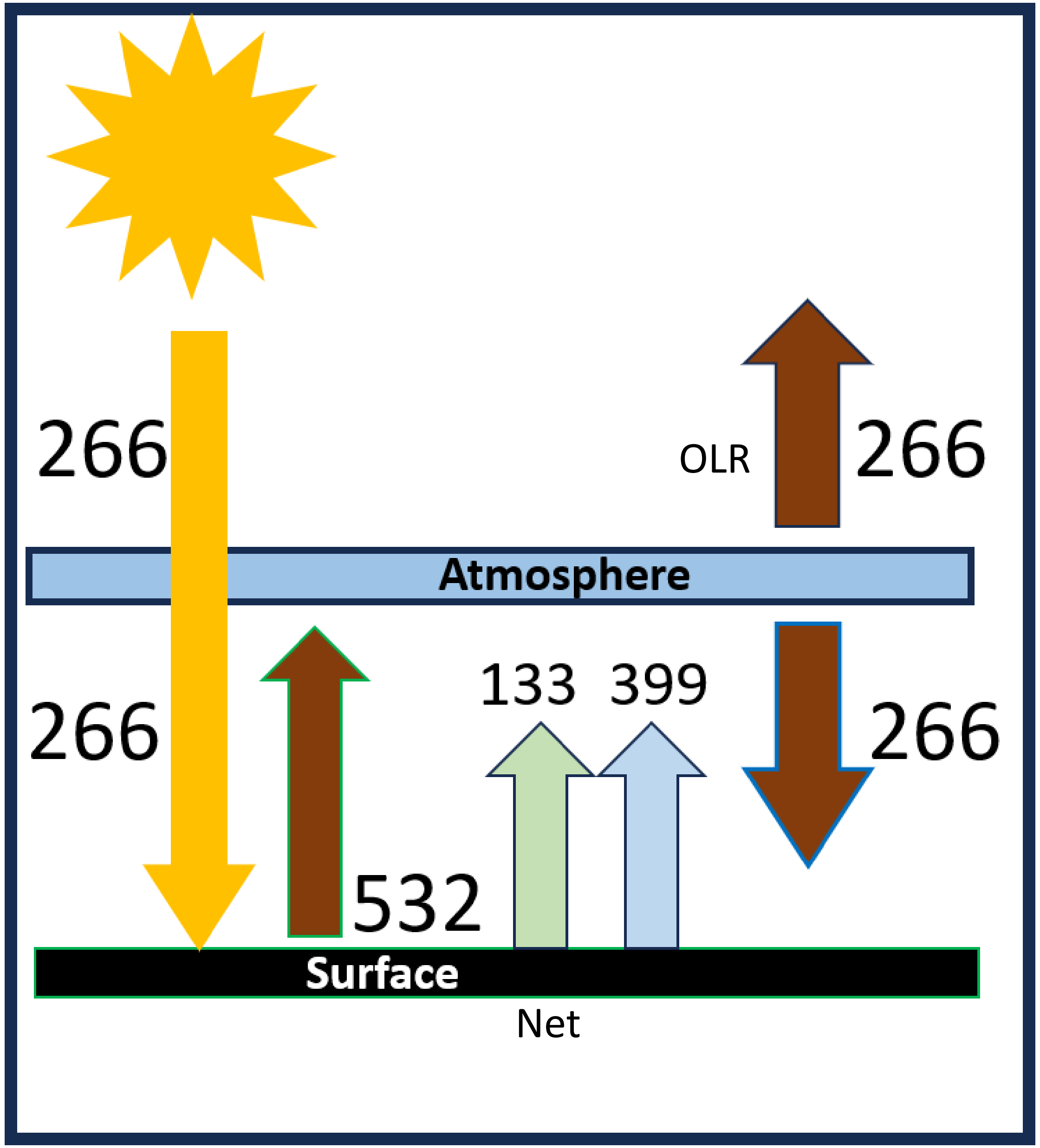
$F_1 = 320\ \mathrm{Wm^{-2}}$

$F_{eq} = 240\ \mathrm{Wm^{-2}}$

$F_2 = 160\ \mathrm{Wm^{-2}}$

$G = 80\ \mathrm{Wm^{-2}}$

Trenberth's Earth-like Geometry

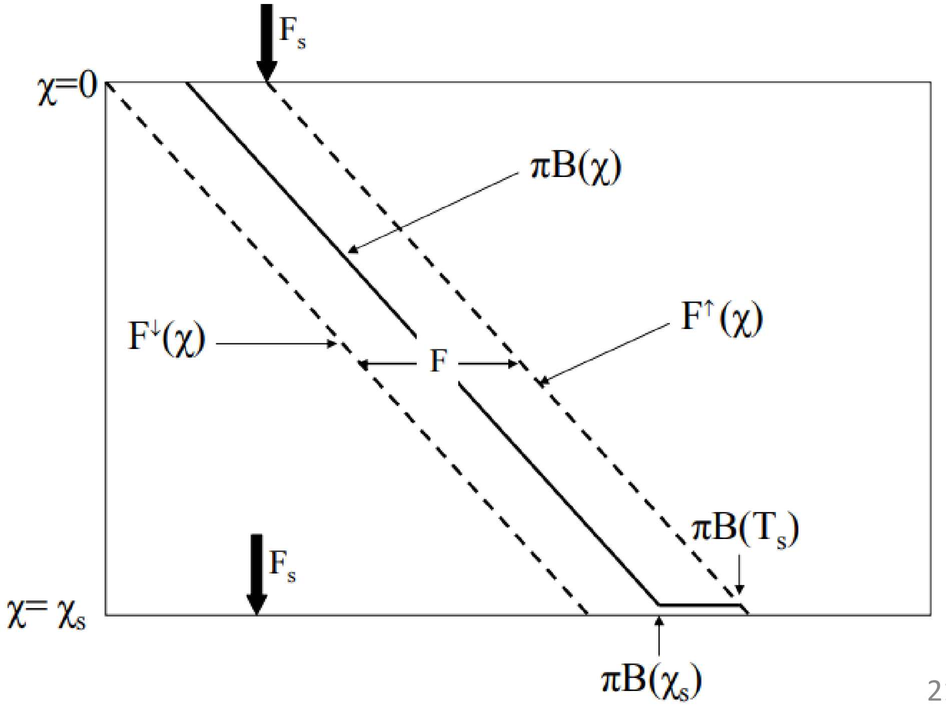


University Lecture Notes

Manchester University

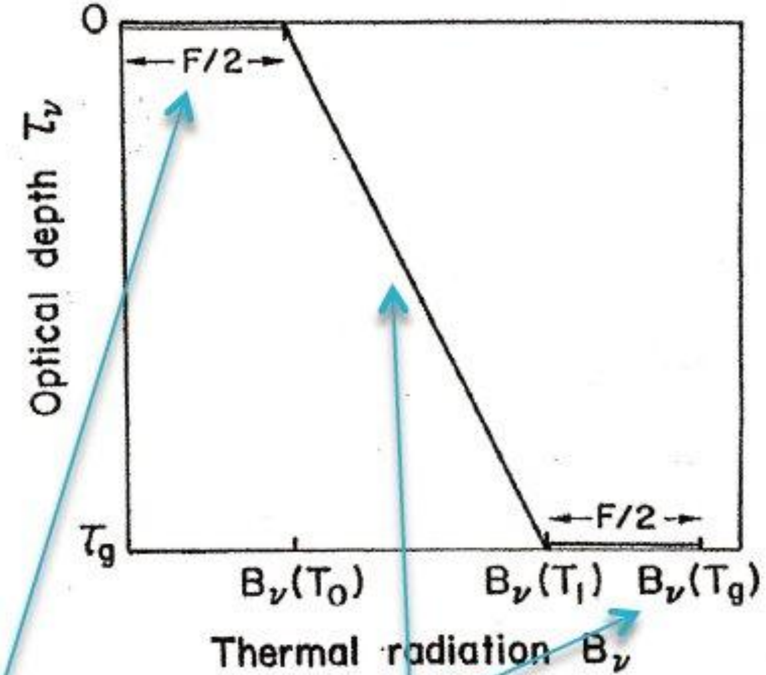
This expression implies that there is a temperature discontinuity between the surface and the cooler lowest layer of the atmosphere.

$$\boxed{\pi B(T_s) = \pi B(\chi_s) + F_s/2.}$$



University of Arizona, 2015

Final TP Profile



Expressions for intensity:

$$I^+ = B(T) + \frac{1}{2\pi}F$$

$$I^- = B(T) - \frac{1}{2\pi}F$$

Boundary Conditions:

At the Ground: $$I_g^+ = B(T_g) = B(T_1) + \frac{1}{2\pi}F,$$

Discontinuity

東京大学 University of Tokyo

惑星大気学_放射(2022).docx

(3.10)(3.11)より

$$\underbrace{B^*(T_S)}_{\text{地表面}} = \underbrace{B^*(\tau_s^*)}_{\text{大気下端}} + \frac{F^0}{2}$$

放射平衡では大気下端の温度と地表面温度は不連続になる。($B^* = \sigma T^4$ に注意)

From (3.10)(3.11)

$$\underbrace{B^*(T_S)}_{\text{surface}} = \underbrace{B^*(\tau_s^*)}_{\text{bottom of atmosphere}} + \frac{F^0}{2}$$

A temperature discontinuity exists at the surface.（Note that $B^* = \sigma T^4$）

University of Toronto

"We will use "Schwarzchild's Equation for Fluxes" in our simple greenhouse atmospheric model."

Two-Stream Model Solution - 1

- We have thus solved for the upward and downward flux densities:

$$\boxed{\begin{aligned} F_{up} &= \sigma T_e^4(\chi^* + 2)/2 \\ F_{down} &= \sigma T_e^4 \chi^*/2 \end{aligned}}$$

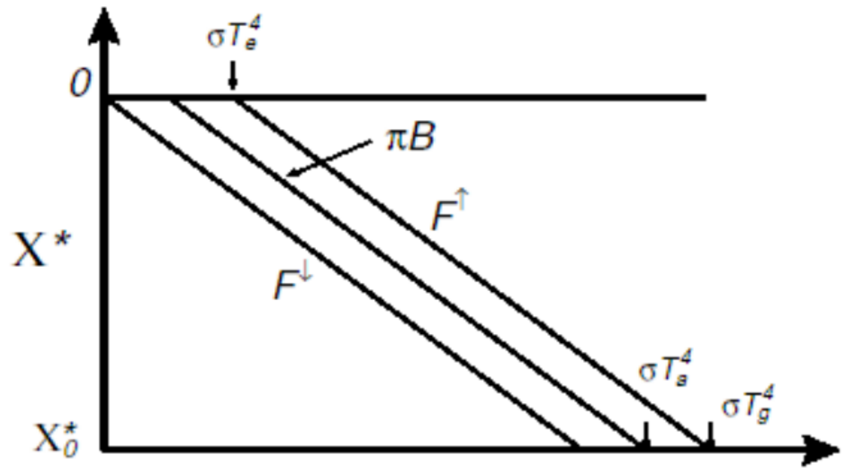
- Finally, we can use the original equations to solve for:

$$\boxed{\pi B = \sigma T_a^4 = \frac{\sigma T_e^4(\chi^* + 1)}{2}}$$

where T_a is the atmospheric temperature.

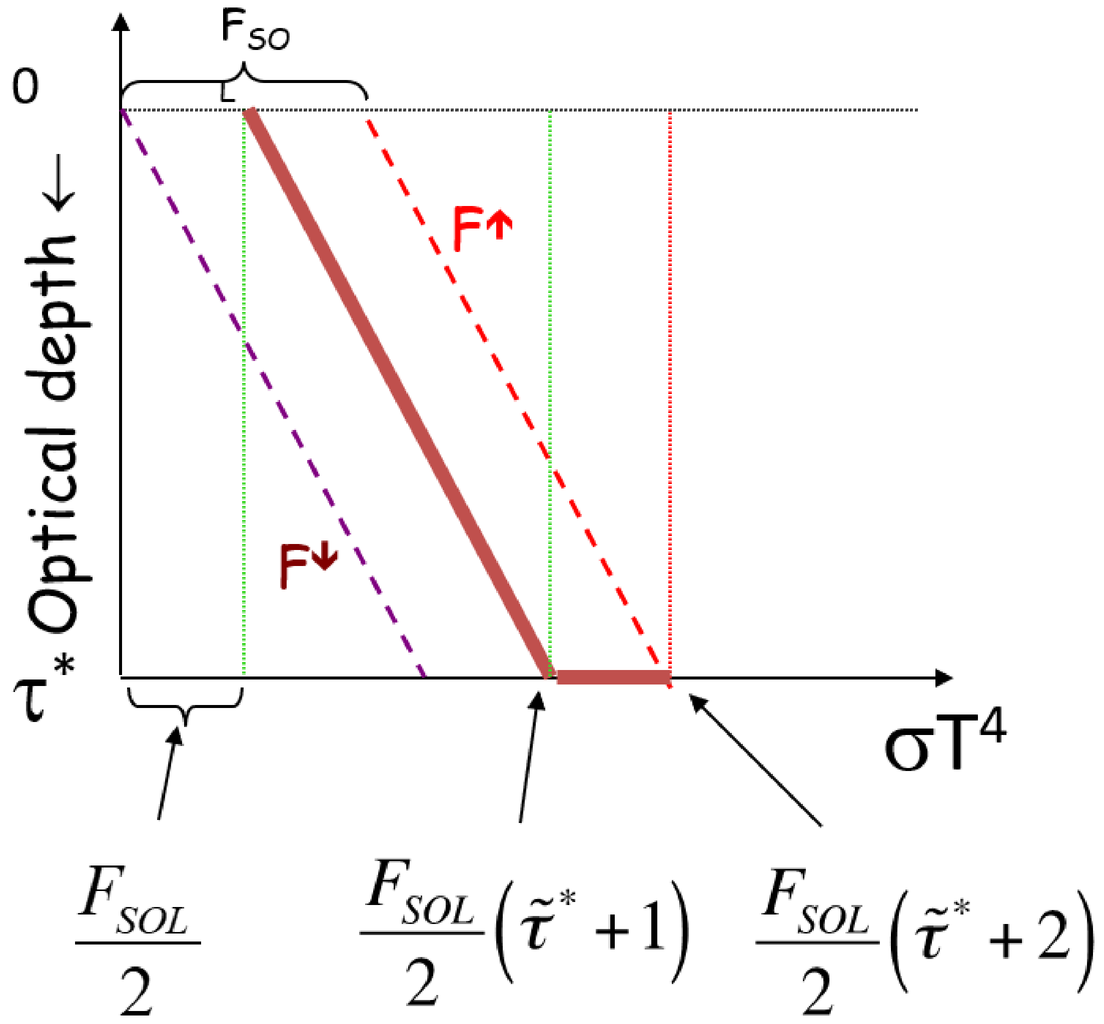
- These three solutions can be plotted against χ^*.
  - → Gives three parallel lines that show how the flux densities and the blackbody function vary with χ^* and hence with altitude.

Two-Stream Model Solution - 3



- If the central line represents the atmospheric temperature and the line for F_{up} must match the ground temperature, then there is a temperature discontinuity at the ground.

Colorado State Univ. (2013)



$$\sigma T^4(\tilde{\tau}) = \frac{F_{SOL}}{2}(\tilde{\tau}+1)$$

$$\sigma T_s^4 = \frac{F_{SOL}}{2}(\tilde{\tau}^*+2)$$

Harvard (2018)

We have got the temperature structure in the atmosphere as a function of τ. Now consider energy balance at the surface (looks familiar?),

$$B(T_s) = F_0 + F^{\downarrow}(\tau_s)$$

From their definitions, we have

$$F^{\downarrow} = \frac{1}{2}\left(\bar{F} - F\right)$$

As the net flux F is constant and equal to F_0, and use Eq. (14), we have:

$$B(T_s) = B(\tau_s) + \frac{F_0}{2}$$

Note the jump at the surface.

University exam task (for A+)

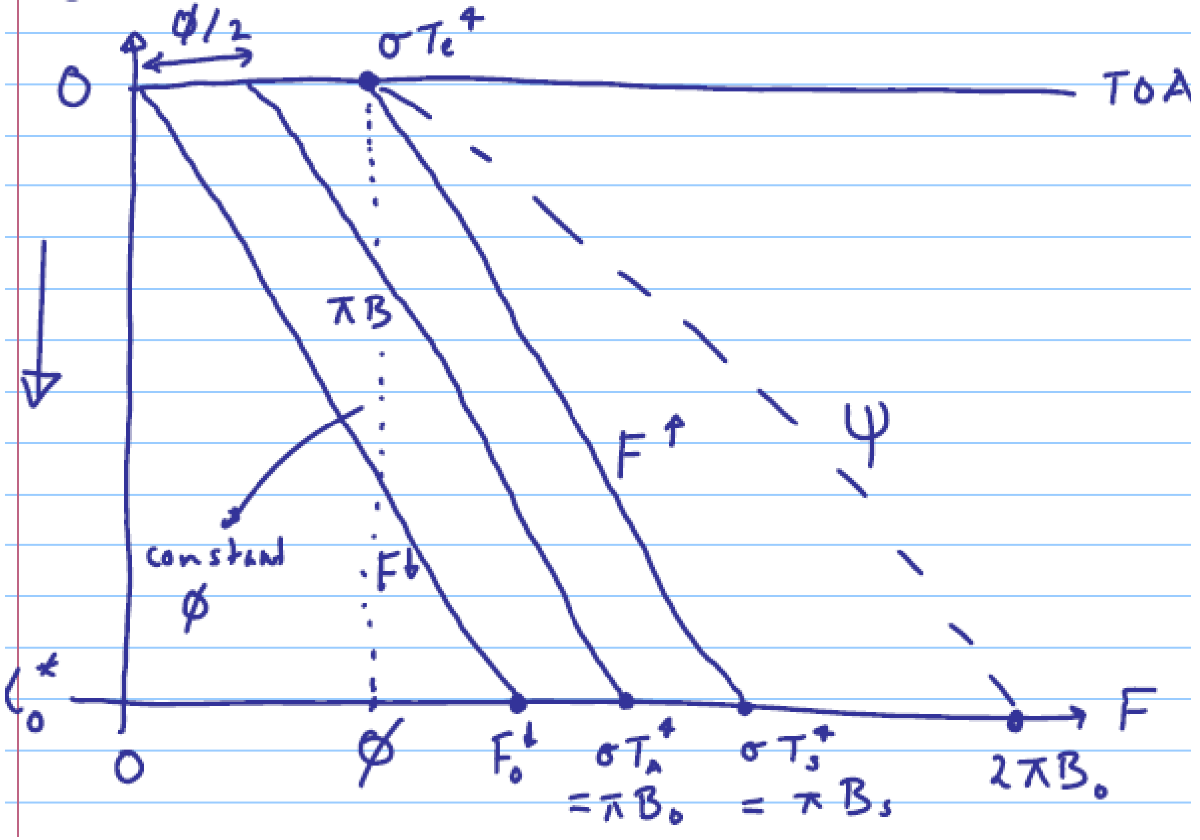
$$\phi = F^{\uparrow} - F^{\downarrow} = \sigma T_e^4$$

$$F^{\uparrow} = \frac{\phi}{2}(\chi^* + 2)$$

$$F^{\downarrow} = \frac{\phi}{2}\chi^*$$

$$\pi B = \frac{\phi}{2}(\chi^* + 1)$$

Let's plot these with χ^* on the y axis.



Data

CERES EBAF Ed2.8, Ed4.2.1

CERES_EBAF-Surface_Ed2.8 Data Quality Summary (March 27, 2015)

Table 4-1. Global annual mean fluxes using data from March 2000 through February 2010 (W m^{-2}).

	Flux Component	Ed3A SYN1deg -Month	EBAF-Surface Ed2.6r	EBAF-Surface Ed2.7	EBAF-Surface Ed2.8	EBAF-TOA Ed2.8
TOA	Incoming solar	339.9	339.9	339.9	339.8	339.8
	LW (all-sky)	237.3	239.7	239.6	239.6	239.6
	SW (all-sky)	98.5	99.6	99.6	99.6	99.6
	Net (all-sky)	4.06	0.64	0.69	0.63	0.59
	LW (clear-sky)	263.7	265.8	265.7	265.7	265.8
	SW (clear-sky)	52.5	52.5	52.6	52.6	52.6
	Net (clear-sky)	23.6	21.6	21.6	21.6	21.5
Surface	LW down (all-sky)	341.8	343.7	345.1	345.1	
	LW up (all-sky)	397.6	398.1	398.1	398.0	
	SW down (all-sky)	187.2	186.7	186.5	186.4	
	SW up (all-sky)	23.3	24.1	24.1	24.1	
	Net (all-sky)	108.1	108.3	109.4	109.4	
	LW down (clear-sky)	313.5	314.1	315.8	316.0	
	LW up (clear-sky)	396.6	398.3	398.4	398.0	
	SW down (clear-sky)	242.4	243.4	244.1	243.9	
	SW up (clear-sky)	28.7	29.6	29.7	29.7	
	Net (clear-sky)	130.6	129.6	131.8	132.2	

Eq. (1) Surface Net (clear-sky) = TOA LW (clear-sky) /2

132.2 = 265.7 /2

CERES_EBAF-Surface_Ed2.8 Data Quality Summary (March 27, 2015)

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	Net (all-sky)	108.1	108.3	109.4	109.4	
	LW down (clear-sky)	313.5	314.1	315.8	316.0	
	LW up (clear-sky)	396.6	398.3	398.4	398.0	
	SW down (clear-sky)	242.4	243.4	244.1	243.9	
	SW up (clear-sky)	28.7	29.6	29.7	29.7	
	Net (clear-sky)	130.6	129.6	131.8	$\boxed{132.2}$	

Eq. (1) Surface Net (clear-sky) = TOA LW (clear-sky) /2 – TOA Net (all-sky)

132.2 = 265.7 /2 – 0.65 Wm^{-2}

CERES_EBAF-Surface_Ed2.8 Data Quality Summary (March 27, 2015)

Table 4-1. Global annual mean fluxes using data from March 2000 through February 2010 ($W\ m^{-2}$).

	Flux Component	Ed3A SYN1deg -Month	EBAF-Surface Ed2.6r	EBAF-Surface Ed2.7	EBAF-Surface Ed2.8	EBAF-TOA Ed2.8
TOA	Incoming solar	339.9	339.9	339.9	339.8	339.8
	LW (all-sky)	237.3	239.7	239.6	239.6	239.6
	SW (all-sky)	98.5	99.6	99.6	99.6	99.6
	Net (all-sky)	4.06	0.64	0.69	0.63	0.59
	LW (clear-sky)	263.7	265.8	265.7	265.7	265.8
	SW (clear-sky)	52.5	52.5	52.6	52.6	52.6
	Net (clear-sky)	23.6	21.6	21.6	21.6	21.5
Surface	LW down (all-sky)	341.8	343.7	345.1	345.1	
	LW up (all-sky)	397.6	398.1	398.1	398.0	
	SW down (all-sky)	187.2	186.7	186.5	186.4	
	SW up (all-sky)	23.3	24.1	24.1	24.1	
	Net (all-sky)	108.1	108.3	109.4	109.4	
	LW down (clear-sky)	313.5	314.1	315.8	316.0	
	LW up (clear-sky)	396.6	398.3	398.4	398.0	
	SW down (clear-sky)	242.4	243.4	244.1	243.9	
	SW up (clear-sky)	28.7	29.6	29.7	29.7	
	Net (clear-sky)	130.6	129.6	131.8	132.2	

Eq. (1) **Surface Net (clear-sky) = TOA LW (clear-sky) /2 – TOA Net (all-sky)**

132.2 = 265.7 /2 – 0.65 Wm^{-2}

But this 0.63 Wm^{-2} is an adjustment

CERES_EBAF-Surface_Ed2.8 Data Quality Summary (March 27, 2015)

Table 4-1. Global annual mean fluxes using data from March 2000 through February 2010 ($W\ m^{-2}$).

	Flux Component	Ed3A SYN1deg -Month	EBAF-Surface Ed2.6r	EBAF-Surface Ed2.7	EBAF-Surface Ed2.8	EBAF-TOA Ed2.8
TOA	Incoming solar	339.9	339.9	339.9	339.8	339.8
	LW (all-sky)	237.3	239.7	239.6	239.6	239.6
	SW (all-sky)	98.5	99.6	99.6	99.6	99.6
	Net (all-sky)	4.06	0.64	0.69	0.63	0.59
	LW (clear-sky)	263.7	265.8	265.7	265.7	265.8
	SW (clear-sky)	52.5	52.5	52.6	52.6	52.6
	Net (clear-sky)	23.6	21.6	21.6	21.6	21.5
Surface	LW down (all-sky)	341.8	343.7	345.1	345.1	
	LW up (all-sky)	397.6	398.1	398.1	398.0	
	SW down (all-sky)	187.2	186.7	186.5	186.4	
	SW up (all-sky)	23.3	24.1	24.1	24.1	
	Net (all-sky)	108.1	108.3	109.4	109.4	
	LW down (clear-sky)	313.5	314.1	315.8	316.0	
	LW up (clear-sky)	396.6	398.3	398.4	398.0	
	SW down (clear-sky)	242.4	243.4	244.1	243.9	
	SW up (clear-sky)	28.7	29.6	29.7	29.7	
	Net (clear-sky)	130.6	129.6	131.8	132.2	

Eq. (1) Surface Net (clear-sky) = TOA LW (clear-sky) /2 – TOA Net (all-sky)

132.2 = 265.7 /2 – 0.65 Wm^{-2}

The plane-parallel equation is EXACT

CERES EBAF Ed2.8, 192 months (Mar 2000 – Feb 2016)
F. Rose et al., 27th STM (2017)

Clear Sky	Ed2.8
TOA SW Insolation	339.87
TOA SW Up	*52.50*
TOA LW Up	*265.59*
SFC SW Down	244.06
SFC SW Up	29.74
SFC LW Down	316.27
SFC LW Up	398.40

Eq. (1) $\quad \mathbf{A - E = \Delta A = A_0/2}$ **(clear-sky)**

SFC SW dn – SW up + LW dn – LW up = TOA LW /2

(244.06 – 29.74 + 316.27) – 398.40 = 132.19 = 265.59/2 – 0.60 Wm^{-2}

Eq. (1) Summary

Globally averaged, the surface has a net surplus of radiant energy and it equals half of the TOA LW in the clear-sky.

There is more:

- „Globally averaged, the surface has a net surplus of radiant energy **while the atmosphere has a net loss. To make up for this imbalance, sensible (conduction & convection) and latent heat (evaporation) are transferred from the surface to the atmosphere**.” (website)
- The net surplus at the surface is the difference of absorbed and emitted radiant energy, balanced by turbulent fluxes (CONV in general): Surface Net = CONV = TOA LW/2 (clear-sky).
- But there is also a net surplus of radiant energy at the surface, **compared to the TOA**.
  This is called the **greenhouse effect**, G = Surface LW up – TOA LW up.
- This surplus is also balanced by convection in the atmosphere, CONV = G (clear).
- But as we have one convection, **CONV = G = TOA LW/2 (clear)**.
- As Surface LW up = TOA LW up + G, it follows that

  Surface LW up (clear-sky) = (3/2) TOA LW up (clear-sky).
- Let’s see on your data.

Table 4-1. Global annual mean fluxes using data from March 2000 through February 2010 ($W\ m^{-2}$).

	Flux Component	Ed3A SYN1deg -Month	EBAF-Surface Ed2.6r	EBAF-Surface Ed2.7	EBAF-Surface Ed2.8	EBAF-TOA Ed2.8
TOA	Incoming solar	339.9	339.9	339.9	339.8	339.8
	LW (all-sky)	237.3	239.7	239.6	239.6	239.6
	SW (all-sky)	98.5	99.6	99.6	99.6	99.6
	Net (all-sky)	4.06	0.64	0.69	0.63	0.59
	LW (clear-sky)	263.7	265.8	265.7	265.7	265.8
	SW (clear-sky)	52.5	52.5	52.6	52.6	52.6
	Net (clear-sky)	23.6	21.6	21.6	21.6	21.5
Surface	LW down (all-sky)	341.8	343.7	345.1	345.1	
	LW up (all-sky)	397.6	398.1	398.1	398.0	
	SW down (all-sky)	187.2	186.7	186.5	186.4	
	SW up (all-sky)	23.3	24.1	24.1	24.1	
	Net (all-sky)	108.1	108.3	109.4	109.4	
	LW down (clear-sky)	313.5	314.1	315.8	316.0	
	LW up (clear-sky)	396.6	398.3	398.4	398.0	
	SW down (clear-sky)	242.4	243.4	244.1	243.9	
	SW up (clear-sky)	28.7	29.6	29.7	29.7	
	Net (clear-sky)	130.6	129.6	131.8	132.2	

Surface Net (clear-sky) = 132.2 = SFC LW up – TOA LW up ≡ G = 398.0 – 265.7 = 132.3, difference 0.1 Wm^{-2}

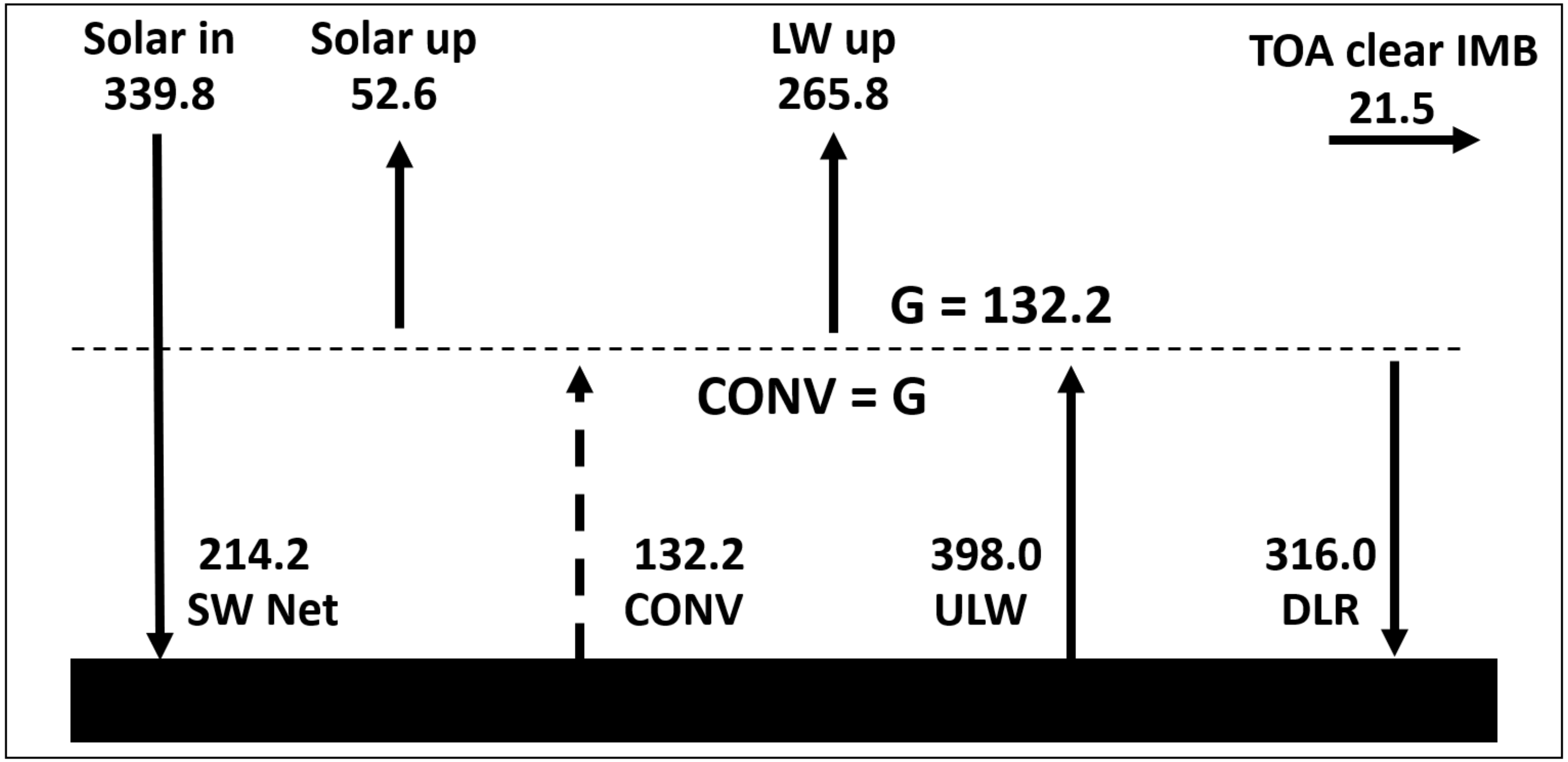
Table 4-1. Global annual mean fluxes using data from March 2000 through February 2010 ($W\ m^{-2}$).

	Flux Component	Ed3A SYN1deg -Month	EBAF-Surface Ed2.6r	EBAF-Surface Ed2.7	EBAF-Surface Ed2.8	EBAF-TOA Ed2.8
TOA	Incoming solar	339.9	339.9	339.9	339.8	339.8
	LW (all-sky)	237.3	239.7	239.6	239.6	239.6
	SW (all-sky)	98.5	99.6	99.6	99.6	99.6
	Net (all-sky)	4.06	0.64	0.69	0.63	0.59
	LW (clear-sky)	263.7	265.8	265.7	265.7	265.8
	SW (clear-sky)	52.5	52.5	52.6	52.6	52.6
	Net (clear-sky)	23.6	21.6	21.6	21.6	21.5
Surface	LW down (all-sky)	341.8	343.7	345.1	345.1	
	LW up (all-sky)	397.6	398.1	398.1	398.0	
	SW down (all-sky)	187.2	186.7	186.5	186.4	
	SW up (all-sky)	23.3	24.1	24.1	24.1	
	Net (all-sky)	108.1	108.3	109.4	109.4	
	LW down (clear-sky)	313.5	314.1	315.8	316.0	
	LW up (clear-sky)	396.6	398.3	398.4	398.0	
	SW down (clear-sky)	242.4	243.4	244.1	243.9	
	SW up (clear-sky)	28.7	29.6	29.7	29.7	
	Net (clear-sky)	130.6	129.6	131.8	132.2	

Surface Net (clear-sky) = 132.2 = SFC LW up – TOA LW up ≡ G = 398.0 – 265.7 = 132.3, difference 0.1 Wm^{-2}

But the EBAF-TOA LW up is 265.8 Wm^{-2}, using this value, G = 132.2 Wm^{-2}.

The equation is EXACT again



Surface radiative surplus from G balanced by CONV

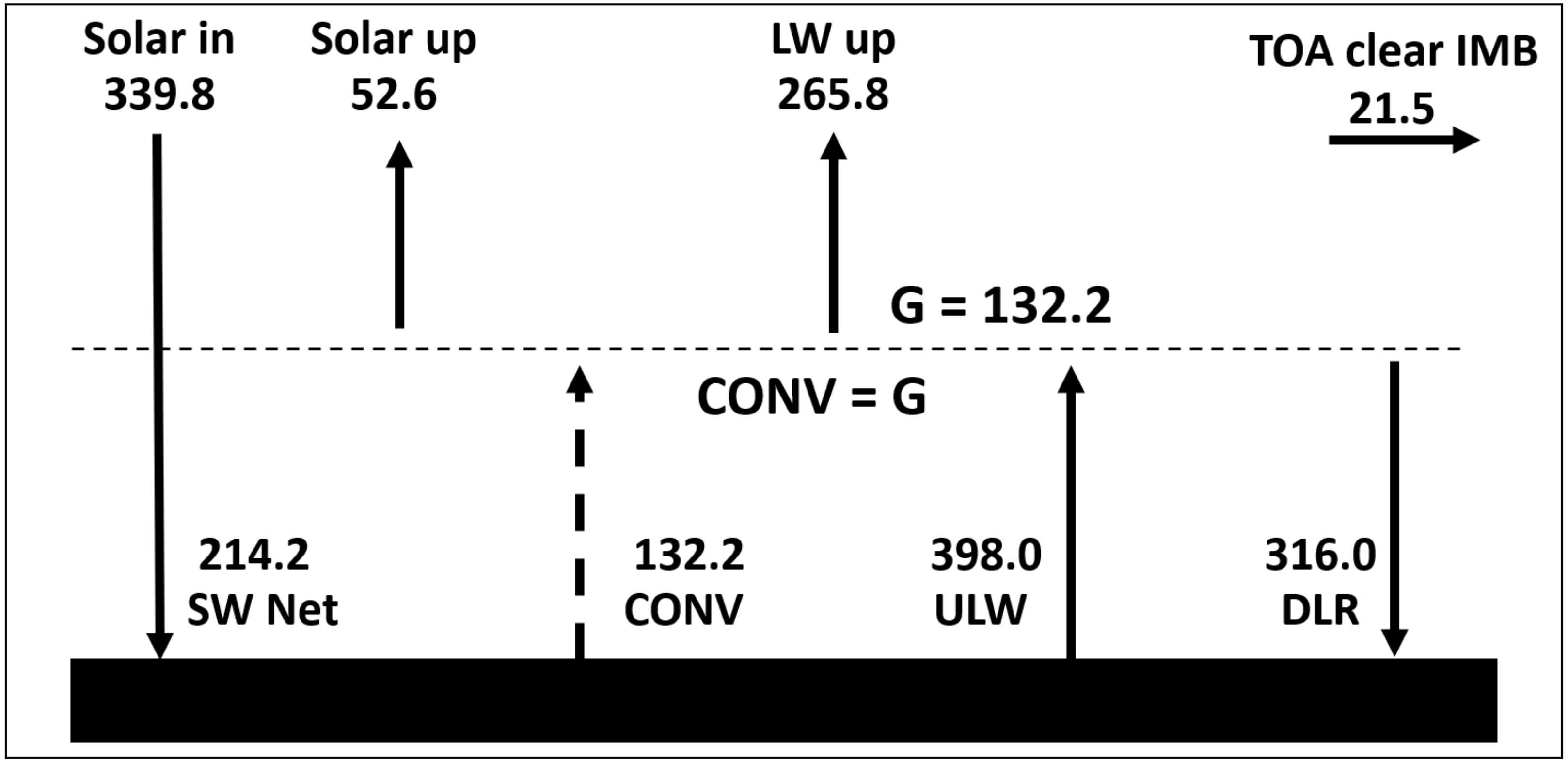
G = CONV = TOA LW/2

=> ULW = (3/2) TOA LW

Surface Net (clear-sky) = 132.2 = SFC LW up – TOA LW up ≡ G = 398.0 – 265.7 = 132.3, difference 0.1 Wm^{-2}

But the EBAF-TOA LW up is 265.8 Wm^{-2}, using this value, G = 132.2 Wm^{-2}.

The equation is EXACT again



Surface radiative surplus from G balanced by CONV

G = CONV = TOA LW/2

=> ULW = (3/2) TOA LW

=> ULW + CONV = 2 TOA LW (Eq.2)

Surface Net (clear-sky) = 132.2 = SFC LW up – TOA LW up ≡ G (clear-sky) = 398.0 – 265.8 = 132.2 Wm^{-2}

=> Surface LW up ≡ TOA LW up (clear-sky) + G (clear-sky) = (3/2) TOA LW up (clear-sky)

=> Surface LW up + Surface Net (clear-sky) = Surface Total (clear-sky) = 2 × TOA LW up (clear-sky) **(Eq.2)**

CERES EBAF Ed2.8 Global means (Mar 2000 – Feb 2016)

Clear-sky

Rose et al., 27th STM (2017)

Eq. (1) $A - E = A_0/2$

530.59 – 398.40 = 265.59 /2 – 0.60 Wm^{-2}

Eq. (2) $A = 2A_0$

530.59 = 2 × 265.59 – 0.59 Wm^{-2}

Clear Sky	Ed2.8
TOA SW Insolation	339.87
TOA SW Up	*52.50*
TOA LW Up	*265.59*
SFC SW Down	244.06
SFC SW Up	29.74
SFC LW Down	316.27
SFC LW Up	398.40

CERES EBAF Ed2.8 Global means (Mar 2000 – Feb 2016)

Clear-sky

Eq. (1) $\mathbf{A} - \mathbf{E} = \mathbf{A_0/2}$

530.59 – 398.40 = 265.59 /2 – 0.60 Wm^{-2}

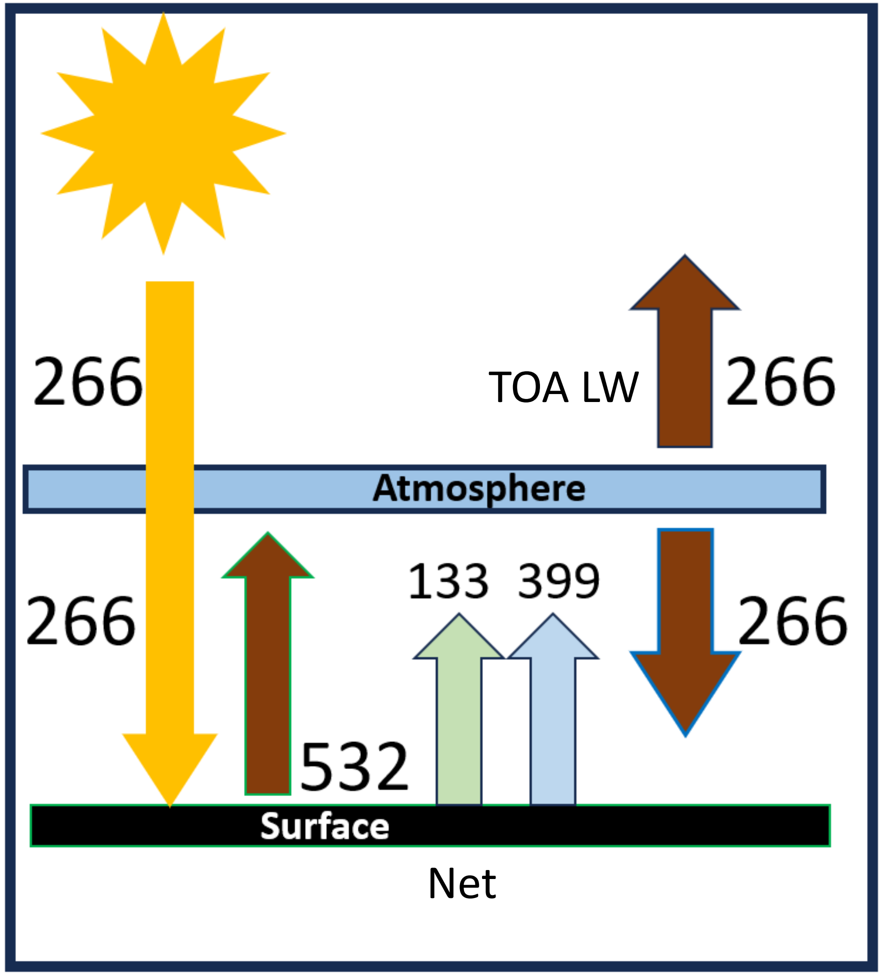
Eq. (2) $\mathbf{A} = \mathbf{2A_0}$

530.59 = 2 × 265.59 – 0.59 Wm^{-2}

Trenberth's ratios:

$\Delta A : A_0 : E : A$ = 1 : 2 : 3 : 4 (clear-sky) justified within EEI

Trenberth's Earth-like geometry



CERES EBAF Ed2.8 Global means (Mar 2000 – Feb 2016)

Clear-sky

Eq. (1) $A - E = A_0/2$

530.59 – 398.40 = 265.59 /2 – 0.60 Wm^{-2}

Eq. (2) $A = 2A_0$

530.59 = 2 × 265.59 – 0.59 Wm^{-2}

Trenberth's ratios:

$\Delta A : A_0 : E : A$ = **1 : 2 : 3 : 4 (clear-sky)**

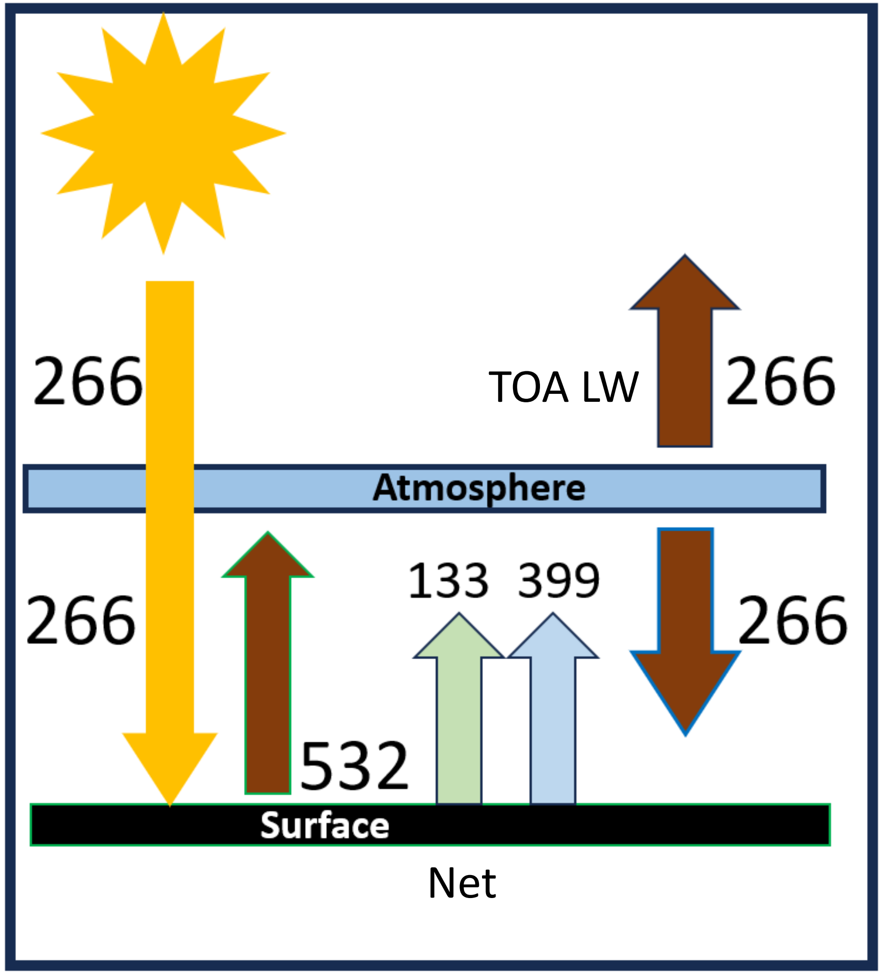
justified within EEI =>

$E = (3/2)\,A_0$ 398.40 = (3/2) × 265.59 + 0.015

$G = OLR/2$ 398.40 – 265.59 = 132.81 = 265.59/2 – 0.015

$g = 1/3$ (398.40 – 265.59) / 398.40 = 0.33336

Trenberth's Earth-like geometry



CERES EBAF Ed2.8 Global means (Mar 2000 – Feb 2016)

Clear-sky

Eq. (1) $A - E = A_0/2$

530.59 – 398.40 = 265.59 /2 – 0.60 Wm^{-2}

Eq. (2) $A = 2A_0$

530.59 = 2 × 265.59 – 0.59 Wm^{-2}

Trenberth's ratios:

$\Delta A : A_0 : E : A$ = **1 : 2 : 3 : 4 (clear-sky)**

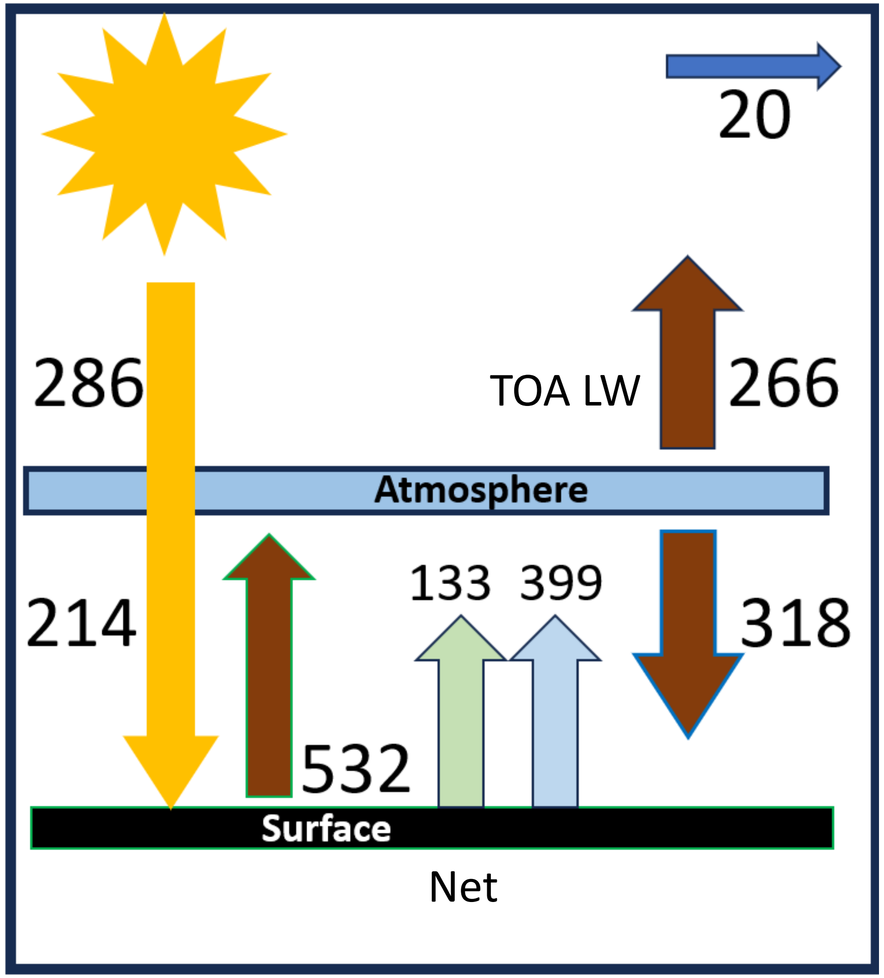
justified within EEI =>

$E = (3/2)\,A_0$ 398.40 = (3/2) × 265.59 + 0.015

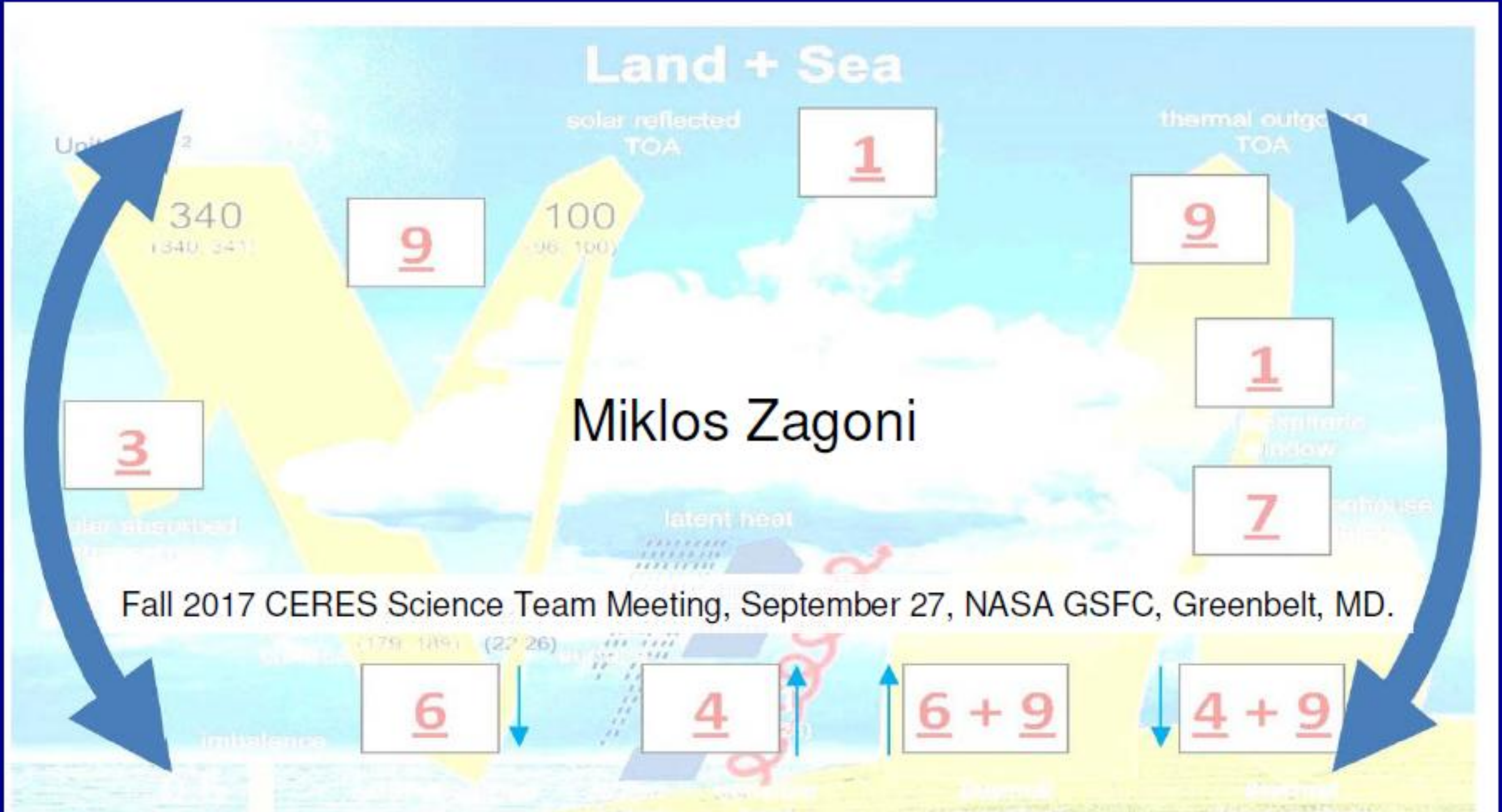
$G = OLR/2$ 398.40 – 265.59 = 132.81 = 265.59/2 – 0.015

$g = 1/3$ (398.40 – 265.59) / 398.40 = 0.33336

More realistic geometry



Patterns in the CERES Global Mean Data



https://ceres.larc.nasa.gov/documents/STM/2017-09/30_Patterns_CERES_Zagoni_Fall2017_v3.pdf

Pattern 1. SFC energy in = 2 × TOA LW out

Clear-sky	Ed2.8
TOA SW in	339.87
TOA SW up	52.50
TOA LW up	265.59
SFC SW down	244.06
SFC SW up	29.74
SFC SW in (down – up)	214.32
SFC LW down	316.27
SFC SW + LW absorbed	530.59
SFC LW up	398.40
SFC SW + LW net	132.19
G = SFC LW up – TOA LW up	132.81
2TOA LW up	531.18
Diff	–0.59

Clear-sky, Ed2.8
Surface energy absorbed
SW + LW (Wm^{-2}):

(SW down – SW up) + LW down
= (244.06 – 29.74) + 316.27
= 214.32 + 316.27
= 530.59

TOA LW out = 265.59
2 × TOA LW out =
= 531.18

Diff = –0.59 Wm^{-2}

$$\boxed{214.32 + 316.27 = 2 \times 265.59 - 0.59}$$

Clear-sky, Ed2.8, time period 2001-2015, climate year:

	SW down	SW up	SW abs	LW abs	E(SFC)	OLR	2OLR	Diff
CLIM 1	255.35	31.34	224.01	306.27	530.28	262.43	524.86	5.42
CLIM 2	251.86	29.86	222	307.98	529.98	262.78	525.56	4.42
CLIM 3	246.89	30.09	216.8	311.09	527.89	263.42	526.84	1.05
CLIM 4	242.26	31.63	210.63	315.14	525.77	265.01	530.02	-4.25
CLIM 5	237.33	32.21	205.12	320.27	525.39	267.18	534.36	-8.97
CLIM 6	233.41	28.67	204.74	325.70	530.44	269.05	538.1	-7.66
CLIM 7	231.63	25.72	205.91	328.35	534.26	269.75	539.5	-5.24
CLIM 8	233.65	24.42	209.23	327.05	536.28	269.12	538.24	-1.96
CLIM 9	239.24	25.80	213.44	321.82	535.26	267.58	535.16	0.1
CLIM 10	247.06	29.92	217.14	315.28	532.42	265.25	530.5	1.92
CLIM 11	253.97	33.73	220.24	309.57	529.81	263.26	526.52	3.29
CLIM 12	256.35	33.52	222.83	306.74	529.57	262.38	524.76	4.81
Average	244.08	29.74	214.34	316.27	530.61	265.60	531.20	-0.59

$$214.34 + 316.27 = 530.61 = 2 \times 265.60 - 0.59$$

E(SFC in, clear-sky) = (SW down – SW up) + LW in = 2OLR – EEI

Pattern 2. SFC Net = G

Clear-sky	Ed2.8
TOA SW in	339.87
TOA SW up	52.50
TOA LW up	265.59
SFC SW down	244.06
SFC SW up	29.74
SFC SW in	214.32
SFC LW in	316.27
SFC SW + LW absorbed	530.59
SFC LW up	398.40
SFC Net	**132.19**
G	**132.81**
Diff	–0.62

SFC Net Flux (non-radiative)
= SFC (SW in + LW in)
– SFC LW up

SFC Net = 214.32 + 316.27
– 398.40
= **132.19**

G = SFC LW up – TOA LW up
= ULW – OLR =
= **132.81**

Diff ($W\ m^{-2}$)
= –0.62

Clear-sky, Ed2.8, climate year, 2001-2015

	ULW	OLR	G	Net SFC	ULW+G	2OLR	Diff
CLIM 1	388.3	262.43	125.87	141.98	514.17	524.86	10.69
CLIM 2	389.89	262.78	127.11	140.09	517	525.56	8.56
CLIM 3	393.31	263.42	129.89	134.57	523.2	526.84	3.64
CLIM 4	398.51	265.01	133.5	127.26	532.01	530.02	-1.99
CLIM 5	403.29	267.18	136.11	122.09	539.4	534.36	-5.04
CLIM 6	407.64	269.05	138.59	122.8	546.23	538.1	-8.13
CLIM 7	409.1	269.75	139.35	125.17	548.45	539.5	-8.95
CLIM 8	407.83	269.12	138.71	128.46	546.54	538.24	-8.30
CLIM 9	403.85	267.58	136.27	131.41	540.12	535.16	-4.96
CLIM 10	397.76	265.25	132.51	134.65	530.27	530.5	0.23
CLIM 11	392.27	263.26	129.01	137.54	521.28	526.52	5.24
CLIM 12	389.00	262.38	126.62	140.57	515.62	524.76	9.14
average	**398.39**	**265.60**	**132.80**	**132.22**	**531.19**	**531.20**	**0.01**

ULW – G = OLR (def.); **Data**: G = Net SFC (= SH+LH)

ULW + G = 2OLR, Diff = **0.01** (!!!) W/m^2 =>

G = OLR/2 <=> g = G/ULW = 1/3

CERES_EBAF_Ed4.2.1_Subset_200101-202412.nc

283	411,5403	270,6774	140,8629	0,34228
284	410,9629	270,4989	140,464	0,34179
285	407,6057	268,9713	138,6344	0,34012
286	402,3427	266,4363	135,9064	0,33779
287	396,1337	264,3171	131,8166	0,33276
288	392,9634	263,6637	129,300	0,32904
289	sfc-lw-up-cl	toa-lw-cl	G	g
290	**398,9818**	**265,9905**	**132,9913**	**0,33333**

Clear-sky	N	Geometry	CERES
Surface LW up	3	399	398.9818
TOA LW up	2	266	265.9905
G	1	133	132.9913
g	1/3	1/3	0.33333

G (clear-sky) = TOA LW up (clear-sky)/2 – 0.004 Wm^{-2}

Surface LW up = (3/2) TOA LW up – 0.004 Wm^{-2}

CERES Science

- Globally averaged, the surface has a net surplus of radiant energy, balanced by convection, equals half of the effective emission in the clear-sky.

$$\mathrm{CONV} = \mathrm{OLR}/2$$

- Globally averaged, the surface has a net surplus compared to TOA, the greenhouse effect, equals half of the effective emission in the clear-sky.

$$G = \mathrm{CONV} = \mathrm{OLR}/2$$

Concluding:

$$\boxed{g(\text{clear}) = 1/3.}$$

Originally there is no net surplus of radiant energy at the surface. There is water at the surface, exposed to the Sun, cooling both radiatively and convectively. Evaporation supplies water vapor into the atmosphere, generating a surplus of radiant energy: the greenhouse effect. It is the hydrological cycle that sets both convective cooling and radiative heating at their limit of OLR/2 to close the balance at the surface.

Extended version

All-sky

Creating the all-sky equations

Eq. (1) **SFC Net = $A – E = A_0/2$** (clear-sky, net)

Eq. (2) **SFC Tot = $A = 2A_0$** (clear-sky, total)

Separating atmospheric radiation transfer from the longwave cloud effect (LWCRE):

Eq. (3) **SFC Net = $A – E = (A_0 – L)/2$** (all-sky, net, incl LWCRE)

Eq. (4) **SFC Tot = $A = 2A_0 + L$** (all-sky, tot, incl LWCRE)

The four equations

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})

Eq. (1) SFC SW down – SW up + LW down – LW up (clear) = TOA LW (clear)/2

240.8680 – 29.0724 + 317.4049 – 398.5211 = 266.0122 /2

Eq. (2) SFC SW down – SW up + LW down (clear) = 2 × TOA LW (clear)

240.8680 – 29.0724 + 317.4049 = 2 × 266.0122

Eq. (3) SFC SW down – SW up + LW down – LW up (all) = [TOA LW (all) – LWCRE]/2

186.8544 – 23.1629 + 345.0108 – 398.7550 = (240.2450 – 25.7672)/2

Eq. (4) SFC SW down – SW up + LW down (all) = 2 × TOA LW (all) + LWCRE

186.8544 – 23.1629 + 345.0108 = 2 × 240.2450 + 25.7672

The four equations

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})

Eq. (1)	**SFC SW down – SW up + LW down – LW up (clear)**	**= TOA LW (clear)/2**		
	240.8680 – 29.0724 + 317.4049 – 398.5211	**= 266.0122 /2**	**– 2.3267**	
Eq. (2)	**SFC SW down – SW up + LW down (clear)**	**= 2 × TOA LW (clear)**		
	240.8680 – 29.0724 + 317.4049	**= 2 × 266.0122**	**– 2.8238**	
Eq. (3)	**SFC SW down – SW up + LW down – LW up (all)**	**= [TOA LW (all) – LWCRE]/2**		
	186.8544 – 23.1629 + 345.0108 – 398.7550	**= (240.2450 – 25.7672)/2**	**+ 2.7083**	
Eq. (4)	**SFC SW down – SW up + LW down (all)**	**= 2 × TOA LW (all) + LWCRE**		
	186.8544 – 23.1629 + 345.0108	**= 2 × 240.2450 + 25.7672**	**+ 2.4450**	

The four equations

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})

Eq. (1) SFC SW down – SW up + LW down – LW up (clear) = TOA LW (clear)/2

240.8680 – 29.0724 + 317.4049 – 398.5211 = 266.0122 /2 – 2.3267

–2.5753

Eq. (2) SFC SW down – SW up + LW down (clear) = 2 × TOA LW (clear)

240.8680 – 29.0724 + 317.4049 = 2 × 266.0122 – 2.8238

Eq. (3) SFC SW down – SW up + LW down – LW up (all) = [TOA LW (all) – LWCRE]/2

186.8544 – 23.1629 + 345.0108 – 398.7550 = (240.2450 – 25.7672)/2 + 2.7083

+2.5767

Eq. (4) SFC SW down – SW up + LW down (all) = 2 × TOA LW (all) + LWCRE

186.8544 – 23.1629 + 345.0108 = 2 × 240.2450 + 25.7672 + 2.4450

The four equations

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})

Eq. (1)	**SFC SW down – SW up + LW down – LW up (clear) = TOA LW (clear)/2**		
	240.8680 – 29.0724 + 317.4049 – 398.5211 = 266.0122 /2	– 2.3267	–2.5753
Eq. (2)	**SFC SW down – SW up + LW down (clear) = 2 × TOA LW (clear)**		
	240.8680 – 29.0724 + 317.4049 = 2 × 266.0122	– 2.8238	
Eq. (3)	**SFC SW down – SW up + LW down – LW up (all) = [TOA LW (all) – LWCRE]/2**		
	186.8544 – 23.1629 + 345.0108 – 398.7550 = (240.2450 – 25.7672)/2	+ 2.7083	+2.5767
Eq. (4)	**SFC SW down – SW up + LW down (all) = 2 × TOA LW (all) + LWCRE**		
	186.8544 – 23.1629 + 345.0108 = 2 × 240.2450 + 25.7672	+ 2.4450	
	Mean		0.0007

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2}) **Mean: 0.0007**

Eq. (1)	**SFC SW down**	**– SW up**	**+ LW down**	**– LW up**	**(clear)**	**= TOA LW (clear)/2**	
	240.8680	**– 29.0724**	**+ 317.4049**	**– 398.5211**		**= 266.0122 /2**	**– 2.3267**
Eq. (2)	**SFC SW down**	**– SW up**	**+ LW down**		**(clear)**	**= 2 × TOA LW (clear)**	
	240.8680	**– 29.0724**	**+ 317.4049**			**= 2 × 266.0122**	**– 2.8238**
Eq. (3)	**SFC SW down**	**– SW up**	**+ LW down**	**– LW up**	**(all)**	**= [TOA LW (all) – LWCRE]/2**	
	186.8544	**– 23.1629**	**+ 345.0108**	**– 398.7550**		**= (240.2450 – 25.7672)/2**	**+ 2.7083**
Eq. (4)	**SFC SW down**	**– SW up**	**+ LW down**		**(all)**	**= 2 × TOA LW (all) + LWCRE**	
	186.8544	**– 23.1629**	**+ 345.0108**			**= 2 × 240.2450 + 25.7672**	**+ 2.4450**

CERES EBAF Ed4.1	Version 3, 22 years	(April 2000 – March 2022) (Wm^{-2})	**Mean:** **0.0007**
CERES EBAF Ed4.2	Version 4, 22 years	(April 2000 – March 2022) (Wm^{-2})	**0.5994**

Eq. (1)	**SFC SW down**	**– SW up**	**+ LW down**	**– LW up**	**(clear)**	**= TOA LW (clear)/2**	
	240.8680	**– 29.0724**	**+ 317.4049**	**– 398.5211**		**= 266.0122 /2**	**– 2.3267**
	241.0969	**– 29.7521**	**+ 317.8744**	**– 398.5890**		**= 265.9594 /2**	**– 2.3495**

Eq. (2)	**SFC SW down**	**– SW up**	**+ LW down**		**(clear)**	**= 2 × TOA LW (clear)**	
	240.8680	**– 29.0724**	**+ 317.4049**			**= 2 × 266.0122**	**– 2.8238**
	241.0969	**– 29.7521**	**+ 317.8744**			**= 2 × 265.9594**	**– 2.6996**

Eq. (3)	**SFC SW down**	**– SW up**	**+ LW down**	**– LW up**	**(all)**	**= [TOA LW (all) – LWCRE]/2**	
	186.8544	**– 23.1629**	**+ 345.0108**	**– 398.7550**		**= (240.2450 – 25.7672)/2**	**+ 2.7083**
	187.1451	**– 23.4950**	**+ 346.1057**	**– 398.4220**		**= (240.3317 – 25.6277)/2**	**+ 3.9818**

Eq. (4)	**SFC SW down**	**– SW up**	**+ LW down**		**(all)**	**= 2 × TOA LW (all) + LWCRE**	
	186.8544	**– 23.1629**	**+ 345.0108**			**= 2 × 240.2450 + 25.7672**	**+ 2.4450**
	187.1451	**– 23.4950**	**+ 346.1057**			**= 2 × 240.3317 + 25.6277**	**+ 3.4647**

CERES EBAF Ed4.1	Version 3, 22 years	(April 2000 – March 2022) (Wm^{-2})	**Mean:** **0.0007**
CERES EBAF Ed4.2	Version 4, 22 years	(April 2000 – March 2022) (Wm^{-2})	**0.5994**
CERES EBAF Ed4.2	Version 4, 24 years	(April 2000 – March 2024) (Wm^{-2})	**0.7147**

Eq. (1)	**SFC SW down**	**– SW up**	**+ LW down**	**– LW up**	**(clear)**	**= TOA LW (clear)/2**	
	240.8680	**– 29.0724**	**+ 317.4049**	**– 398.5211**		**= 266.0122 /2**	**– 2.3267**
	241.0969	**– 29.7521**	**+ 317.8744**	**– 398.5890**		**= 265.9594 /2**	**– 2.3495**
	241.0514	**– 29.7043**	**+ 318.0984**	**– 398.7742**		**= 265.9748 /2**	**– 2.3161**

Eq. (2)	**SFC SW down**	**– SW up**	**+ LW down**	**(clear)**	**= 2 × TOA LW (clear)**	
	240.8680	**– 29.0724**	**+ 317.4049**		**= 2 × 266.0122**	**– 2.8238**
	241.0969	**– 29.7521**	**+ 317.8744**		**= 2 × 265.9594**	**– 2.6996**
	241.0514	**– 29.7043**	**+ 318.0984**		**= 2 × 265.9748**	**– 2.5042**

Eq. (3)	**SFC SW down**	**– SW up**	**+ LW down**	**– LW up**	**(all)**	**= [TOA LW (all) – LWCRE]/2**	
	186.8544	**– 23.1629**	**+ 345.0108**	**– 398.7550**		**= (240.2450 – 25.7672)/2**	**+ 2.7083**
	187.1451	**– 23.4950**	**+ 346.1057**	**– 398.4220**		**= (240.3317 – 25.6277)/2**	**+ 3.9818**
	187.1756	**– 23.4607**	**+ 346.3158**	**– 398.6162**		**= (240.3894 – 25.5854)/2**	**+ 4.0126**

Eq. (4)	**SFC SW down**	**– SW up**	**+ LW down**	**(all)**	**= 2 × TOA LW (all) + LWCRE**	
	186.8544	**– 23.1629**	**+ 345.0108**		**= 2 × 240.2450 + 25.7672**	**+ 2.4450**
	187.1451	**– 23.4950**	**+ 346.1057**		**= 2 × 240.3317 + 25.6277**	**+ 3.4647**
	187.1756	**– 23.4607**	**+ 346.3158**		**= 2 × 240.3894 + 25.5854**	**+ 3.6665**

			Mean:
CERES EBAF Ed4.1	Version 3, 22 years	(April 2000 – March 2022) (Wm^{-2})	0.0007
CERES EBAF Ed4.2	Version 4, 22 years	(April 2000 – March 2022) (Wm^{-2})	0.5994
CERES EBAF Ed4.2	Version 4, 24 years	(April 2000 – March 2024) (Wm^{-2})	0.7147
CERES EBAF Ed4.2.1	Version 1, 22 years	(April 2000 – March 2022) (Wm^{-2})	0.5995

Eq. (1)	SFC SW down	– SW up	+ LW down	– LW up	(clear)	= TOA LW (clear)/2	
	240.8680	– 29.0724	+ 317.4049	– 398.5211		= 266.0122 /2	– 2.3267
	241.0969	– 29.7521	+ 317.8744	– 398.5890		= 265.9594 /2	– 2.3495
	241.0514	– 29.7043	+ 318.0984	– 398.7742		= 265.9748 /2	– 2.3161
	241.0969	– 29.7521	+ 317.8744	– 398.5890		= 265.9594 /2	– 2.3495

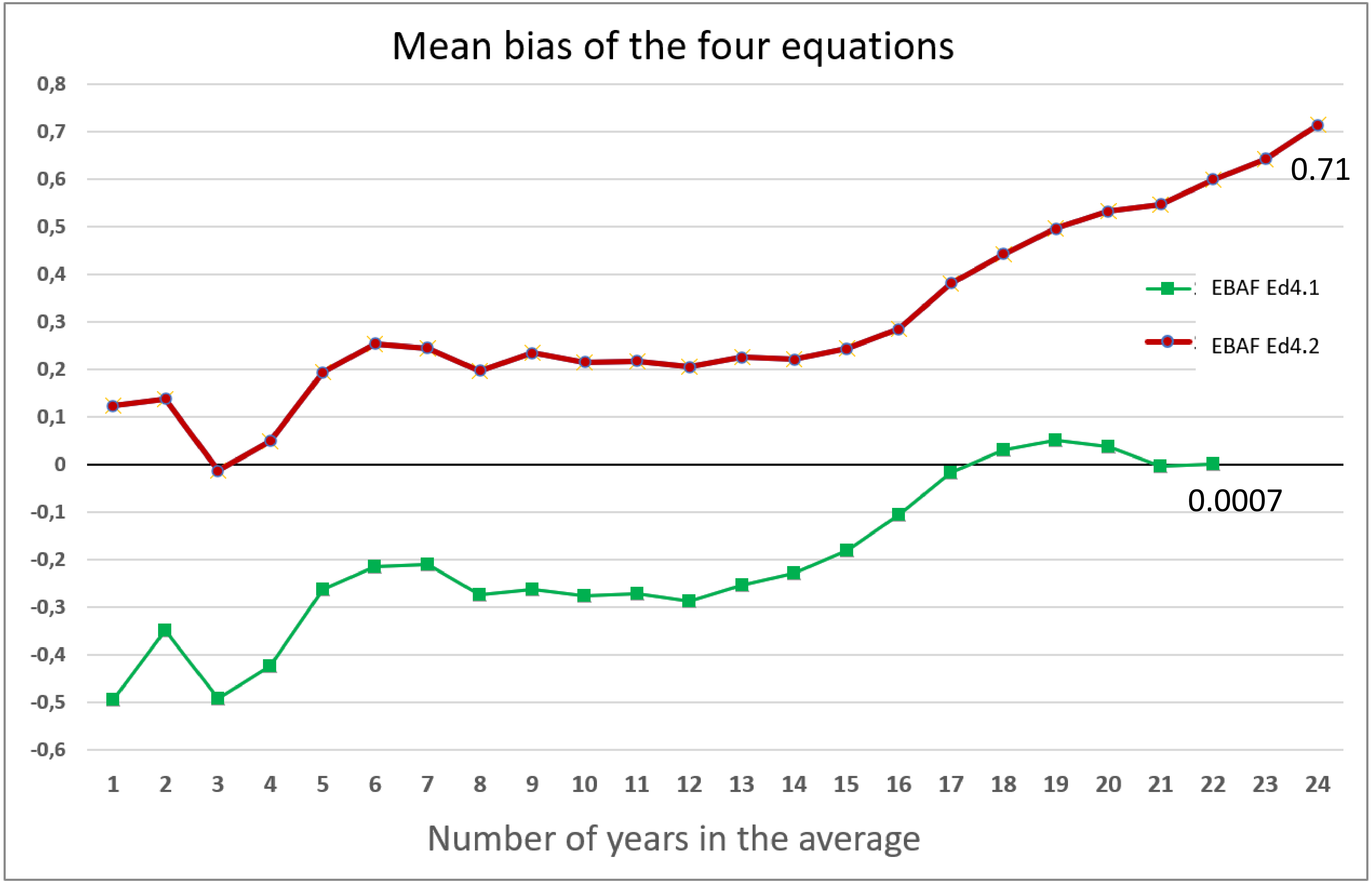
Eq. (2)	SFC SW down	– SW up	+ LW down	(clear)	= 2 × TOA LW (clear)	
	240.8680	– 29.0724	+ 317.4049		= 2 × 266.0122	– 2.8238
	241.0969	– 29.7521	+ 317.8744		= 2 × 265.9594	– 2.6996
	241.0514	– 29.7043	+ 318.0984		= 2 × 265.9748	– 2.5042
	241.0969	– 29.7521	+ 317.8744		= 2 × 265.9594	– 2.6996

Eq. (3)	SFC SW down	– SW up	+ LW down	– LW up	(all)	= [TOA LW (all) – LWCRE]/2	
	186.8544	– 23.1629	+ 345.0108	– 398.7550		= (240.2450 – 25.7672)/2	+ 2.7083
	187.1451	– 23.4950	+ 346.1057	– 398.4220		= (240.3317 – 25.6277)/2	+ 3.9818
	187.1756	– 23.4607	+ 346.3158	– 398.6162		= (240.3894 – 25.5854)/2	+ 4.0126
	187.1451	– 23.4950	+ 346.1059	– 398.4220		= (240.3317 – 25.6277)/2	+ 3.9818

Eq. (4)	SFC SW down	– SW up	+ LW down	(all)	= 2 × TOA LW (all) + LWCRE	
	186.8544	– 23.1629	+ 345.0108		= 2 × 240.2450 + 25.7672	+ 2.4450
	187.1451	– 23.4950	+ 346.1057		= 2 × 240.3317 + 25.6277	+ 3.4647
	187.1756	– 23.4607	+ 346.3158		= 2 × 240.3894 + 25.5854	+ 3.6665
	187.1451	– 23.4950	+ 346.1059		= 2 × 240.3317 + 25.6277	+ 3.4649

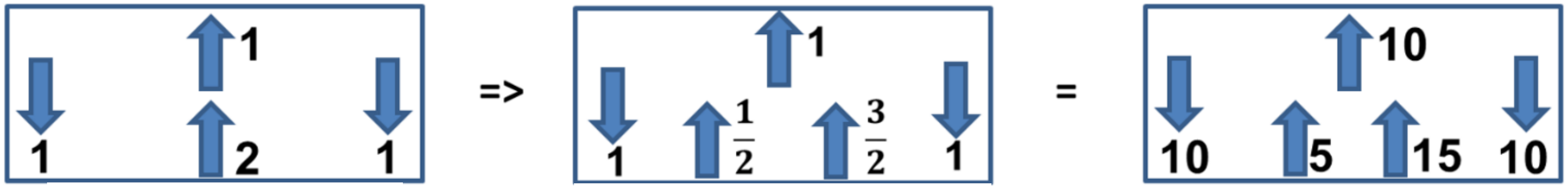
CERES EBAF Ed4.1	Version 3, 22 years	(April 2000 – March 2022) (Wm^{-2})	**Mean:**	**0.0007**
CERES EBAF Ed4.2	Version 4, 22 years	(April 2000 – March 2022) (Wm^{-2})		**0.5994**
CERES EBAF Ed4.2	Version 4, 24 years	(April 2000 – March 2024) (Wm^{-2})		**0.7147**
CERES EBAF Ed4.2.1	Version 1, 22 years	(April 2000 – March 2022) (Wm^{-2})		**0.5995**
CERES EBAF Ed4.2.1	Version 1, 24 years	(April 2000 – March 2024) (Wm^{-2})		**0.7036**

Eq. (1)	**SFC SW down**	**– SW up**	**+ LW down**	**– LW up**	**(clear)**	**= TOA LW (clear)/2**	
	240.8680	**– 29.0724**	**+ 317.4049**	**– 398.5211**		**= 266.0122 /2**	**– 2.3267**
	241.0969	**– 29.7521**	**+ 317.8744**	**– 398.5890**		**= 265.9594 /2**	**– 2.3495**
	241.0514	**– 29.7043**	**+ 318.0984**	**– 398.7742**		**= 265.9748 /2**	**– 2.3161**
	241.0969	**– 29.7521**	**+ 317.8744**	**– 398.5890**		**= 265.9594 /2**	**– 2.3495**
	241.0362	**–29.6972**	**+ 318.1063**	**– 398.8026**		**= 265.9732/2**	**– 2.3439**
Eq. (2)	**SFC SW down**	**– SW up**	**+ LW down**		**(clear)**	**= 2 × TOA LW (clear)**	
	240.8680	**– 29.0724**	**+ 317.4049**			**= 2 × 266.0122**	**– 2.8238**
	241.0969	**– 29.7521**	**+ 317.8744**			**= 2 × 265.9594**	**– 2.6996**
	241.0514	**– 29.7043**	**+ 318.0984**			**= 2 × 265.9748**	**– 2.5042**
	241.0969	**– 29.7521**	**+ 317.8744**			**= 2 × 265.9594**	**– 2.6996**
	241.0362	**–29.6972**	**+ 318.1063**			**=2×265.9732**	**– 2.5012**
Eq. (3)	**SFC SW down**	**– SW up**	**+ LW down**	**– LW up**	**(all)**	**= [TOA LW (all) – LWCRE]/2**	
	186.8544	**– 23.1629**	**+ 345.0108**	**– 398.7550**		**= (240.2450 – 25.7672)/2**	**+ 2.7083**
	187.1451	**– 23.4950**	**+ 346.1057**	**– 398.4220**		**= (240.3317 – 25.6277)/2**	**+ 3.9818**
	187.1756	**– 23.4607**	**+ 346.3158**	**– 398.6162**		**= (240.3894 – 25.5854)/2**	**+ 4.0126**
	187.1451	**– 23.4950**	**+ 346.1059**	**– 398.4220**		**= (240.3317 – 25.6277)/2**	**+ 3.9818**
	187.1513	**– 23.4547**	**+ 346.3226**	**– 398.6131**		**= (240.3894 – 25.5835)/2**	**+ 4.0032**
Eq. (4)	**SFC SW down**	**– SW up**	**+ LW down**		**(all)**	**= 2 × TOA LW (all) + LWCRE**	
	186.8544	**– 23.1629**	**+ 345.0108**			**= 2 × 240.2450 + 25.7672**	**+ 2.4450**
	187.1451	**– 23.4950**	**+ 346.1057**			**= 2 × 240.3317 + 25.6277**	**+ 3.4647**
	187.1756	**– 23.4607**	**+ 346.3158**			**= 2 × 240.3894 + 25.5854**	**+ 3.6665**
	187.1451	**– 23.4950**	**+ 346.1059**			**= 2 × 240.3317 + 25.6277**	**+ 3.4649**
	187.1513	**– 23.4547**	**+ 346.3226**			**= 2 × 240.3894 + 25.5835**	**+ 3.6565**

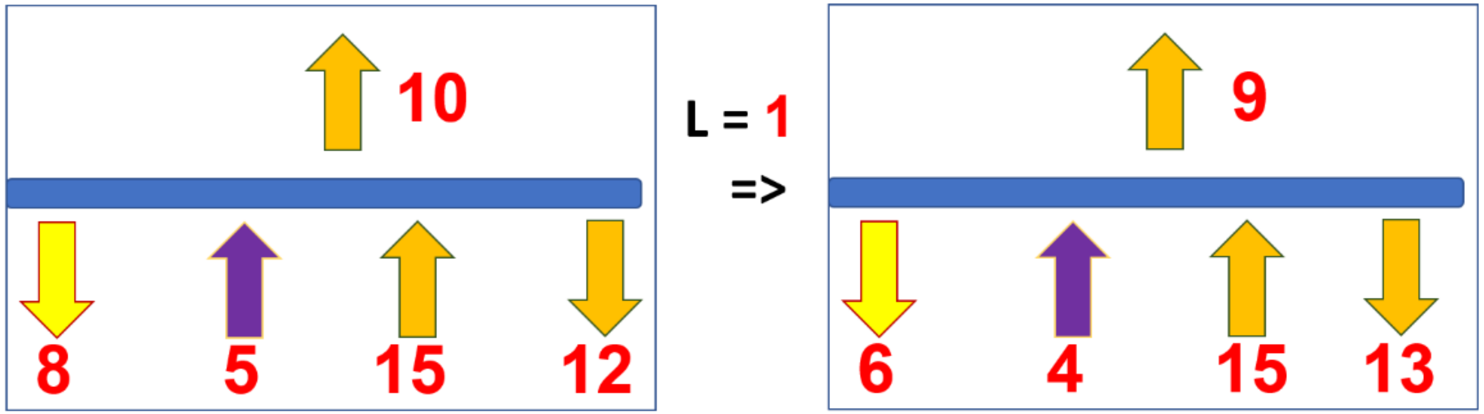

Mean bias of the four equations
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
-0,5
-0,6
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Number of years in the average
EBAF Ed4.1
EBAF Ed4.2
0.71
0.0007

The N-numbers, as solution of the equations

Pure geometry: No reference to GHGs



Eq.(2) $A = 2A_0$ (Clear-sky) Eq.(1) $\Delta A = A - E = A_0/2$



$8 + 12 - 15 = 10\ /\ 2$

$8 + 12 = 10 \times 2$

$6 + 13 - 15 = (9 - 1)/2$

$6 + 13 = 9 \times 2 + 1$

Eq. (1) SFC Net = $A_0/2$

Eq. (2) SFC Tot = $2A_0$

Clear-sky

Eq. (3) SFC Net = $(A_0 - L)/2$

Eq. (4) SFC Tot = $2A_0 + L$

All-sky

The flux components with LWCRE = 1

TOA LW	clear-sky =	**10**	TOA LW	all-sky =	**9**
SFC LW up	clear-sky =	**15**	SFC LW up	all-sky =	**15**
SFC LW down	clear-sky =	**12**	SFC LW down	all-sky =	**13**
SFC LW net	clear-sky =	**–3**	SFC LW net	all-sky =	**–2**
SFC SW net	clear-sky =	**8**	SFC SW net	all-sky =	**6**
SFC SW+LW net	clear-sky =	**5**	SFC SW+LW net	all-sky =	**4**
SFC SW+LW total	clear-sky =	**20**	SFC SW+LW total	all-sky =	**19**
G greenhouse effect	clear-sky =	**5**	G greenhouse effect	all-sky =	**6**
SWCRE (surface)	=	**–2**	LWCRE (surface. TOA)	=	**1**

Fit model to observation: CERES EBAF Ed4.2.1, 24 years, Jan 2001 — Dec 2024 data

Best fit: 1 unit = 1 = LWCRE = 26.68 ± 0.01 Wm^{-2}

CERES EBAF Ed4.2.1, January 2001 – December 2024

Eq. (1) 8 + 12 – 15 = 10/2; Eq. (2) 8 + 12 = 10 × 2; 1 = 26.68 Wm^{-2}

		N	N × Unit	EBAF Ed4.2.1	Difference (Wm^{-2})
Clear-Sky TOA	LW	40/4	266.80	265.99	-0.81
	SW	8/4	53.36	53.75	0.39
	Net	3/4	20.01	20.47	0.46
Clear-Sky Surface	LW down	12	320.16	318.32	-1.84
	LW up	15	400.20	398.98	-1.22
	LW net	–3	–79.98	–80.66	-0.68
	SW down	9	240.12	240.95	0.83
	SW up	1	26.68	29.65	2.97
	SW net	8	213.44	211.30	-2.14
	SW + LW net	5	133.40	130.64	-2.76
	G	5	133.40	132.99	-0.41

CERES EBAF Ed4.2.1, January 2001 – December 2024

Eq. (3) $6 + 13 - 15 = (9 - 1)/2$; Eq. (4) $6 + 13 = 9 \times 2 + 1$ $1 = 26.68\ \mathrm{Wm^{-2}}$

	All-sky	N	N × Unit	EBAF Ed4.2.1	Diff ($\mathrm{Wm^{-2}}$)
TOA	SW insolation	51/4	340.17	340.21	0.04
	SW up	15/4	100.05	98.88	-1.17
	LW up	36/4	240.12	240.45	0.33
	TOT net	0	0	0.88	0.88
Surface	SW down	7	186.76	187.11	0.35
	SW up	1	26.68	23.42	-3.26
	SW net	6	160.08	163.69	3.61
	LW down	13	346.84	346.50	-0.34
	LW up	15	400.20	398.78	-1.42
	LW net	–2	–53.36	-52.28	1.08
	TOT net	4	106.72	111.41	4.69
	CRE				
TOA	SW	–7/4	–46.69	-45.13	1.56
	LW	1	26.68	25.54	-1.14
	TOT	–3/4	-20.01	-19.59	0.42

CERES EBAF Ed4.2.1, 24-yr mean (Jan 2001 – Dec 2024)

Clear-sky	N	Theory	CERES
Surface LW up	15	400.20	398.9818
TOA LW up	10	266.80	265.9905
G	5	133.40	132.9913
g	5/15	1/3	0.33333

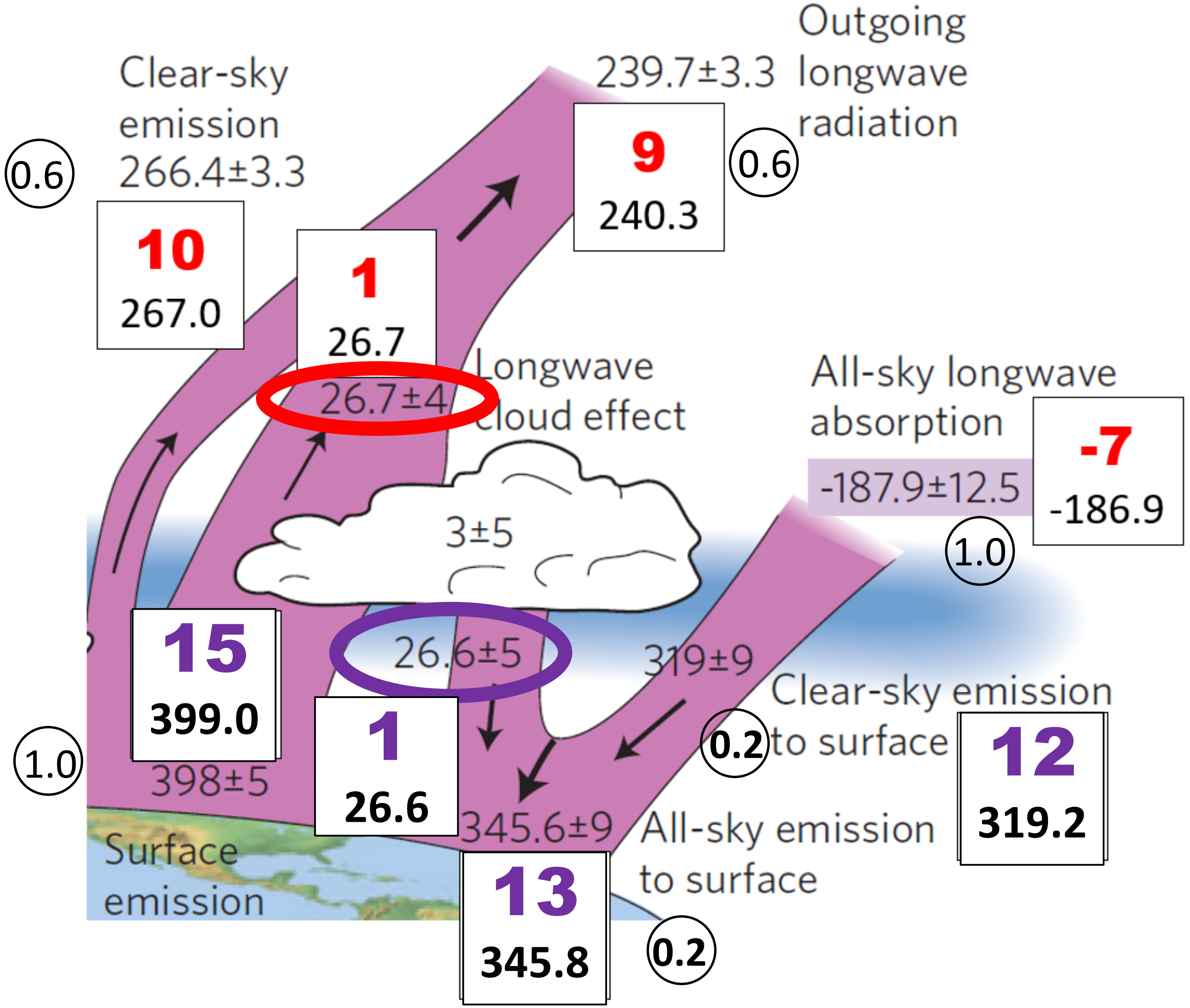
SFC LW up (clear) = (3/2) TOA LW up (clear) – 0.004 Wm^{-2}

All-sky	N	Theory	CERES
Surface LW up	15	400.20	398.72
TOA LW up	9	240.12	240.42
G	6	160.08	158.30
g	6/15	0.4	0.397

An update on Earth's energy balance in light of the latest global observations

Nature Geosci 2012

Graeme L. Stephens[1*], Juilin Li[1], Martin Wild[2], Carol Anne Clayson[3], Norman Loeb[4], Seiji Kato[4], Tristan L'Ecuyer[5], Paul W. Stackhouse Jr[4], Matthew Lebsock[1] and Timothy Andrews[6]



An update on Earth's energy balance in light of the latest global observations

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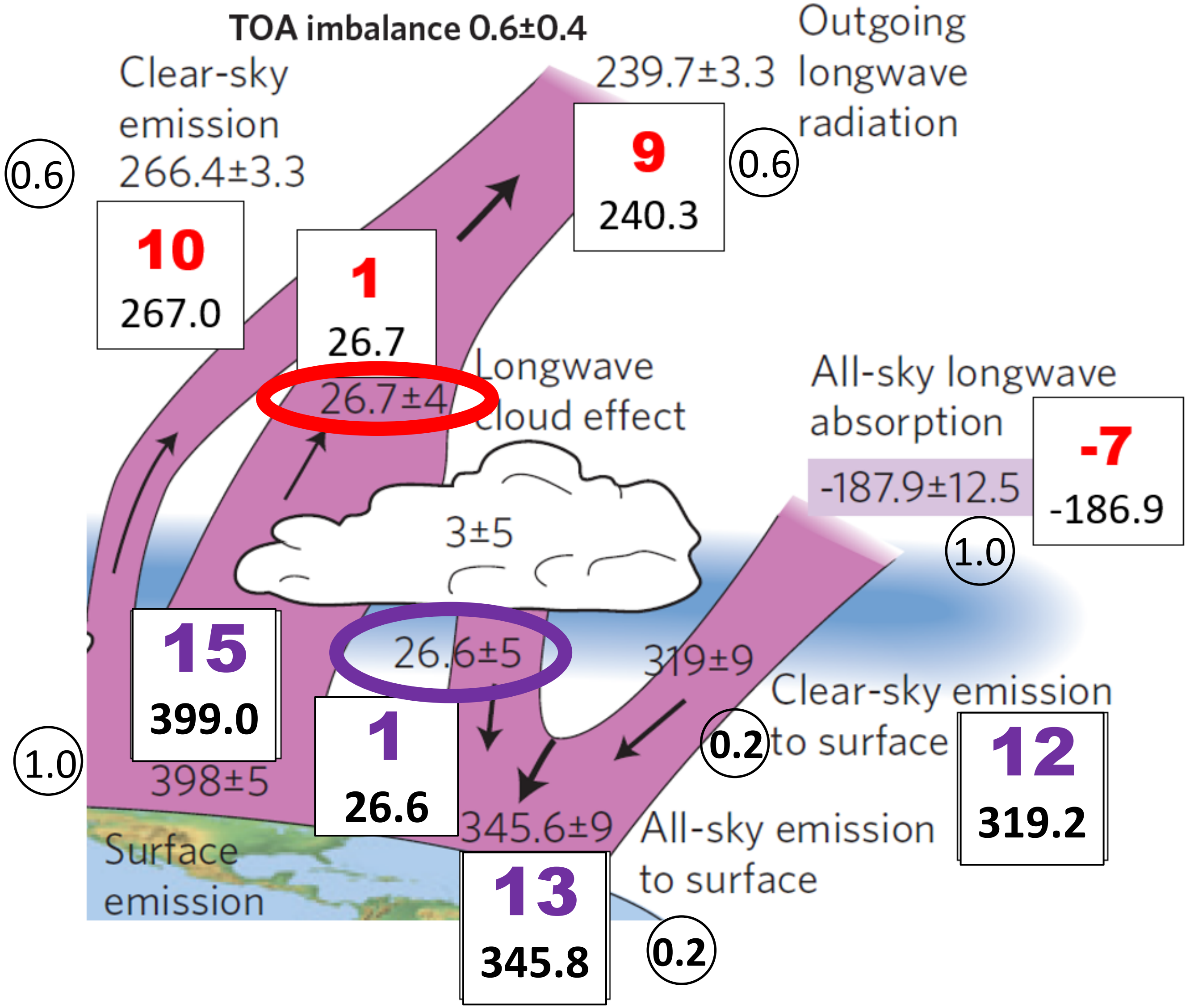
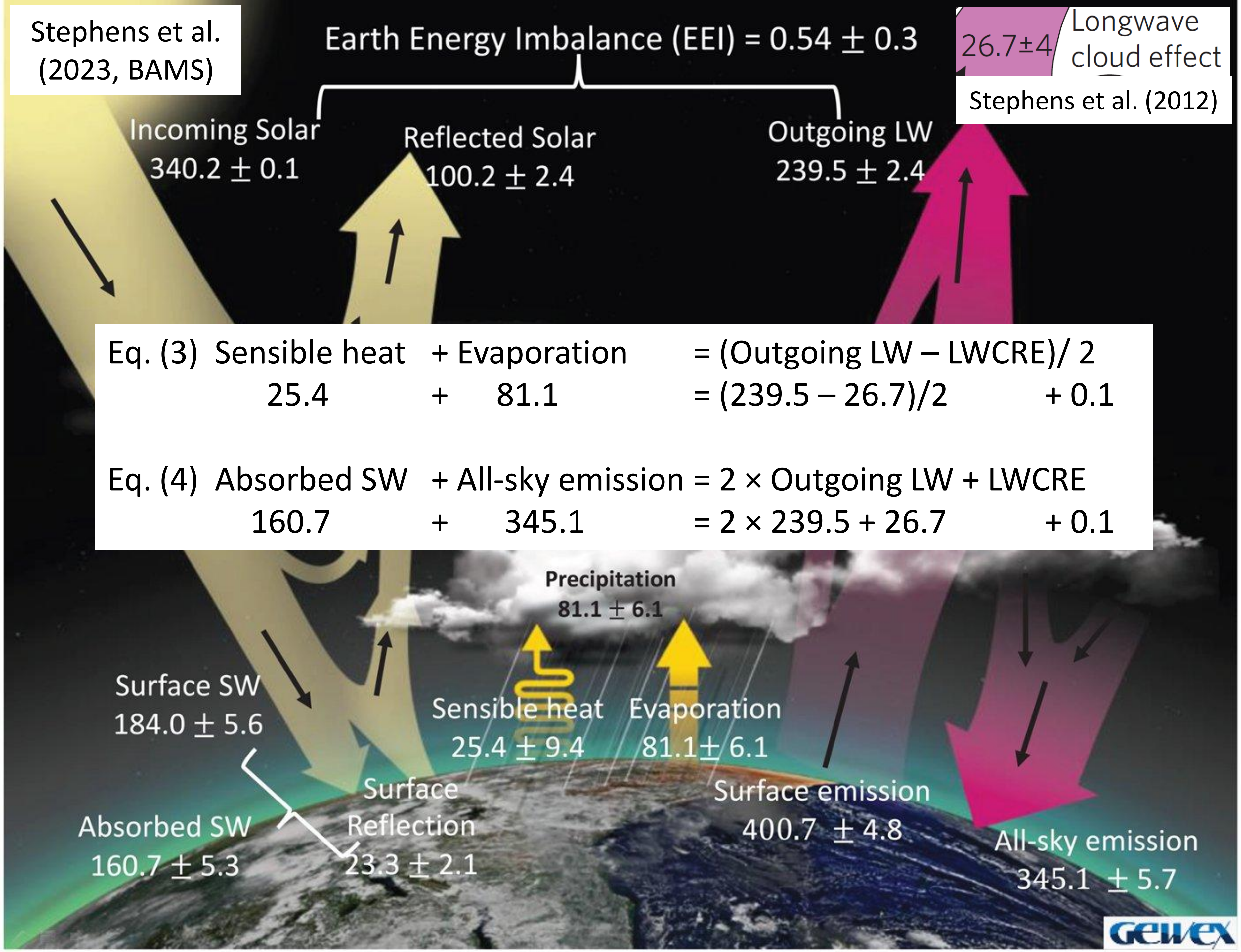


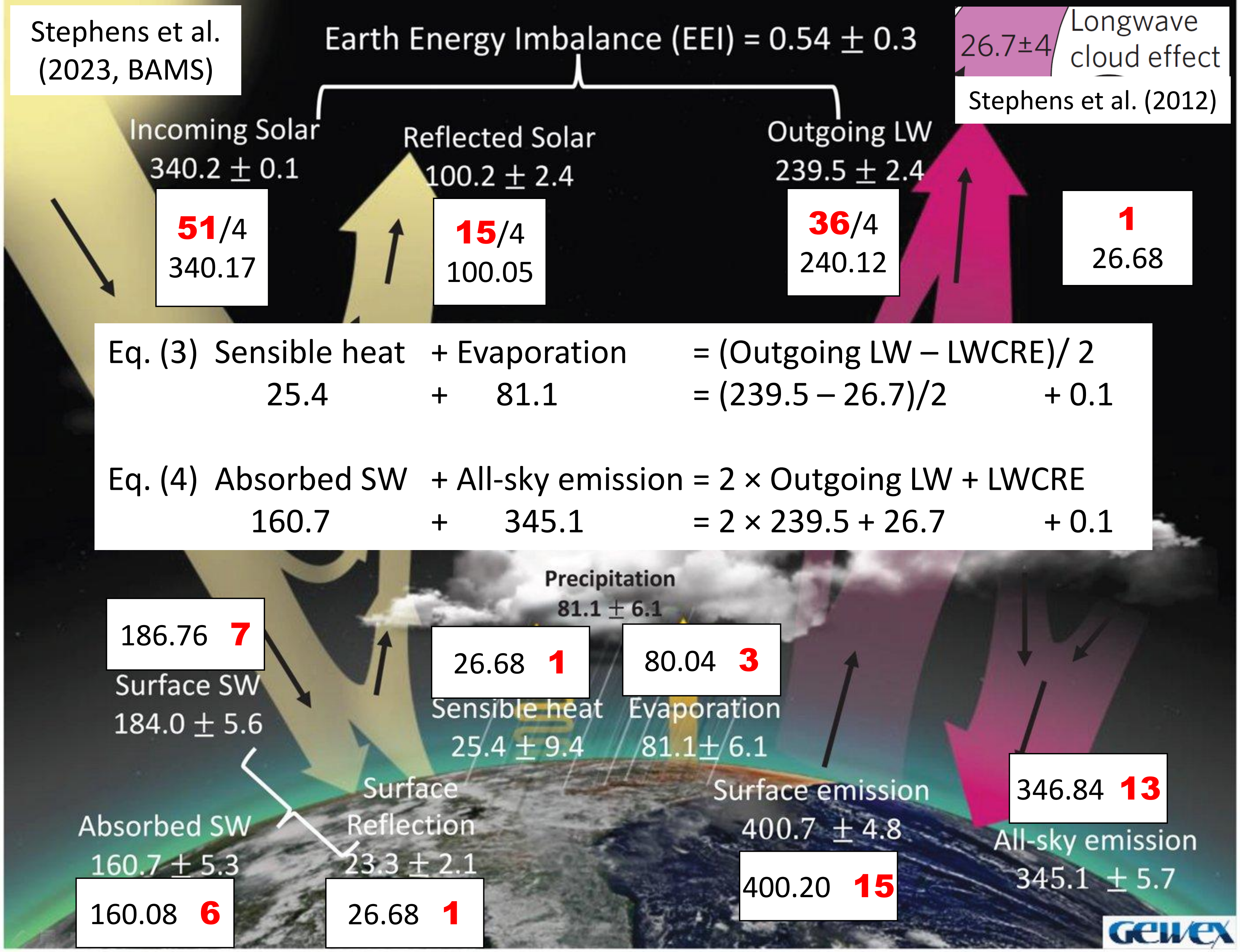


Eq. (3) Sensible heat + Evaporation = (Outgoing LW – LWCRE)/ 2

25.4 + 81.1 = (239.5 – 26.7)/2 + 0.1

Eq. (4) Absorbed SW + All-sky emission = 2 × Outgoing LW + LWCRE

160.7 + 345.1 = 2 × 239.5 + 26.7 + 0.1



Eq. (3) Sensible heat + Evaporation = (Outgoing LW – LWCRE)/ 2

25.4 + 81.1 = (239.5 – 26.7)/2 + 0.1

Eq. (4) Absorbed SW + All-sky emission = 2 × Outgoing LW + LWCRE

160.7 + 345.1 = 2 × 239.5 + 26.7 + 0.1