EBAF Ed4.2.1 Data vs. Theory

Miklos Zagoni
Budapest, Hungary

Spring CERES Science Team Meeting
May 13-15, 2025, NASA LaRC, Hampton, VA.

Remote Presentation

Theory: Schwarzschild (1906)
Plane-parallel, gray, LTE

Ueber das Gleichgewicht der Sonnenatmosphäre

Von

K. Schwarzschild.

Vorgelegt in der Sitzung vom 13. Januar 1906.

On the Equilibrium of the Sun's Atmosphere

by K. Schwarzschild

(*Presented at the meeting of the Berlin Academy of Sciences on January 13, 1906*)

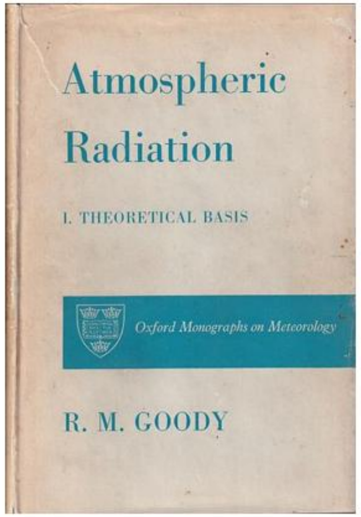
$$E = \frac{A_0}{2}(1 + \bar{\tau}), \qquad A = \frac{A_0}{2}(2 + \bar{\tau}), \qquad B = \frac{A_0}{2}\bar{\tau}. \tag{11}$$

E emission of a layer, A upward beam, B downward beam, A_0 emerging flux at TOA, τ optical depth

Eq. (1) $A - E = \Delta A = A_0 /2$

Net radiation at the surface, independent of τ

R. Goody: Atmospheric radiation. Theoretical basis. Oxford, 1964

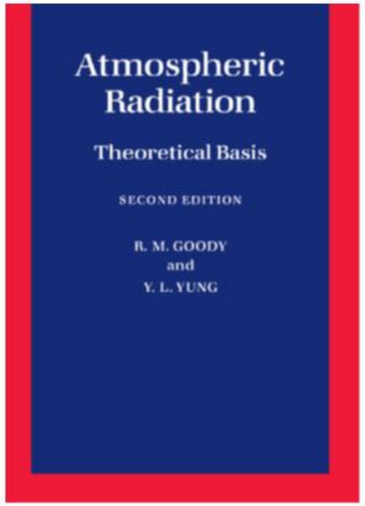


$$F_\nu/2\pi = J_\nu(0) - B_\nu^*(0) = B_\nu^*(\tau_\nu^*) - J_\nu(\tau_\nu^*). \qquad (2.115)$$

Note that if the medium is in thermodynamic equilibrium ($J_\nu = B_\nu$), equation (2.115) requires a discontinuity in B_ν (i.e. a *temperature discontinuity*) at both boundaries.

the step in B always equals $F/2\pi$

Goody – Yung: Atmospheric radiation. Theoretical basis. Oxford, 1989



$$F/2\pi = B(0) - B^*(0) = B^*(\tau_1) - B(\tau_1). \qquad (2.146)$$

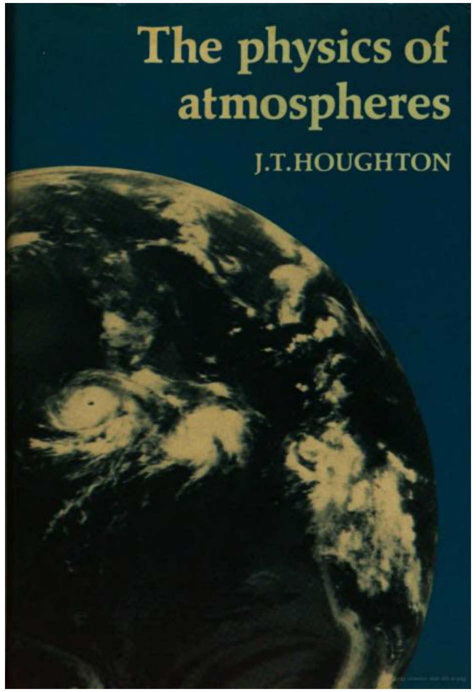
Equation (2.146) requires a discontinuity in the Planck function, implying a discontinuity of temperature, at the boundary.

2. There are discontinuities,

$$\boxed{\Delta B = \frac{F_S}{2\pi}}$$

DISCONTINUITY = OLR/2

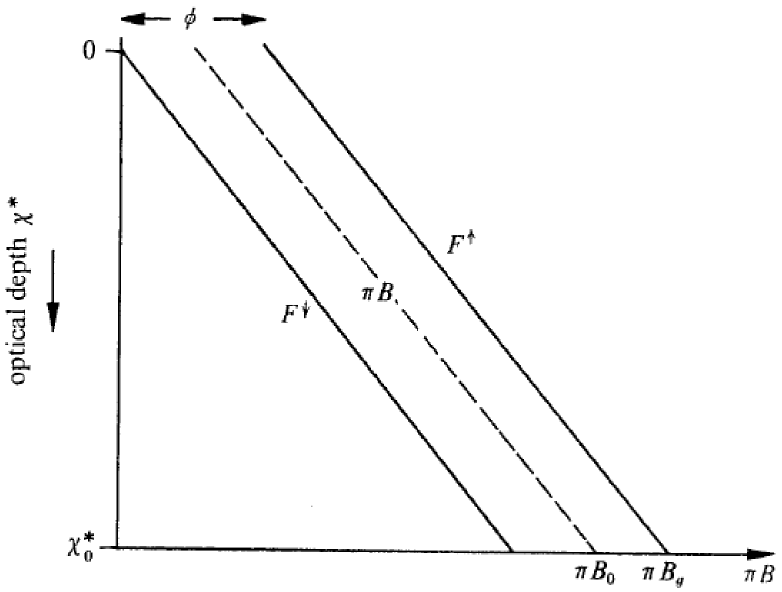
John Houghton: The Physics of Atmospheres
Cambridge (1977, 1986, 2002) Eq. 2.13



At the bottom of the atmosphere where $\chi^* = \chi_0^*$, $F^\uparrow = \pi B_g$, B_g being the black-body function at the temperature of the ground. It is easy to show that there must be a temperature discontinuity at the lower boundary, the black-body function for the air close to the ground being B_0, and

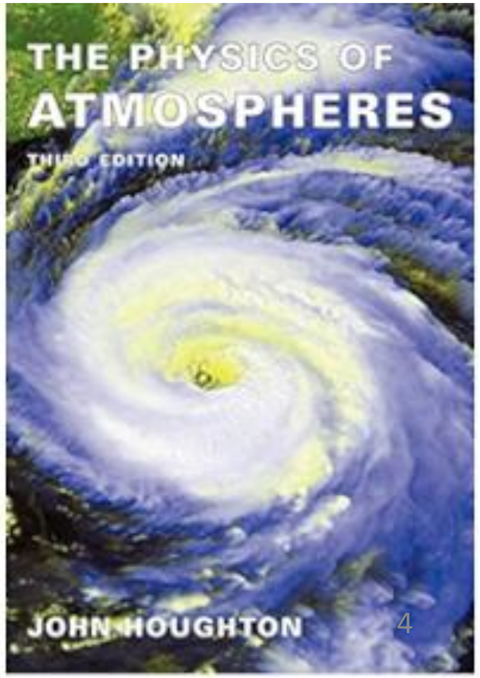
$$\boxed{B_g - B_0 = \frac{\phi}{2\pi}} \qquad (2.13)$$

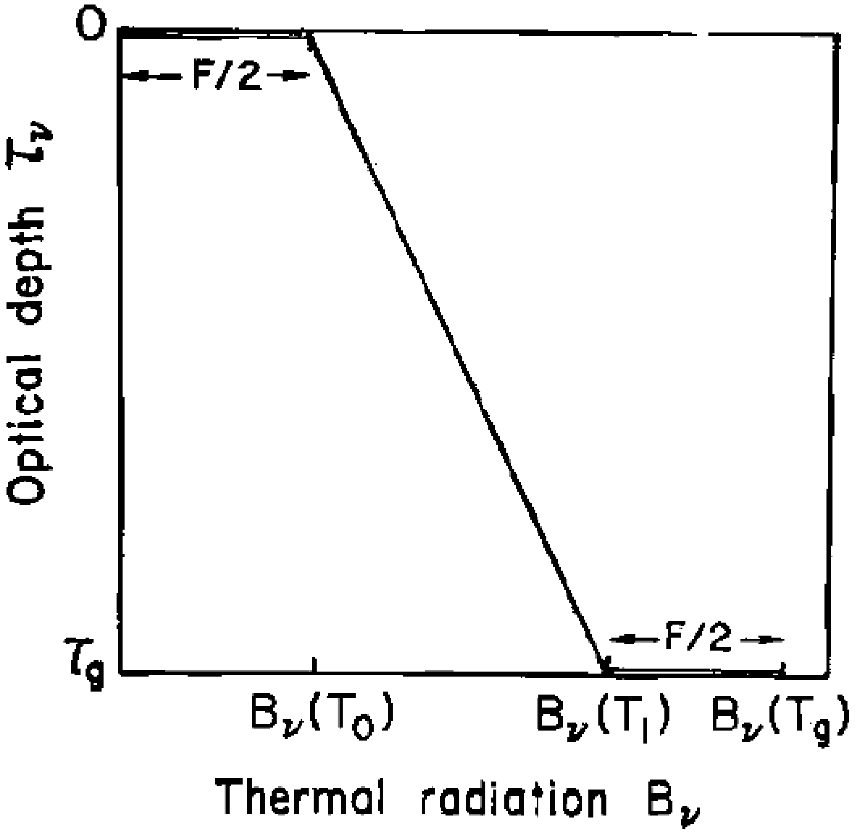
DISCONTINUITY = OLR/2
(independent of the optical depth)



such a steep lapse rate is very unstable with respect to vertical motion, and will soon be destroyed by the process of *convection*

$\boxed{\text{Discontinuity = Convection = OLR/2}}$





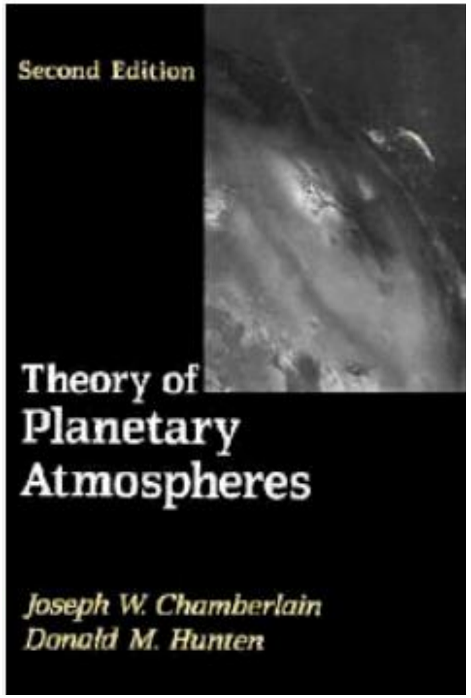
Joseph Chamberlain
Theory of Planetary Atmospheres (1978, 1987)
Academic Press, Fig. 1.4, Eq. 1.2.29

Fig. 1.4 The MRE solution for $T(\tau)$, presented as $B_\nu(T)$ vs. τ. Note the discontinuity at the ground and the finite skin temperature at $\tau = 0$.

DISCONTINUITY = OLR/2
(independent of τ)

Hence the upward intensity at the ground is

$$I_g^{\,+} \equiv B_\nu(T_g) = B_\nu(T_1) + \tfrac{1}{2}F_\nu \qquad (1.2.29)$$



Visconti: Fundamentals of Physics and Chemistry of the Atmospheres
Springer (2001)

$$T^4 = \frac{T_e^4}{2}\left(\frac{3}{2}\tau + 1\right) \tag{3.47}$$

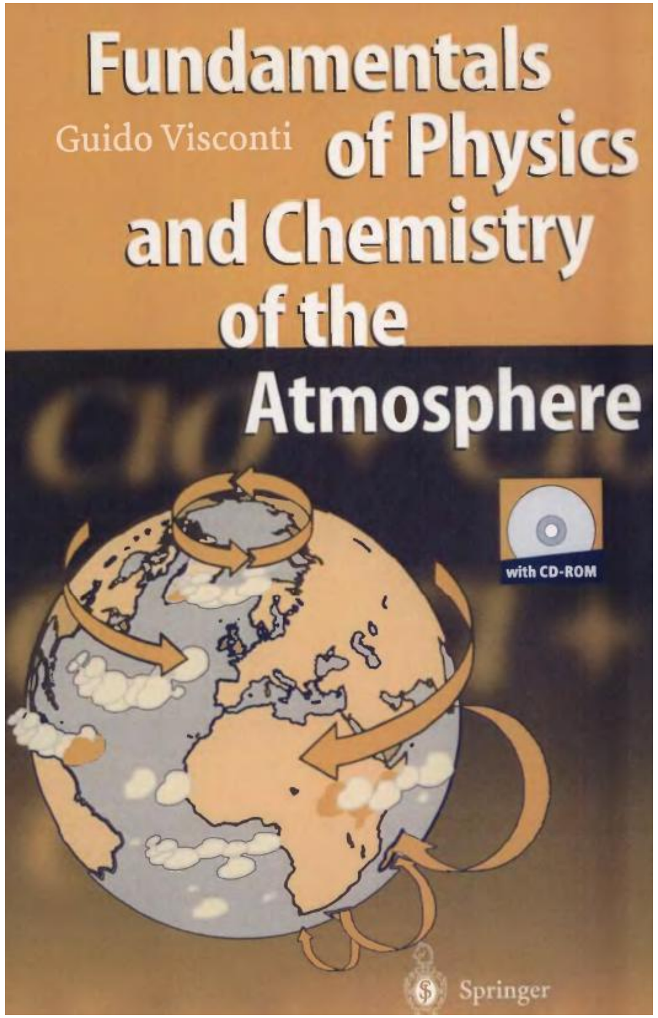
where τ has been replaced by its effective value. An interesting consequence of this solution is to discover what happens at the surface, where the optical thickness is τ^* and the temperature is T_s ; we have

$$F^{\uparrow}(\tau^*) = \sigma T_s^4 = \frac{1}{2}\sigma T_e^4\left(\frac{3}{2}\tau^* + 2\right) \tag{3.48}$$

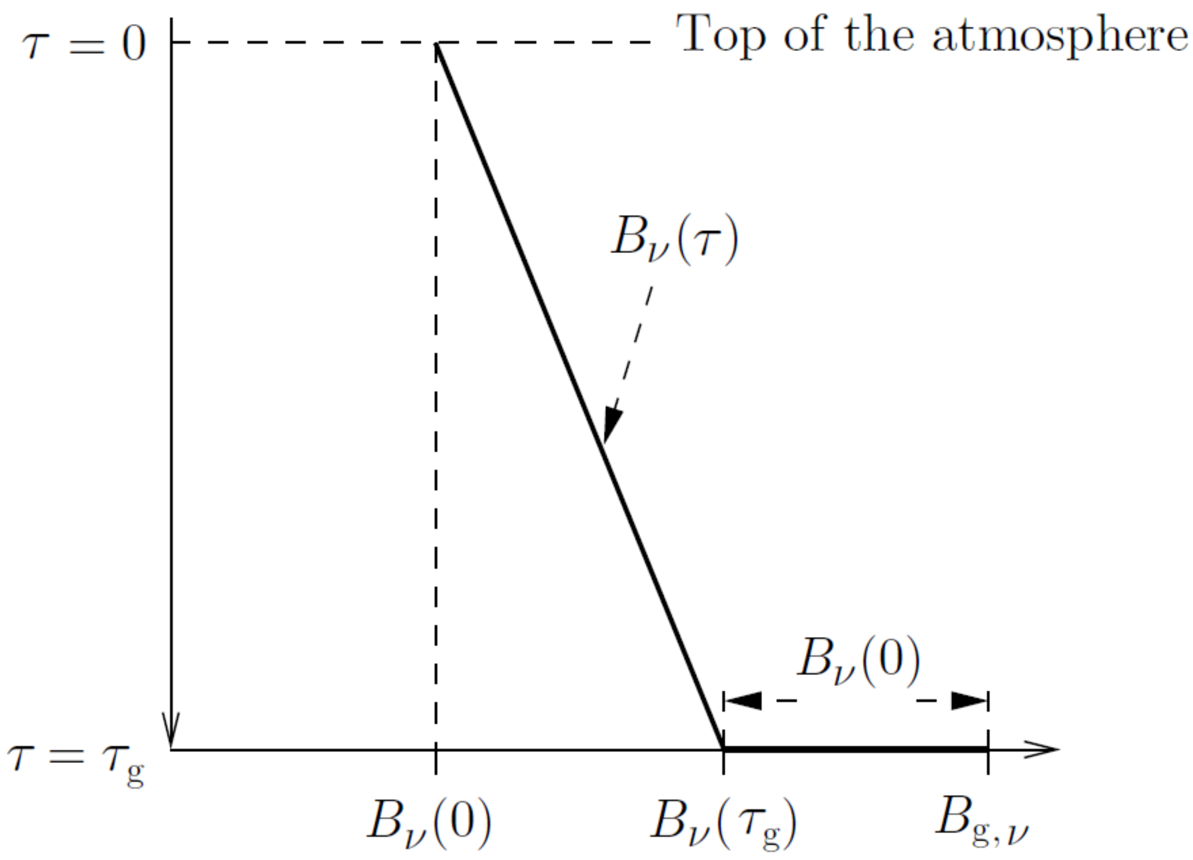
The temperature at the bottom of the atmosphere at τ^* is given by Equation (3.47), so that we have a discontinuity between the air temperature and that of the surface

$$T_s^4 - T(\tau^*) = T_e^4/2 \tag{3.49}$$

DISCONTINUITY = OLR/2



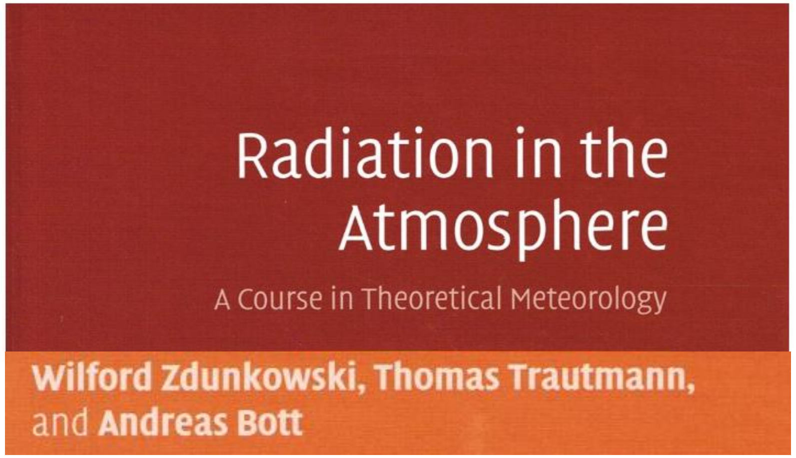
Two-stream methods for the solution of the RTE



Radiation in the Atmosphere

Zdunkowski, Trautmann and Bott

Cambridge (2008) Fig. 6.7



DISCONTINUITY = OLR/2

$$\boxed{B_{g,\nu} = B_\nu(\tau_g) + \frac{1}{2\pi} E_{\text{net},\nu}}$$

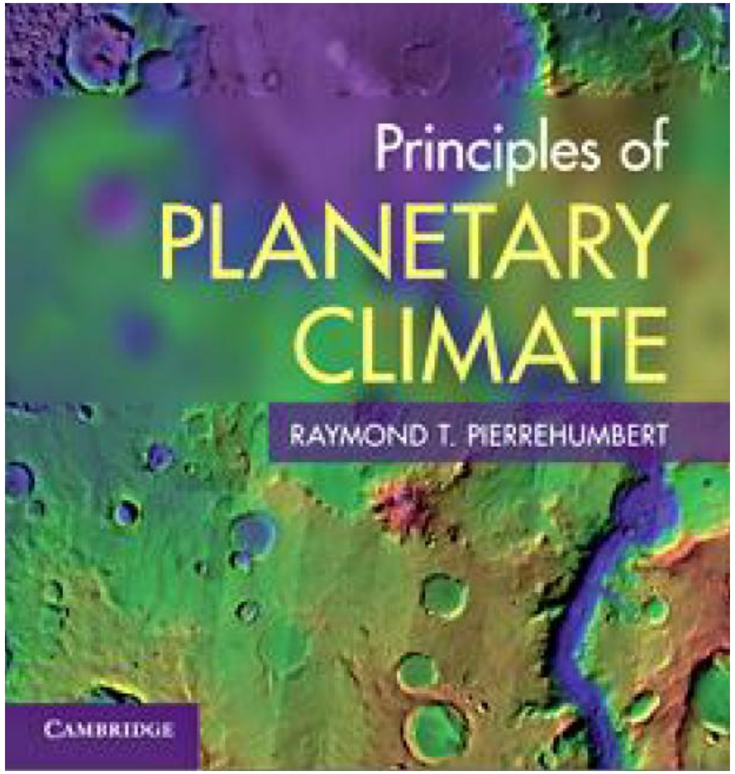
observe a discontinuity of the curve expressing a temperature jump ΔT_g between the surface temperature T_g and the lowest atmospheric layer $T(\tau_g)$, i.e.

$$\Delta T_g = T_g - T(\tau_g) > 0 \tag{6.155}$$

Raymond Pierrehumbert: Principles of Planetary Climate.
Cambridge (2008) Eq. 4.45

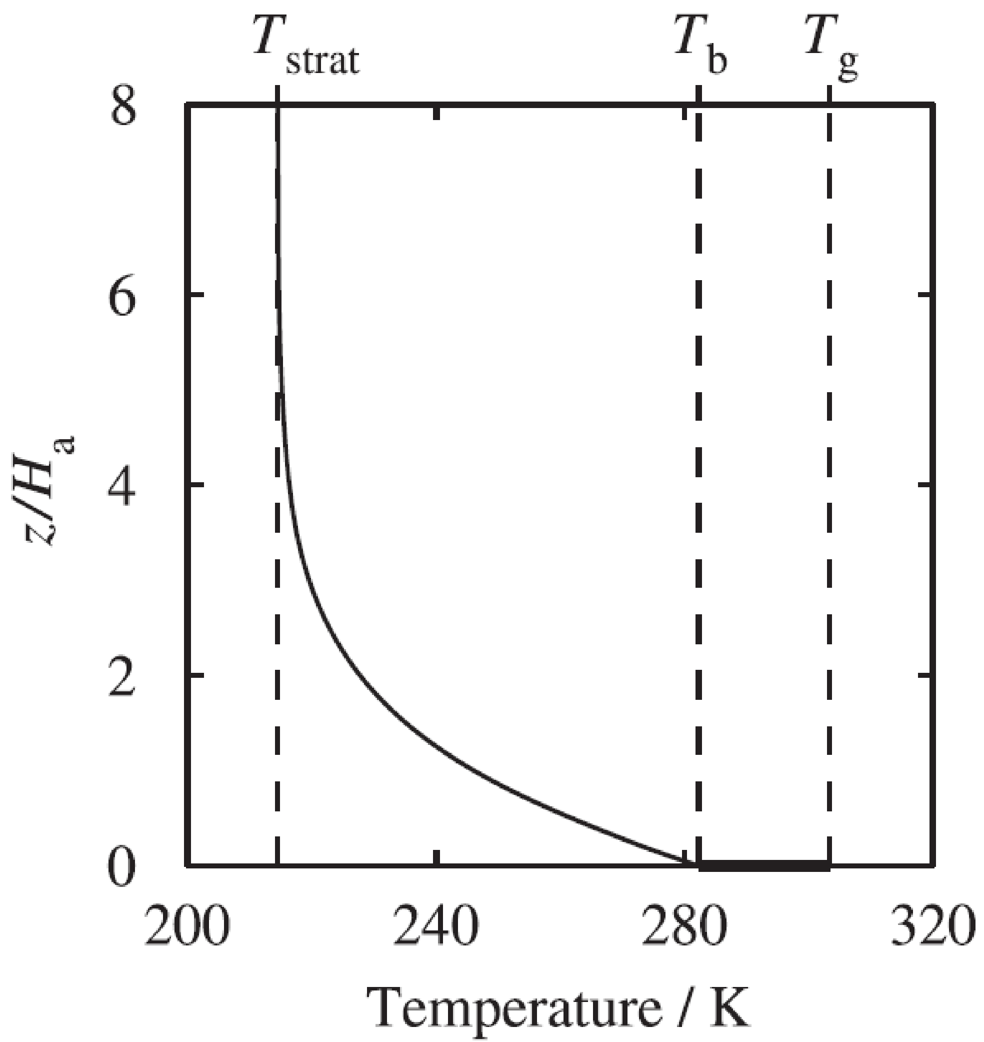
Let's now compare the surface temperature with the temperature of the air in immediate contact with the surface. From Eq. (4.42) we find that the low-level air temperature is determined by $\sigma T(0)^4 = (1 - \alpha)S \cdot \left(\frac{1}{2} + \frac{1}{2}\tau_\infty\right)$. Taking the ratio,

$$\frac{T(0)}{T_g} = \left(\frac{\frac{1}{2} + \frac{1}{2}\tau_\infty}{1 + \frac{1}{2}\tau_\infty}\right)^{1/4} \qquad (4.45)$$



DISCONTINUITY = $(1 - \alpha)S/2$ = OLR/2

Andrews: An Introduction to Atmospheric Physics
Cambridge (2010), pp 85-86.

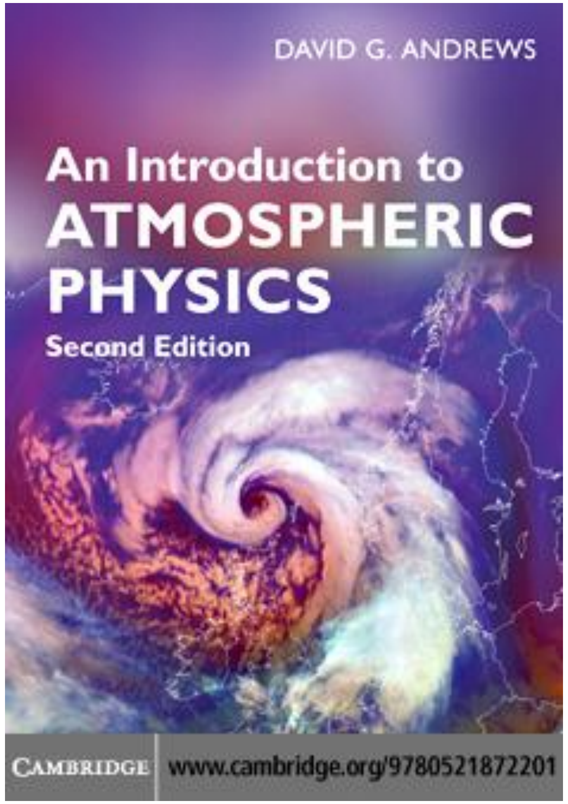


$$T_b \equiv T_e \left(\frac{1 + \chi_g^*}{2} \right)^{1/4}$$

$$T_g \equiv T_e \left(\frac{2 + \chi_g^*}{2} \right)^{1/4}$$

$$\boxed{T_g^4 - T_b^4 = T_e^4/2}$$

DISCONTINUITY = OLR/2



discontinuity between the bottom of the atmosphere and the ground.

Inclusion of convection in the model removes the temperature discontinuity

8.5 Thermal equilibrium 235

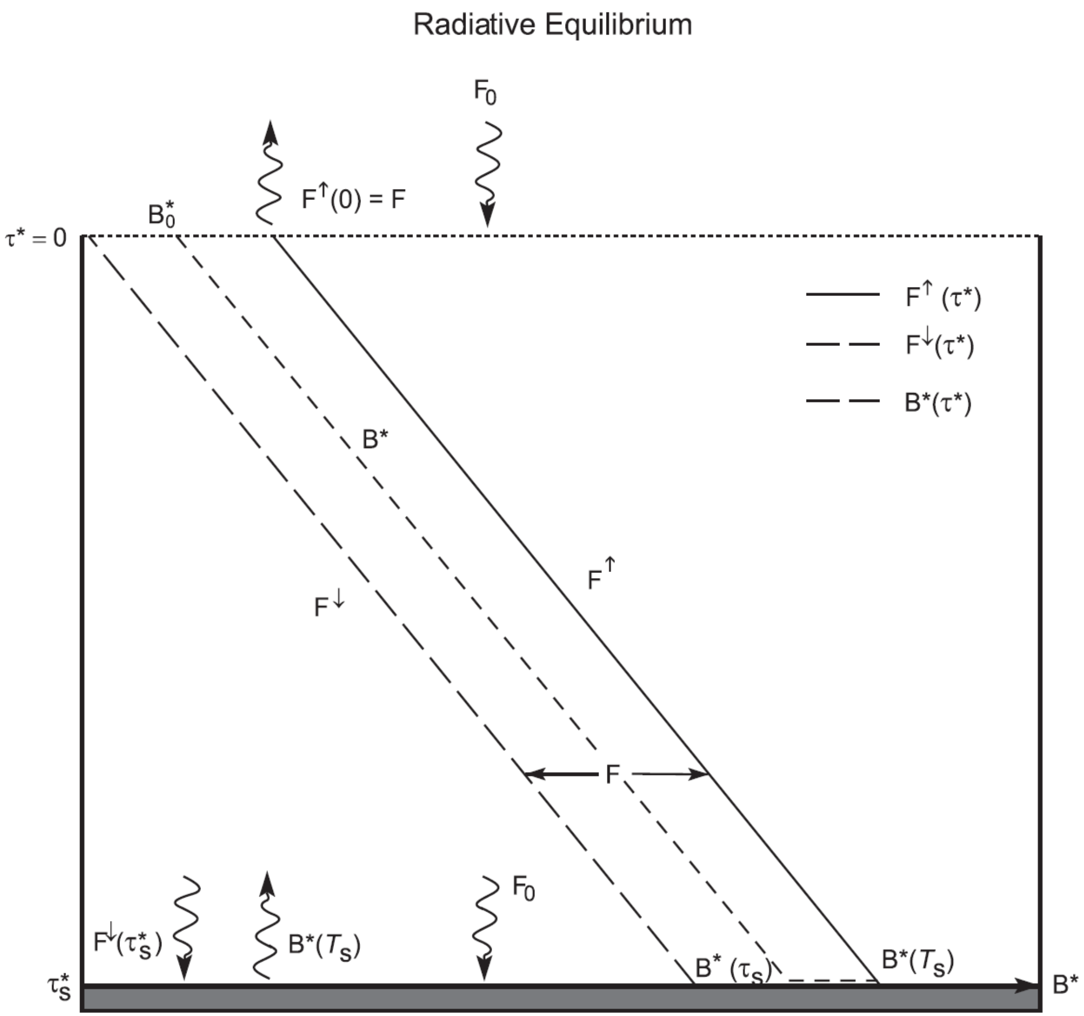
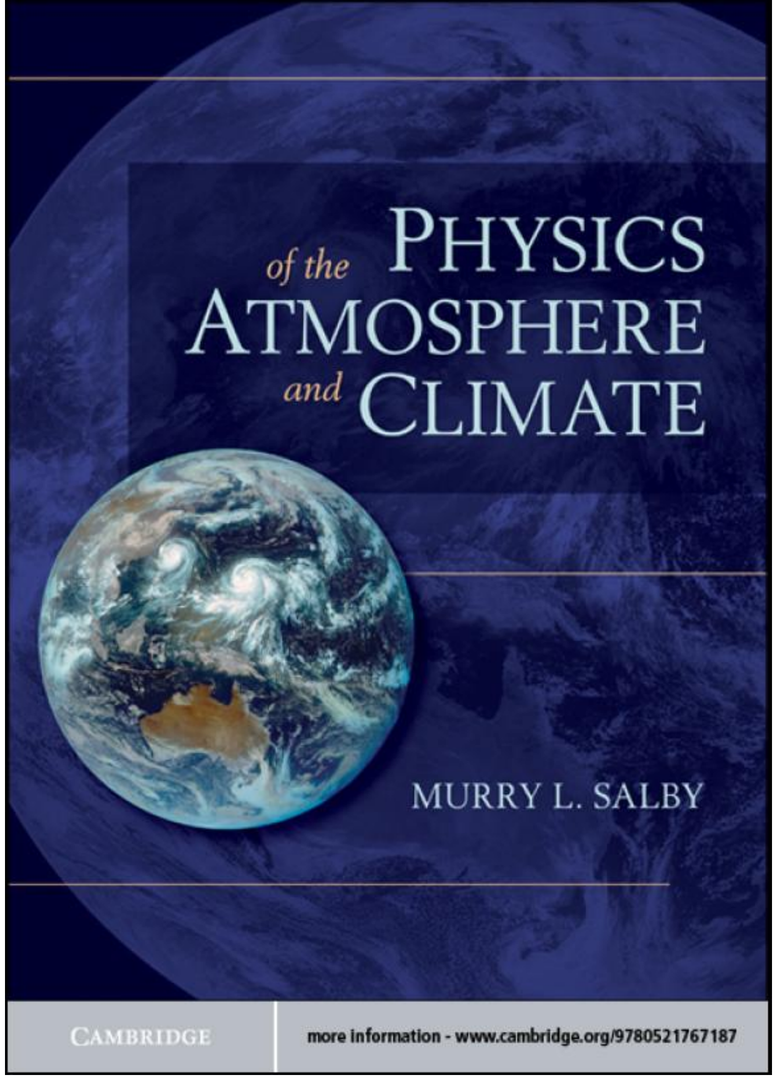


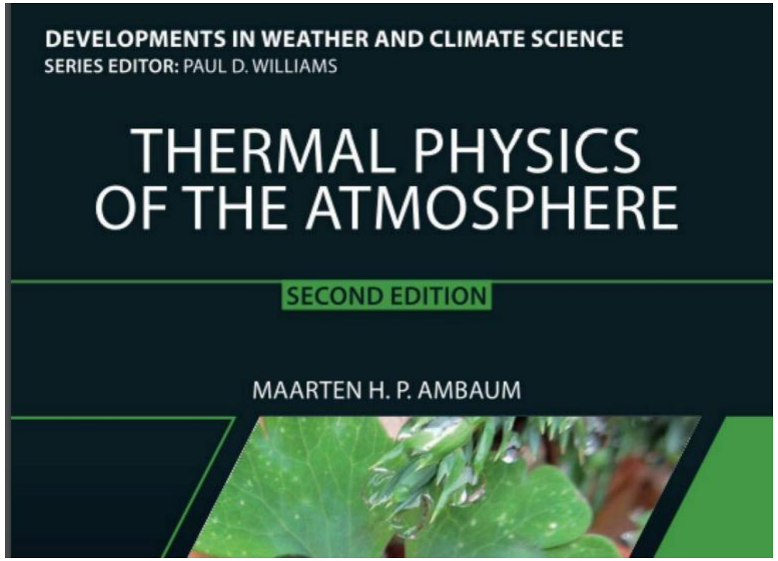
Figure 8.20 Upwelling and downwelling LW fluxes and LW emission in a gray atmosphere that is in radiative equilibrium with an incident SW flux F_0 and a black underlying surface. Note: the emission profile is discontinuous at the surface.



M. Salby: Physics of the Atmosphere and Climate
Cambridge, 2012

$$B^*(T_s) = B^*(\tau_s^*) + \frac{F_0}{2}. \qquad (8.67)$$

According to (8.67), the temperature predicted by radiative equilibrium is discontinuous at the surface, the ground being warmer than the overlying air.



These two budget equations can be combined by eliminating $L_{\downarrow 0}$ to find

$$\blacktriangleright \qquad \sigma T_E^4 = \sigma T_S^4 + S_{n0}/2. \tag{10.56}$$

So we find that under radiative equilibrium the surface temperature is higher than the temperature of the adjoining atmosphere. This temperature discontinuity is unstable in practice and there will be turbulent heat exchange which will remove the temperature discontinuity. This instability of the radiative state is a driver of surface layer turbulence: the radiation will force the lower part of the boundary layer, the surface layer, towards instability and this tendency is compensated by turbulent fluxes near the surface.

Kevin E. Trenberth (2022):

The Changing Flow of Energy Through the Climate System

Sidebar 3.2: How Does a Greenhouse Effect Work?

(page 30: Earth's Energy Balance)

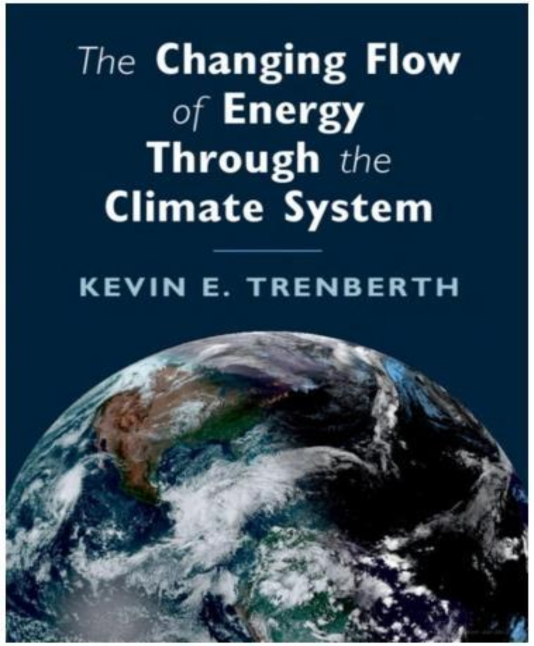
The answer is:

Imagine two plates in space

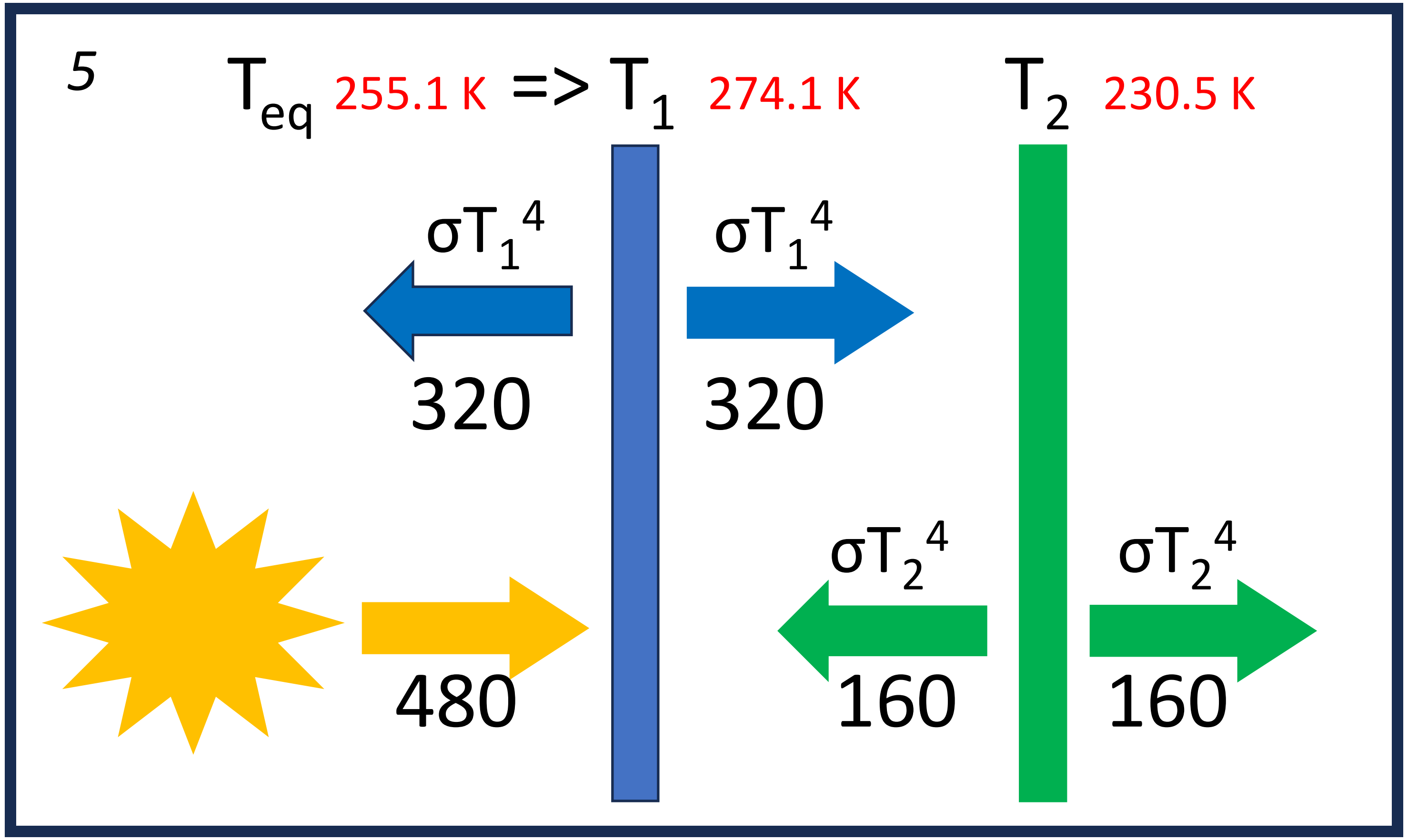
Incoming solar radiation: 240 Wm^{-2}

The plate is two-sided: 480 Wm^{-2}

Five panels, four equations.



The final state



$$G : F_2 : F_{eq} : F_1 : F_{sol} = 1 : 2 : 3 : 4 : 6$$

Trenberth's four equations

(T1) $480\ \mathrm{Wm^{-2}} = 2\sigma T_{eq}^4$

(T2) $480\ \mathrm{Wm^{-2}} = \sigma T_1^4 + \sigma T_2^4$

(T3) $\sigma T_1^4 = 2\sigma T_2^4$

(T4) $480\ \mathrm{Wm^{-2}} = 1.5\ \sigma T_1^4$

$F_{sol} = 480\ \mathrm{Wm^{-2}}$

$F_1 = 320\ \mathrm{Wm^{-2}}$

$F_{eq} = 240\ \mathrm{Wm^{-2}}$

$F_2 = 160\ \mathrm{Wm^{-2}}$

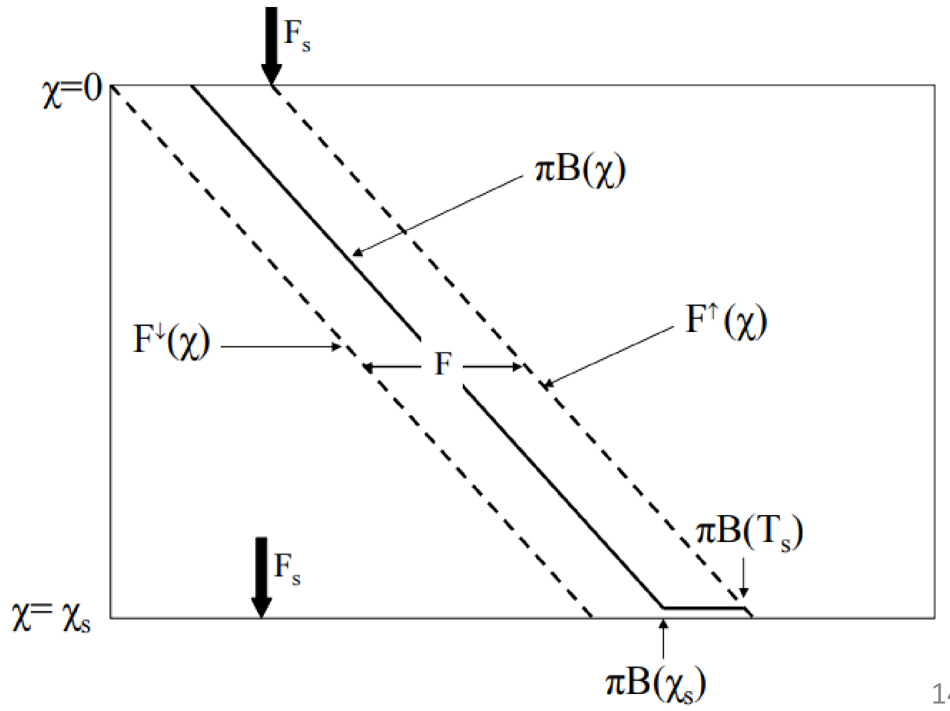
$G = 80\ \mathrm{Wm^{-2}}$

University Lecture Notes

Manchester University

This expression implies that there is a temperature discontinuity between the surface and the cooler lowest layer of the atmosphere.

$$\boxed{\pi B(T_s) = \pi B(\chi_s) + F_s/2.}$$



Southampton University; Tyndall Centre, East Anglia

However, it is very important to notice that the (ground) surface temperature is set by F_{up} through equation (13), i.e.

$$\sigma T_g^4 = [F_{up}]_{z=0} = F_0 (1 + \tau/2) \tag{16}$$

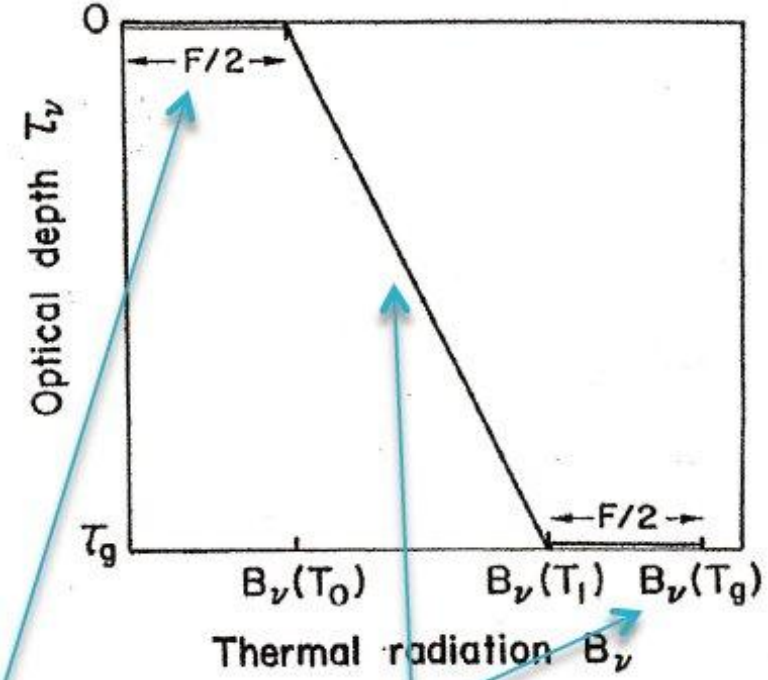
whereas the air temperature just above the ground is set by F_{tot} through equation (11) so that

$$\sigma T_0^4 = B(\tau) = F_0 (\tau + 1)/2$$

ground surface temperature derived above exceeds that of the overlying air in this model, by an amount corresponding to an extra heat flux of $F_0/2$. This calculated ground-air temperature discontinuity may be substantial (10 or 20 °K, or more). It only occurs because we have assumed that the only heat fluxes are those due to radiation, so there is no conduction and no turbulent convection. In the real atmosphere these would operate together, as conduction will transfer heat into the air near the ground, creating an unstable

University of Arizona, 2015

Final TP Profile



Expressions for intensity:

$$I^{+} = B(T) + \frac{1}{2\pi}F$$

$$I^{-} = B(T) - \frac{1}{2\pi}F$$

Boundary Conditions:

At the Ground: $I_g^{+} = B(T_g) = B(T_1) + \frac{1}{2\pi}F,$

Discontinuity

(3.10)(3.11)より

$$\underbrace{B^*(T_S)}_{\text{地表面}} = \underbrace{B^*(\tau_s^*)}_{\text{大気下端}} + \frac{F^0}{2}$$

放射平衡では大気下端の温度と地表面温度は不連続になる。($B^* = \sigma T^4$ に注意)

From (3.10)(3.11)

$$\underbrace{B^*(T_S)}_{\text{surface}} = \underbrace{B^*(\tau_s^*)}_{\text{bottom of atmosphere}} + \frac{F^0}{2}$$

A temperature discontinuity exists at the surface. (Note that $B^* = \sigma T^4$)

University of Toronto

"We will use "Schwarzchild's Equation for Fluxes" in our simple greenhouse atmospheric model."

Two-Stream Model Solution - 1

- We have thus solved for the upward and downward flux densities:

$$\boxed{\begin{aligned} F_{up} &= \sigma T_e^4(\chi^* + 2)/2 \\ F_{down} &= \sigma T_e^4 \chi^*/2 \end{aligned}}$$

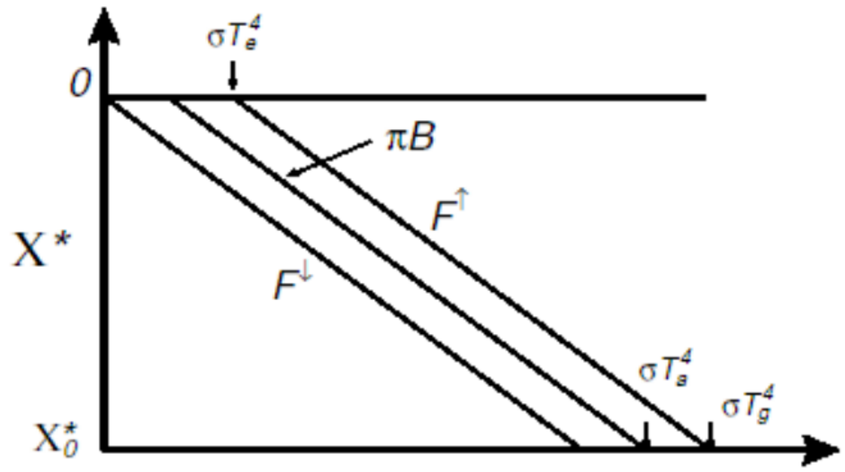
- Finally, we can use the original equations to solve for:

$$\boxed{\pi B = \sigma T_a^4 = \frac{\sigma T_e^4(\chi^* + 1)}{2}}$$

where T_a is the atmospheric temperature.

- These three solutions can be plotted against χ^*.
  → Gives three parallel lines that show how the flux densities and the blackbody function vary with χ^* and hence with altitude.

Two-Stream Model Solution - 3



- If the central line represents the atmospheric temperature and the line for F_{up} must match the ground temperature, then there is a temperature discontinuity at the ground.

Harvard (2018)

We have got the temperature structure in the atmosphere as a function of τ. Now consider energy balance at the surface (looks familiar?),

$$B(T_s) = F_0 + F^{\downarrow}(\tau_s)$$

From their definitions, we have

$$F^{\downarrow} = \frac{1}{2}\left(\bar{F} - F\right)$$

As the net flux F is constant and equal to F_0, and use Eq. (14), we have:

$$B(T_s) = B(\tau_s) + \frac{F_0}{2}$$

Note the jump at the surface.

Graeme L. Stephens: Radiative Transfer Notes AT 622.

Colorado State University, Fort Collins, CO (2003)

Example 6.3: Skin temperatures and temperature discontinuities

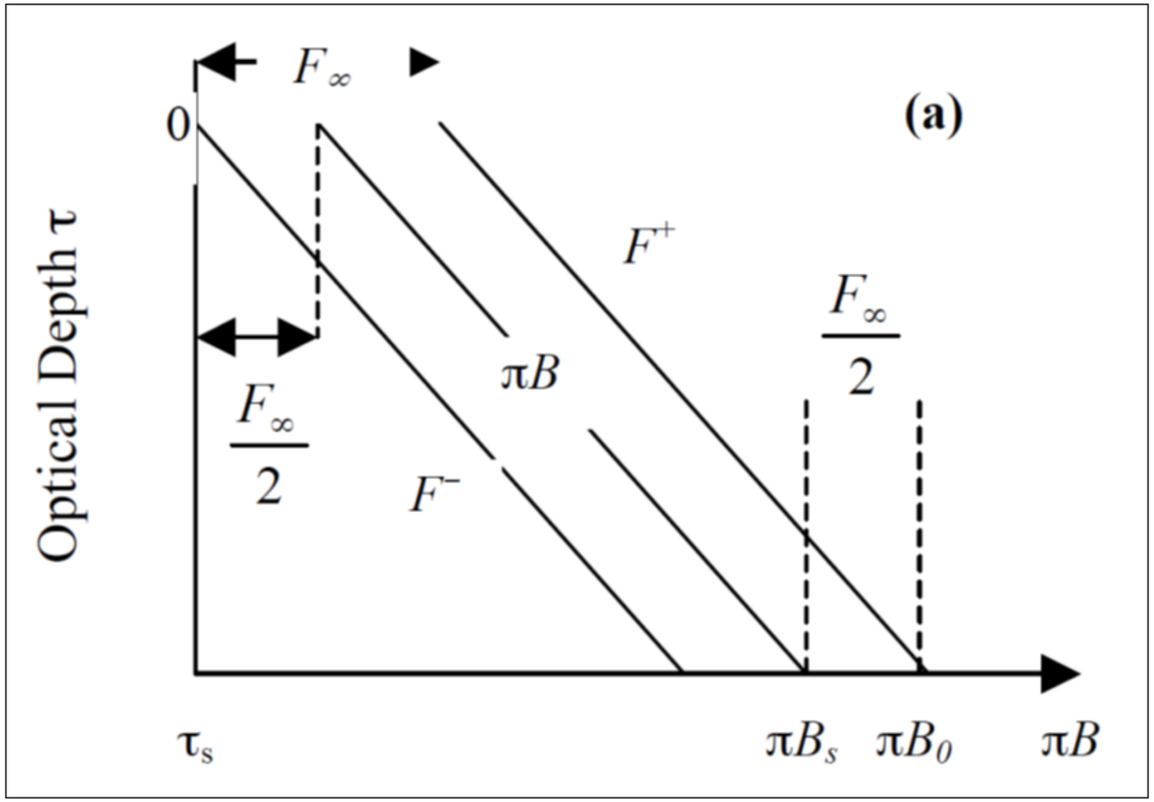
The solutions represented by Eqns. (6.10a) and (6.10b) provide rather interesting insights into the temperature profiles that are predicted by these equations. One of the results of this model is an estimate of the 'skin' temperature, which we think of as a measure of the stratospheric temperature. We obtain this using Eqn. (6.10a) with $\tilde{\tau} = 0$

$$\sigma T^4(\tilde{\tau} = 0) = \frac{F_\infty}{2}$$

and with $F_\infty \approx 235$ Wm^{-2}, it follows that this temperature is $T_{skin} = [117.5/5.68 \times 10^{-8}]^{0.25} = 213$ K.

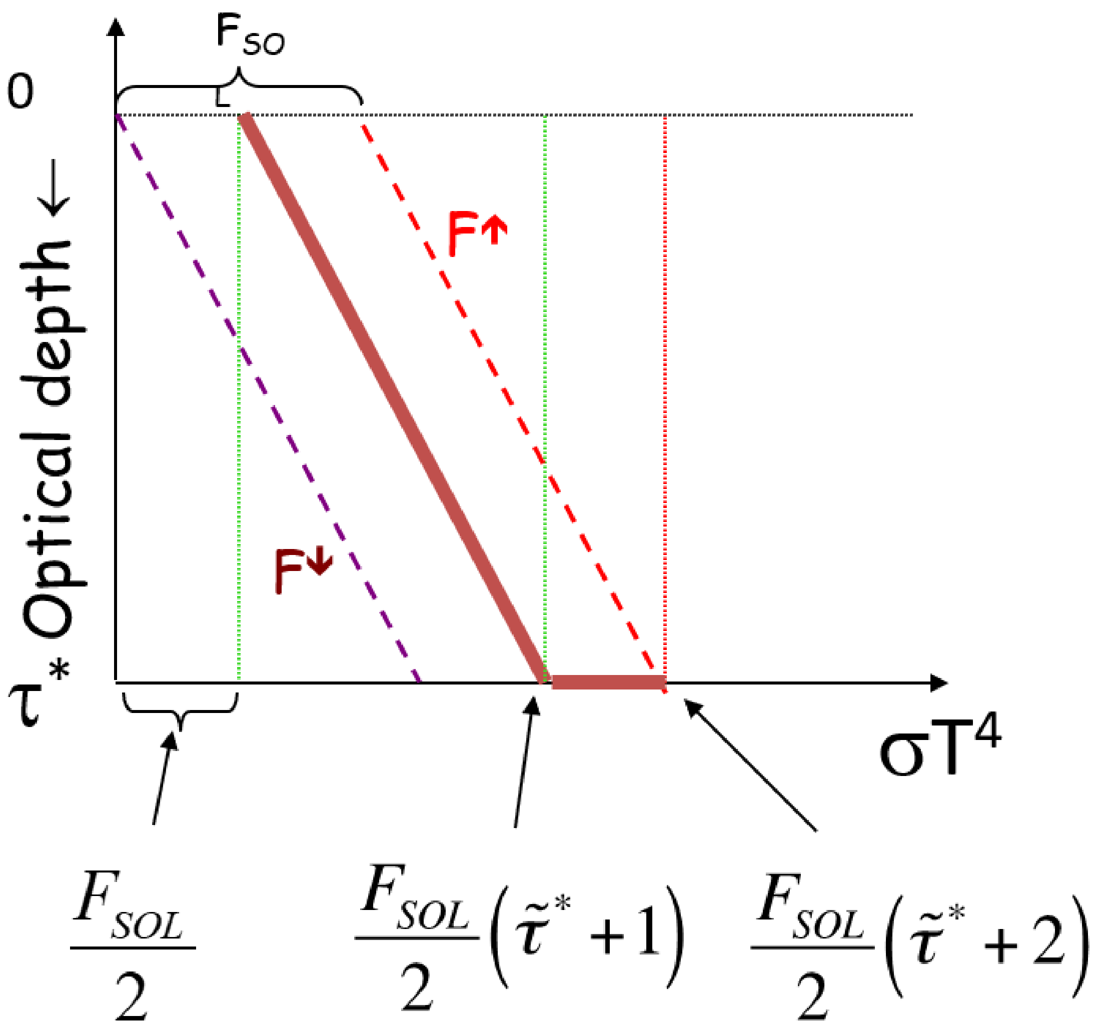
The solutions in Eqns. (6.10a) and (6.10b) predict a discontinuity between the surface temperature T_s and the air temperature just above the ground $T(\tilde{\tau}_s)$. Differencing these equations and with $\tilde{\tau} = \tilde{\tau}_s$,

$$\sigma T_s^4 - \sigma T^4(\tilde{\tau}_s) = \frac{F_\infty}{2}.$$



“This radiative equilibrium profile is unstable w.r.t. vertical motion and is destroyed by convection”

Colorado State Univ. (2013)



$$\sigma T^4\left(\tilde{\tau}\right) = \frac{F_{SOL}}{2}\left(\tilde{\tau} + 1\right)$$

$$\sigma T_s^4 = \frac{F_{SOL}}{2}\left(\tilde{\tau}^* + 2\right)$$

University exam task (for A+)

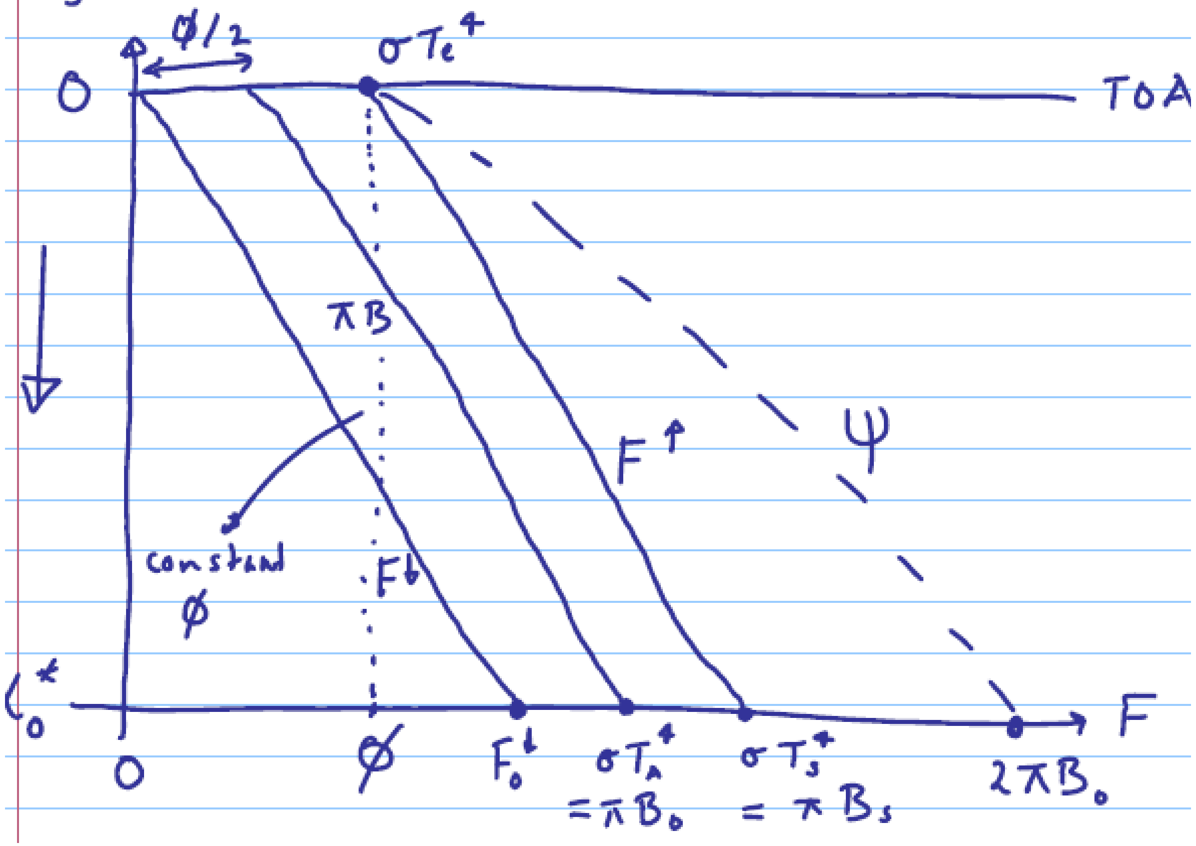
$$\phi = F^{\uparrow} - F^{\downarrow} = \sigma T_e^4$$

$$F^{\uparrow} = \frac{\phi}{2}(\chi^* + 2)$$

$$F^{\downarrow} = \frac{\phi}{2}\chi^*$$

$$\pi B = \frac{\phi}{2}(\chi^* + 1)$$

Let's plot these with χ^* on the y axis.



Early verification

CERES_EBAF-Surface_Ed2.8 Data Quality Summary (March 27, 2015)

Table 4-1. Global annual mean fluxes using data from March 2000 through February 2010 (W m-2).

	Eq. (1)	**A – E**	**=**	**ΔA**	**= $A_0/2$**
SFC	SW dn - SW up + LW dn – LW up			= TOA LW /2	
	(243.9 – 29.7 + 316.0) – 398.0			**= 132.85**	**= 265.7 /2**

Early verification

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CERES EBAF Ed2.8, 192 months (Mar 2000 – Feb 2016)

Eq. (1) **A – E = ΔA = $A_0/2$**

SFC SW dn - SW up + LW dn – LW up = TOA LW /2

(244.06 – 29.74 + 316.27) – 398.40 = 132.19 = 265.59 /2

Early verification

CERES_EBAF-Surface_Ed2.8 Data Quality Summary (March 27, 2015)

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CERES EBAF Ed2.8, 192 months (Mar 2000 – Feb 2016)

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Early verification

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SFC SW dn - SW up + LW dn – LW up = TOA LW /2

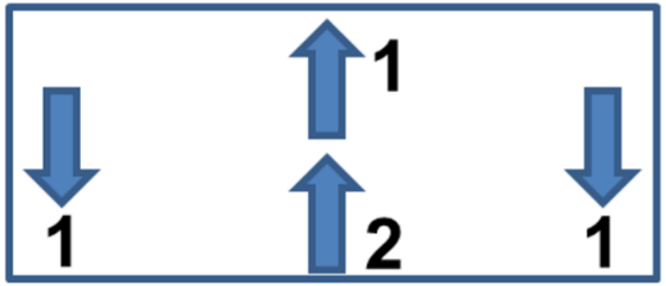
(244.06 – 29.74 + 316.27) – 398.40 = 132.19 = 265.59 /2 – 0.60 Wm^{-2}

Earth heat uptake for July 2005–June 2010 is 0.58 ± 0.38 Wm^{-2}

Eq. (2): Schwarzschild (1906, Eq. 11) at $\bar{\tau} = 2$

$$E = \frac{A_0}{2}(1 + \bar{\tau}), \qquad A = \frac{A_0}{2}(2 + \bar{\tau}), \qquad B = \frac{A_0}{2}\bar{\tau}. \tag{11}$$

Eq. (2) $A = 2A_0$ $E = 3A_0/2;$ $B = A_0$



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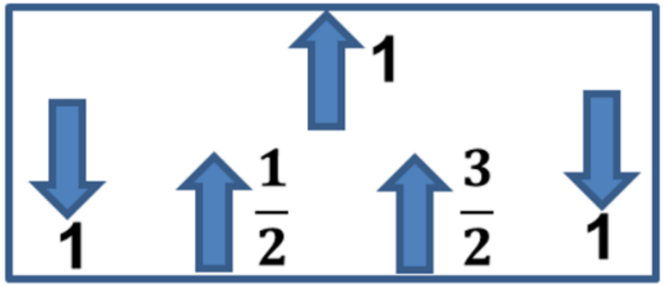
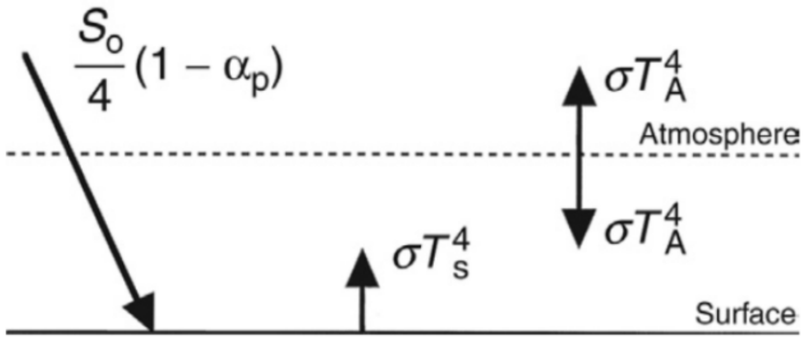


Eq. (2) $A = 2A_0$ Clear-sky Eq.(1) $\Delta A = A - E = A_0/2$



$\sigma T_s^4 = 2\sigma T_A^4$

Hartmann (1994) Fig. 2.3

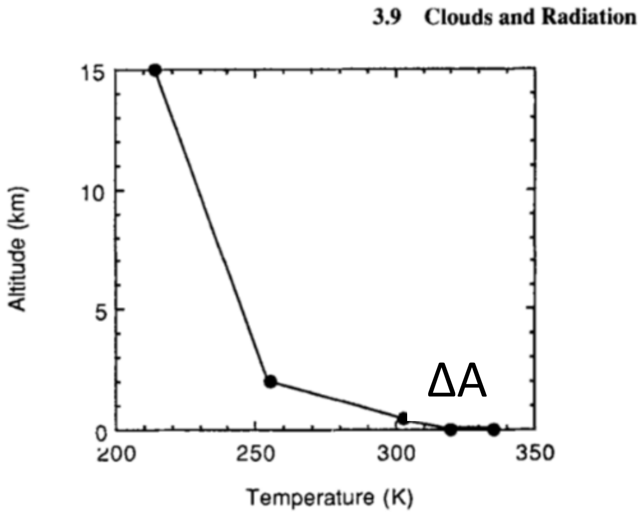


Fig. 3.11

CERES EBAF Ed2.8, Global means (Mar 2000 – Feb 2016) Clear-sky

Eq. (1)	$\mathbf{A} - \mathbf{E}$	$= \Delta\mathbf{A}$	$= \mathbf{A_0/2}$	
	530.59 – 398.40	**= 132.19**	**= 265.59 /2**	**– 0.60 Wm^{-2}**
Eq. (2)	$\mathbf{A}$		$= \mathbf{2A_0}$	
	530.59		**= 2 × 265.59**	

CERES EBAF Ed2.8, Global means (Mar 2000 – Feb 2016) Clear-sky

Eq. (1)	**A – E**	**= ΔA**	**= $A_0/2$**	
	530.59 – 398.40	= 132.19	= 265.59 /2	– 0.60 Wm^{-2}
Eq. (2)	**A**		**= $2A_0$**	
	530.59		= 2 × 265.59	– 0.59 Wm^{-2}

CERES EBAF Ed2.8, Global means (Mar 2000 – Feb 2016) Clear-sky

Eq. (1)	$\mathbf{A - E}$	$\mathbf{= \Delta A}$	$\mathbf{= A_0/2}$	
	530.59 – 398.40	**= 132.19**	**= 265.59 /2**	**– 0.60 Wm^{-2}**
Eq. (2)	$\mathbf{A}$		$\mathbf{= 2A_0}$	
	530.59		**= 2 × 265.59**	**– 0.59 Wm^{-2}**

$\mathbf{\Delta A : A_0 : E : A = 1 : 2 : 3 : 4}$ **(clear-sky) justified within EEI**

CERES EBAF Ed2.8, Global means (Mar 2000 – Feb 2016) Clear-sky

Eq. (1)	$A - E$	$= \Delta A$	$= A_0/2$	
	530.59 – 398.40	= 132.19	= 265.59 /2	– 0.60 Wm^{-2}
Eq. (2)	A		$= 2A_0$	
	530.59		= 2 × 265.59	– 0.59 Wm^{-2}

$\Delta A : A_0 : E : A$ = 1 : 2 : 3 : 4 (clear-sky) justified within EEI

$$E = (3/2)\, A_0$$

$$g = 1/3$$

CERES EBAF Ed2.8, Global means (Mar 2000 – Feb 2016) Clear-sky

Eq. (1)	$A \quad - \quad E$	$= \quad \Delta A$	$= \quad A_0/2$	
	530.59 – 398.40	**= 132.19**	**= 265.59 /2**	**– 0.60 Wm^{-2}**
Eq. (2)	A		$= \quad 2A_0$	
	530.59		**= 2 × 265.59**	**– 0.59 Wm^{-2}**

$\Delta A : A_0 : E : A =$ **1 : 2 : 3 : 4 (clear-sky) justified within EEI**

$E = (3/2)\, A_0$ **398.40 = (3/2) × 265.59/2**

$g = 1/3$ **(398.40 – 265.59) / 398.40**

CERES EBAF Ed2.8, Global means (Mar 2000 – Feb 2016) Clear-sky

Eq. (1)	$A \; - \; E \; = \; \Delta A \; = \; A_0/2$		
	$530.59 - 398.40 = 132.19 = 265.59/2$		$-0.60\ \mathrm{Wm^{-2}}$
Eq. (2)	$A \; = \; 2A_0$		
	$530.59 = 2 \times 265.59$		$-0.59\ \mathrm{Wm^{-2}}$

$\Delta A : A_0 : E : A$ = 1 : 2 : 3 : 4 (clear-sky) justified within EEI

$E = (3/2)\,A_0$ $\quad 398.40 = (3/2) \times 265.59/2 + 0.015$

$g = 1/3$ $\quad (398.40 - 265.59)\,/\,398.40 = 0.33336$

Creating the all-sky versions

Eq. (1) **SFC Net = A – E =** $\mathbf{A_0/2}$ (clear-sky, net)

Eq. (2) **SFC Tot =** **A =** $\mathbf{2A_0}$ (clear-sky, total at $\tau = 2$)

Separating atmospheric radiation transfer from the longwave cloud effect (LWCRE):

Eq. (3) **SFC Net = A – E =** $\mathbf{(A_0 - L)/2}$ (all-sky, net, incl LWCRE)

Eq. (4) **SFC Tot =** **A** = $\mathbf{2}A_0 \mathbf{+ L}$ (all-sky, total at $\tau = 2$ incl LWCRE)

Verification of the four equations

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})

Eq. (1) SFC SW down – SW up + LW down – LW up (clear) = TOA LW (clear)/2

240.8680 – 29.0724 + 317.4049 – 398.5211 = 266.0122 /2

Eq. (2) SFC SW down – SW up + LW down (clear) = 2 × TOA LW (clear)

240.8680 – 29.0724 + 317.4049 = 2 × 266.0122

Eq. (3) SFC SW down – SW up + LW down – LW up (all) = [TOA LW (all) – LWCRE]/2

186.8544 – 23.1629 + 345.0108 – 398.7550 = (240.2450 – 25.7672)/2

Eq. (4) SFC SW down – SW up + LW down (all) = 2 × TOA LW (all) + LWCRE

186.8544 – 23.1629 + 345.0108 = 2 × 240.2450 + 25.7672

Verification of the four equations

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})

Eq. (1)	**SFC**	**SW down – SW up + LW down – LW up**	**(clear)**	**= TOA LW (clear)/2**	
		240.8680 – 29.0724 + 317.4049 – 398.5211		= 266.0122 /2	– 2.3267
Eq. (2)	**SFC**	**SW down – SW up + LW down**	**(clear)**	**= 2 × TOA LW (clear)**	
		240.8680 – 29.0724 + 317.4049		= 2 × 266.0122	– 2.8238
Eq. (3)	**SFC**	**SW down – SW up + LW down – LW up**	**(all)**	**= [TOA LW (all) – LWCRE]/2**	
		186.8544 – 23.1629 + 345.0108 – 398.7550		= (240.2450 – 25.7672)/2	+ 2.7083
Eq. (4)	**SFC**	**SW down – SW up + LW down**	**(all)**	**= 2 × TOA LW (all) + LWCRE**	
		186.8544 – 23.1629 + 345.0108		= 2 × 240.2450 + 25.7672	+ 2.4450

Verification of the four equations

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})

Eq. (1) SFC SW down – SW up + LW down – LW up (clear) = TOA LW (clear)/2

240.8680 – 29.0724 + 317.4049 – 398.5211 = 266.0122 /2 – 2.3267

–2.5753

Eq. (2) SFC SW down – SW up + LW down (clear) = 2 × TOA LW (clear)

240.8680 – 29.0724 + 317.4049 = 2 × 266.0122 – 2.8238

Eq. (3) SFC SW down – SW up + LW down – LW up (all) = [TOA LW (all) – LWCRE]/2

186.8544 – 23.1629 + 345.0108 – 398.7550 = (240.2450 – 25.7672)/2 + 2.7083

+2.5767

Eq. (4) SFC SW down – SW up + LW down (all) = 2 × TOA LW (all) + LWCRE

186.8544 – 23.1629 + 345.0108 = 2 × 240.2450 + 25.7672 + 2.4450

Verification of the four equations

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})

Eq. (1)	SFC SW down – SW up + LW down – LW up (clear)	= TOA LW (clear)/2		
	240.8680 – 29.0724 + 317.4049 – 398.5211	= 266.0122 /2	– 2.3267	–2.5753
Eq. (2)	SFC SW down – SW up + LW down (clear)	= 2 × TOA LW (clear)		
	240.8680 – 29.0724 + 317.4049	= 2 × 266.0122	– 2.8238	
Eq. (3)	SFC SW down – SW up + LW down – LW up (all)	= [TOA LW (all) – LWCRE]/2		
	186.8544 – 23.1629 + 345.0108 – 398.7550	= (240.2450 – 25.7672)/2	+ 2.7083	+2.5767
Eq. (4)	SFC SW down – SW up + LW down (all)	= 2 × TOA LW (all) + LWCRE		
	186.8544 – 23.1629 + 345.0108	= 2 × 240.2450 + 25.7672	+ 2.4450	
		Mean		0.0007

CERES EBAF Ed4.1 Version 3, 22 years (April 2000 – March 2022) (Wm^{-2})
CERES EBAF Ed4.2 Version 4, 22 years (April 2000 – March 2022) (Wm^{-2})
CERES EBAF Ed4.2 Version 4, 24 years (April 2000 – March 2024) (Wm^{-2})

Eq. (1)	**SFC**	**SW down**	**– SW up**	**+ LW down**	**– LW up**	**(clear)**	**= TOA LW (clear)/2**	
		240.8680	– 29.0724	+ 317.4049	– 398.5211		= 266.0122 /2	– 2.3267
		241.0969	– 29.7521	+ 317.8744	– 398.5890		= 265.9594 /2	– 2.3495
		241.0514	– 29.7043	+ 318.0984	– 398.7742		= 265.9748 /2	– 2.3161
Eq. (2)	**SFC**	**SW down**	**– SW up**	**+ LW down**		**(clear)**	**= 2 × TOA LW (clear)**	
		240.8680	– 29.0724	+ 317.4049			= 2 × 266.0122	– 2.8238
		241.0969	– 29.7521	+ 317.8744			= 2 × 265.9594	– 2.6996
		241.0514	– 29.7043	+ 318.0984			= 2 × 265.9748	– 2.5042
Eq. (3)	**SFC**	**SW down**	**– SW up**	**+ LW down**	**– LW up**	**(all)**	**= [TOA LW (all) – LWCRE]/2**	
		186.8544	– 23.1629	+ 345.0108	– 398.7550		= (240.2450 – 25.7672)/2	+ 2.7083
		187.1451	– 23.4950	+ 346.1057	– 398.4220		= (240.3317 – 25.6277)/2	+ 3.9818
		187.1756	– 23.4607	+ 346.3158	– 398.6162		= (240.3894 – 25.5854)/2	+ 4.0126
Eq. (4)	**SFC**	**SW down**	**– SW up**	**+ LW down**		**(all)**	**= 2 × TOA LW (all) + LWCRE**	
		186.8544	– 23.1629	+ 345.0108			= 2 × 240.2450 + 25.7672	+ 2.4450
		187.1451	– 23.4950	+ 346.1057			= 2 × 240.3317 + 25.6277	+ 3.4647
		187.1756	– 23.4607	+ 346.3158			= 2 × 240.3894 + 25.5854	+ 3.6665
							Mean	0.0007
								0.5994
								0.7147

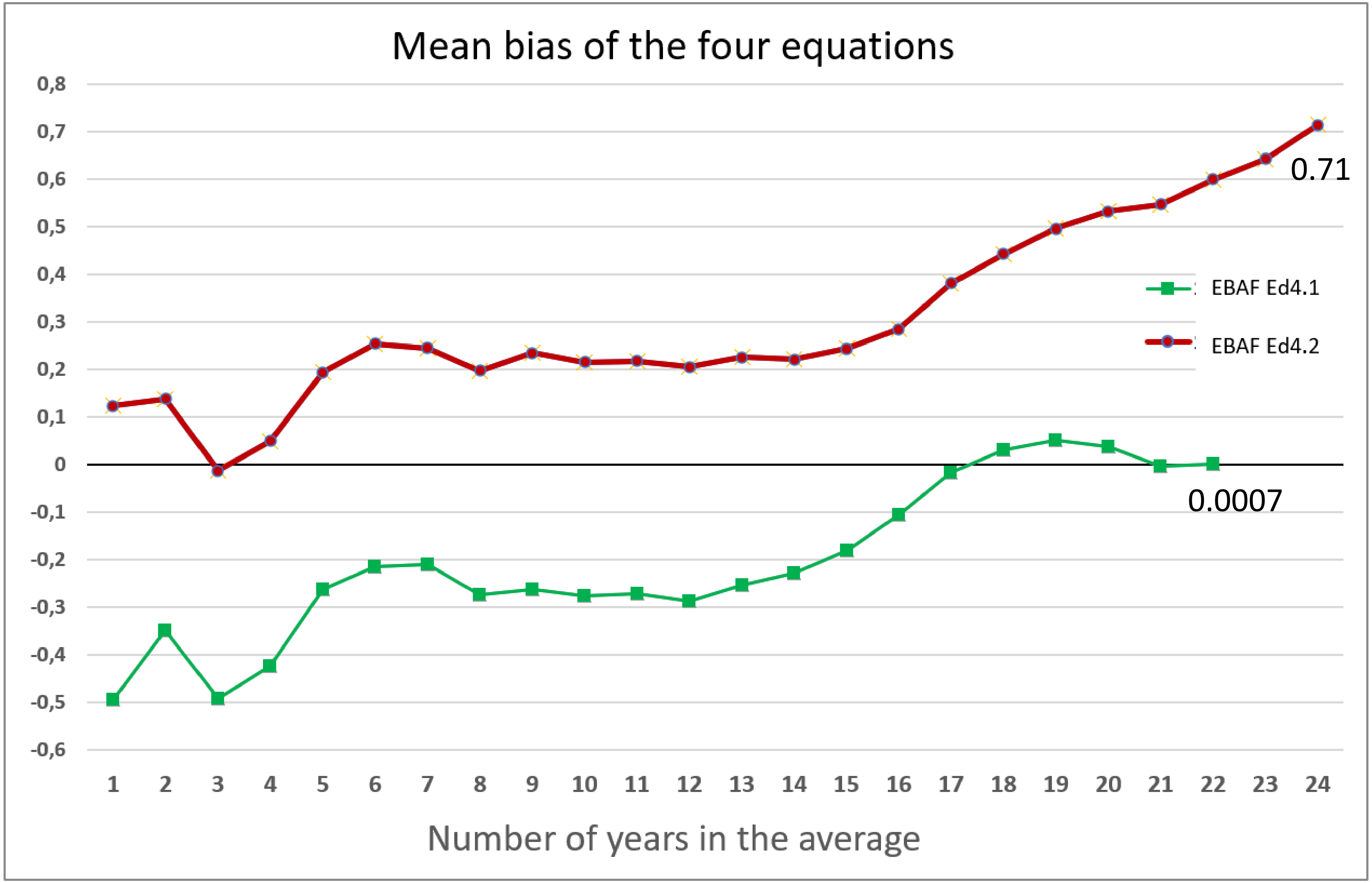
			Mean:
CERES EBAF Ed4.1	Version 3, 22 years	(April 2000 – March 2022) (Wm^{-2})	0.0007
CERES EBAF Ed4.2	Version 4, 22 years	(April 2000 – March 2022) (Wm^{-2})	0.5994
CERES EBAF Ed4.2	Version 4, 24 years	(April 2000 – March 2024) (Wm^{-2})	0.7147
CERES EBAF Ed4.2.1	Version 1, 22 years	(April 2000 – March 2022) (Wm^{-2})	0.5995
CERES EBAF Ed4.2.1	Version 1, 24 years	(April 2000 – March 2024) (Wm^{-2})	0.7036

Eq. (1)	SFC SW down	– SW up	+ LW down	– LW up	(clear)	= TOA LW (clear)/2	
	240.8680	– 29.0724	+ 317.4049	– 398.5211		= 266.0122 /2	– 2.3267
	241.0969	– 29.7521	+ 317.8744	– 398.5890		= 265.9594 /2	– 2.3495
	241.0514	– 29.7043	+ 318.0984	– 398.7742		= 265.9748 /2	– 2.3161
	241.0969	– 29.7521	+ 317.8744	– 398.5890		= 265.9594 /2	– 2.3495
	241.0362	–29.6972	+ 318.1063	– 398.8026		= 265.9732/2	– 2.3439

Eq. (2)	SFC SW down	– SW up	+ LW down	(clear)	= 2 × TOA LW (clear)	
	240.8680	– 29.0724	+ 317.4049		= 2 × 266.0122	– 2.8238
	241.0969	– 29.7521	+ 317.8744		= 2 × 265.9594	– 2.6996
	241.0514	– 29.7043	+ 318.0984		= 2 × 265.9748	– 2.5042
	241.0969	– 29.7521	+ 317.8744		= 2 × 265.9594	– 2.6996
	241.0362	–29.6972	+ 318.1063		=2×265.9732	– 2.5012

Eq. (3)	SFC SW down	– SW up	+ LW down	– LW up	(all)	= [TOA LW (all) – LWCRE]/2	
	186.8544	– 23.1629	+ 345.0108	– 398.7550		= (240.2450 – 25.7672)/2	+ 2.7083
	187.1451	– 23.4950	+ 346.1057	– 398.4220		= (240.3317 – 25.6277)/2	+ 3.9818
	187.1756	– 23.4607	+ 346.3158	– 398.6162		= (240.3894 – 25.5854)/2	+ 4.0126
	187.1451	– 23.4950	+ 346.1059	– 398.4220		= (240.3317 – 25.6277)/2	+ 3.9818
	187.1513	– 23.4547	+ 346.3226	– 398.6131		= (240.3894 – 25.5835)/2	+ 4.0032

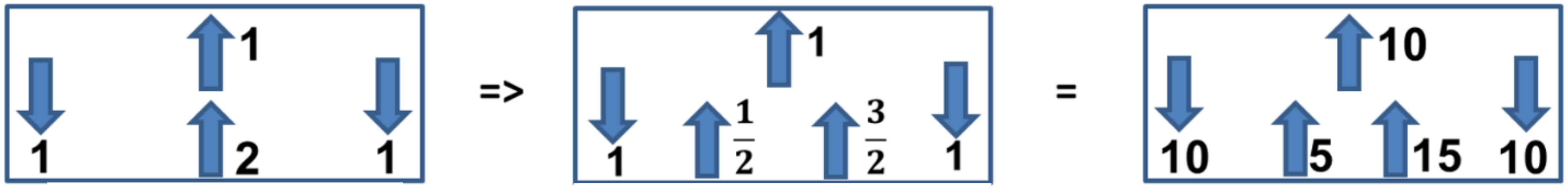
Eq. (4)	SFC SW down	– SW up	+ LW down	(all)	= 2 × TOA LW (all) + LWCRE	
	186.8544	– 23.1629	+ 345.0108		= 2 × 240.2450 + 25.7672	+ 2.4450
	187.1451	– 23.4950	+ 346.1057		= 2 × 240.3317 + 25.6277	+ 3.4647
	187.1756	– 23.4607	+ 346.3158		= 2 × 240.3894 + 25.5854	+ 3.6665
	187.1451	– 23.4950	+ 346.1059		= 2 × 240.3317 + 25.6277	+ 3.4649
	187.1513	– 23.4547	+ 346.3226		= 2 × 240.3894 + 25.5835	+ 3.6565



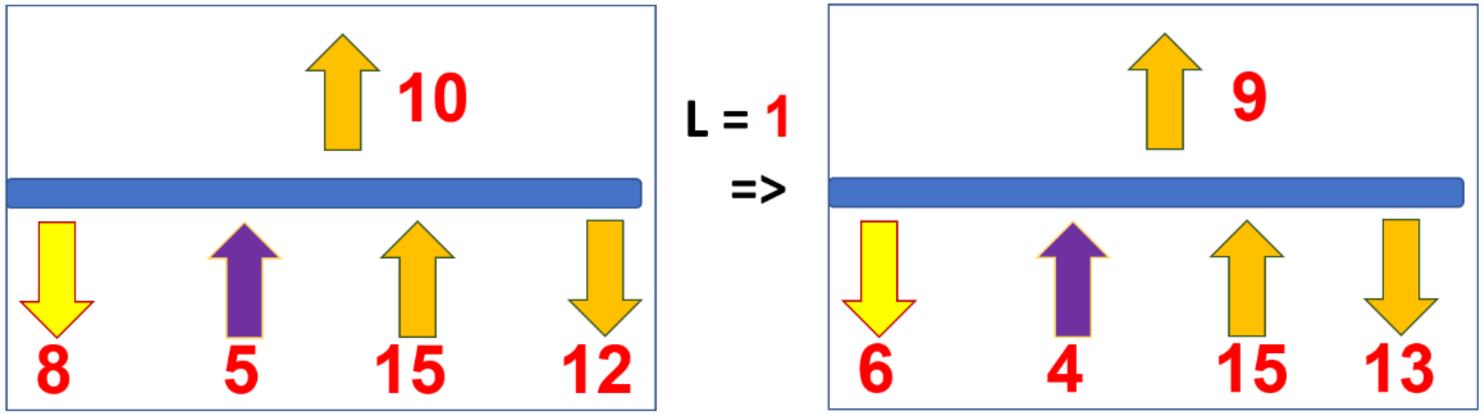
Mean bias of the four equations
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
-0,5
-0,6
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Number of years in the average
EBAF Ed4.1
EBAF Ed4.2
0.71
0.0007

The **N**-numbers, as solution of the equations

Pure geometry: No reference to GHGs



Eq.(2) $A = 2A_0$ (Clear-sky) Eq.(1) $\Delta A = A - E = A_0/2$



$8 + 12 - 15 = 10 / 2$

$8 + 12 = 10 \times 2$

$6 + 13 - 15 = (9 - 1)/2$

$6 + 13 = 9 \times 2 + 1$

Eq. (1) SFC Net = $A_0/2$

Eq. (2) SFC Tot = $2A_0$

Clear-sky

Eq. (3) SFC Net = $(A_0 - L)/2$

Eq. (4) SFC Tot = $2A_0 + L$

All-sky

The flux components with LWCRE = 1

TOA LW	clear-sky	=	10
SFC LW up	clear-sky	=	15
SFC LW down	clear-sky	=	12
SFC LW net	clear-sky	=	–3
SFC SW net	clear-sky	=	8
SFC SW+LW net	clear-sky	=	5
SFC SW+LW total	clear-sky	=	20
G greenhouse effect	clear-sky	=	5
SWCRE (surface)		=	–2

TOA LW	all-sky	=	9
SFC LW up	all-sky	=	15
SFC LW down	all-sky	=	13
SFC LW net	all-sky	=	–2
SFC SW net	all-sky	=	6
SFC SW+LW net	all-sky	=	4
SFC SW+LW total	all-sky	=	19
G greenhouse effect	all-sky	=	6
LWCRE (surface, TOA)		=	1

Fit model to observation: CERES EBAF Ed4.2.1, 24 years, Oct 2000 — Sep 2024 data

Best fit: 1 unit = 1 = LWCRE = 26.68 ± 0.01 Wm^{-2}

CERES_EBAF_Ed4.2.1, 24 years (288 monthly means, October 2000 – September 2024), **1** = 26.68 Wm^{-2}

253		98,7074	240,9554	2,5637	52,90444	265,6999	23,62213	-45,80284	24,74439	-21,05843	342,2266	189,6414	244,0378	23,24179	29,07158	347,8019	318,5667	400,204	400,0479	166,3996	214,9664	-52,4021	-81,48115	113,9975	133,4851
254		104,4346	238,2724	4,9966	56,10619	263,6071	27,98941	-48,32811	25,33461	-22,99351	347,703	193,2171	250,9092	26,74921	32,85757	342,1113	312,8091	394,5234	394,4719	166,4678	218,0516	-52,41211	-81,66286	114,0557	136,3887
255		107,5056	237,4099	6,2549	56,67256	262,7762	31,7214	-50,83266	25,3662	-25,46644	351,1706	192,5235	252,8553	26,60167	32,78657	339,0085	310,4919	390,9289	391,2567	165,9218	220,0688	-51,92038	-80,76502	114,0014	139,3037
256		104,8217	237,3957	9,253	55,32596	262,7736	33,37041	-49,4954	25,37786	-24,11755	351,4704	193,3327	251,7359	24,80234	30,81571	337,4441	309,6986	389,7022	390,0338	168,5303	220,9203	-52,25814	-80,33506	116,2722	140,5853
257		100,4988	238,7799	9,4367	54,44646	263,5489	30,72001	-46,05215	24,76886	-21,28329	348,7156	193,3844	248,0479	23,31385	29,34392	338,2945	311,0346	390,9992	391,1939	170,0706	218,7039	-52,70467	-80,15937	117,3659	138,5445
258		97,7993	238,5701	7,3783	54,25489	263,9326	25,56023	-43,54433	25,36258	-18,18176	343,7478	191,4149	243,3389	23,41029	29,4502	342,2765	314,8845	394,8565	395,0477	168,0046	213,8886	-52,58004	-80,16295	115,4246	133,7257
259		96,0976	240,0821	1,7949	55,47027	265,1029	17,40201	-40,62751	25,02076	-15,60676	337,9749	189,9422	239,2045	25,3512	31,49526	346,2696	318,1993	399,4394	399,9435	164,591	207,7094	-53,16977	-81,7442	111,4212	125,9651
260		96,7908	241,504	-5,2819	55,2164	266,8023	10,99409	-41,57462	25,29841	-16,27623	333,0126	183,9398	234,7318	25,06589	31,84753	350,5461	322,1613	403,8929	403,9753	158,8739	202,8843	-53,34674	-81,81413	105,5272	121,0702
261		94,2918	243,1035	-7,3893	52,42343	268,6725	8,910278	-41,86861	25,56879	-16,29981	330,0056	180,1227	230,7089	22,13565	28,73714	354,5281	327,1277	408,0301	408,2962	157,987	201,9718	-53,50202	-81,16852	104,485	120,8033
262		93,4224	244,7928	-8,6565	50,33212	270,0281	9,199315	-43,09057	25,23519	-17,85538	329,559	176,719	228,1416	19,37508	25,5568	358,5741	330,8475	409,2198	409,7949	157,3439	202,5847	-50,64572	-78,9473	106,6982	123,6374
263		91,8107	243,9578	-3,8613	48,96224	269,1224	13,8224	-42,84861	25,16452	-17,68407	331,9068	179,6492	230,4227	18,21585	24,25596	357,4669	330,2098	408,4352	408,8898	161,4333	206,1667	-50,96832	-78,68001	110,465	127,4867
264		93,3346	242,9631	0,2387	50,08413	267,9279	18,52429	-43,25048	24,96487	-18,28564	336,5364	184,3139	235,454	19,46911	25,1872	353,2803	324,7163	405,0963	405,4862	164,8448	210,2668	-51,81599	-80,76996	113,0288	129,4969
265		98,9496	240,6711	2,7428	53,15376	265,7502	23,45969	-45,7957	25,07917	-20,71652	342,3639	188,9254	243,5483	23,14738	28,91193	347,9407	318,2406	400,0871	400,2213	165,7781	214,6363	-52,14635	-81,9804	113,6317	132,6559
266		104,689	238,4728	4,6625	56,56578	263,3417	27,91626	-48,12286	24,8689	-23,25397	347,8242	192,7777	250,6648	26,84423	32,96877	341,3409	311,8542	393,6637	393,7872	165,9335	217,696	-52,32288	-81,93311	113,6106	135,763
267		106,2429	237,2914	7,6908	56,83059	262,3939	32,00045	-49,41195	25,10263	-24,30932	351,2256	193,5878	252,4548	26,50303	32,52414	337,9436	309,7894	390,5557	391,0644	167,0848	219,9305	-52,61208	-81,27502	114,4727	138,6555
268		103,8612	238,1718	9,5616	55,28458	262,8087	33,50086	-48,57627	24,63689	-23,93938	351,5945	194,0472	251,648	24,57954	30,43088	336,9802	309,3247	389,6776	390,4656	169,4676	221,2172	-52,6974	-81,14087	116,7702	140,0763
269		100,4361	238,4082	10,014	54,60859	263,3044	30,94511	-45,82726	24,89607	-20,93118	348,8583	192,7375	247,5771	23,16995	29,1373	338,9588	311,3165	391,546	392,3134	169,5676	218,4398	-52,5872	-80,99689	116,9804	137,4429
270		96,8389	238,8378	8,301	53,75555	264,0556	26,1667	-43,08337	25,21774	-17,86562	343,9779	192,2131	244,1617	23,32921	29,47409	343,0176	315,1904	395,9231	396,7707	168,8839	214,6875	-52,90546	-81,5801	115,9784	133,1074
271		95,4387	238,812	3,9093	55,06451	264,4027	18,69326	-40,37437	25,59056	-14,7838	338,1602	190,388	239,2811	25,28007	31,40031	346,5129	318,7782	399,8491	400,3909	165,108	207,8808	-53,33624	-81,61266	111,7717	126,2681
272		96,28	240,0879	-3,3144	55,04652	266,2265	11,78083	-41,23367	26,13864	-15,09504	333,0536	183,5718	234,0113	24,81425	31,73598	352,2384	324,2188	405,2886	405,2398	158,7576	202,2753	-53,05022	-81,02094	105,7074	121,2543
273		94,1155	243,9189	-7,9531	52,32	269,1089	8,65178	-41,79574	25,18998	-16,60576	330,0803	179,1258	229,6274	21,93086	28,53834	356,7723	329,5032	409,651	409,7618	157,1949	201,0891	-52,87865	-80,25866	104,3163	120,8304
274		91,9415	245,5909	-7,9875	50,36101	270,4723	8,71242	-41,58075	24,88147	-16,69928	329,5454	177,1177	226,662	19,22236	25,33484	360,2484	333,5309	411,4499	412,0339	157,8954	201,3271	-51,20147	-78,50298	106,6939	122,8241
275		90,0442	246,0138	-4,0719	48,86162	270,3517	12,77289	-41,1827	24,33787	-16,84485	331,9859	180,6093	229,242	18,09754	23,79961	359,6151	332,9008	410,9144	411,3944	162,5118	205,4424	-51,29924	-78,49373	111,2125	126,9487
276		91,8499	244,6051	0,1319	49,77739	269,1724	17,63781	-42,07256	24,56721	-17,50534	336,5875	184,6852	234,2685	19,22868	24,57043	356,68	329,0005	408,1161	408,4055	165,4566	209,698	-51,43609	-79,40533	114,0205	130,2928
277		97,7124	242,5307	2,2283	52,5686	266,8806	23,02255	-45,1436	24,35001	-20,79361	342,4721	189,1095	242,8158	22,70358	28,31689	351,3488	321,9916	402,7723	402,7569	166,4059	214,499	-51,42344	-80,76534	114,9825	133,7337
278		104,1191	239,2973	4,4884	56,20588	264,3373	27,36126	-47,91288	25,04019	-22,87269	347,905	191,8966	249,4565	26,3176	32,44026	346,1155	316,5586	396,904	396,9132	165,579	217,0163	-50,78852	-80,3543	114,7905	136,6619
279		106,3374	238,2565	6,7748	56,67485	263,6084	31,08583	-49,66216	25,35186	-24,31027	351,3696	193,2553	252,2531	26,6874	32,55133	342,2156	313,8424	393,6581	394,238	166,5679	219,7019	-51,44246	-80,39542	115,1254	139,3064
280		104,8924	238,8159	8,0029	55,36269	264,0092	32,33899	-49,52936	25,19333	-24,33604	351,7114	192,6077	251,3724	24,66777	30,7223	340,859	313,0428	392,1826	392,9261	167,9399	220,65	-51,32364	-79,88313	116,6163	140,7669
281		100,8297	239,1887	8,742	54,03152	264,4475	30,28127	-46,79798	25,2587	-21,53926	348,7605	191,705	247,7225	22,9062	28,99872	343,1651	315,163	393,9589	394,9054	168,7988	218,7237	-50,79385	-79,74237	118,0049	138,9815
282		97,8548	239,2097	6,7147	53,50908	264,6825	25,58821	-44,34574	25,47272	-18,87301	343,7798	190,219	243,4278	23,24954	29,32313	346,3315	318,2617	396,8664	397,6888	166,9694	214,1047	-50,53487	-79,42713	116,4346	134,6776
283		95,8494	239,8717	2,2767	54,33109	265,5461	18,1211	-41,5185	25,67441	-15,8441	337,9982	188,7794	238,7513	24,79462	30,9199	349,8272	322,0793	401,4675	402,0357	163,9848	207,8314	-51,64028	-79,95622	112,3445	127,8752
284		96,4398	242,4165	-5,8927	55,20716	267,5814	10,17486	-41,23293	25,16492	-16,068	332,9631	183,6233	234,0934	25,32066	32,34639	353,8289	325,9371	406,3748	406,2236	158,3027	201,7469	-52,54584	-80,28638	105,7568	121,4606
285		94,759	244,133	-8,8701	52,08978	269,4231	8,509075	-42,66945	25,28998	-17,37947	330,0214	178,1862	229,7249	21,79689	28,61031	358,2435	330,6667	410,082	410,3212	156,3893	201,1147	-51,83855	-79,65453	104,5508	121,4601
286		93,1732	245,3037	-8,7732	50,29143	270,6774	8,735243	-42,88205	25,37369	-17,50835	329,7037	175,2004	226,4833	18,91136	25,26239	360,3	333,1862	411,0113	411,5403	156,289	201,2209	-50,71124	-78,35405	105,5778	122,8668
287		90,9307	245,6309	-4,5747	49,09739	270,4989	12,39063	-41,83353	24,86801	-16,96552	331,9866	178,1705	227,5554	17,85514	23,61805	359,8003	332,7403	410,4758	410,9629	160,3154	203,9373	-50,6755	-78,22261	109,6399	125,7147
288	month	92,647	244,5557	-0,3599	49,99052	268,9713	17,88118	-42,65653	24,41561	-18,24092	336,843	182,7636	232,7271	19,10348	24,47001	356,3938	328,276	407,2179	407,6057	163,6601	208,257	-50,82407	-79,32978	112,8361	128,9273
289	**mean**	**98,91**	**240,42**	**0,88**	**53,76**	**265,98**	**20,46**	**-45,15**	**25,56**	**-19,59**	**340,21**	**187,12**	**240,97**	**23,44**	**29,67**	**346,44**	**318,25**	**398,72**	**398,92**	**163,68**	**211,30**	**-52,28**	**-80,67**	**111,40**	**130,63**
290	name	sw-a	lw-a	net-a	sw-cl	lw-cl	net-cl	cre-sw	cre-lw	cre-net	solar	sw-dn-a	sw-dn-c	sw-up-a	sw-up-c	lw-dn-a	lw-dn-c	lw-up-a	lw-up-c	net-sw-a	net-sw-c	net-lw-a	net-lw-c	net-tot-a	net-tot-c
291	**N**	**15**/4	**9**	**0**	**2**	**10**	**3**/4	**–7**/4	**1**	**–3**/4	**51**/4	**7**	**9**	**1**	**1**	**13**	**12**	**15**	**15**	**6**	**8**	**–2**	**–3**	**4**	**5**
292	N×unit	100,05	240,12	0	53,36	266,8	20,01	–46,69	26,68	–20,01	340,17	186,76	240,12	26,68	26,68	346,84	320,16	400,2	400,2	160,08	213,44	–53,36	–80,04	106,72	133,4
293	diff	-1,14	0,30	0,88	0,40	-0,82	0,45	1,54	-1,12	0,42	0,04	0,36	0,85	-3,24	2,99	-0,40	-1,91	-1,48	-1,28	3,60	-2,14	1,08	0,63	4,68	-2,77

CERES EBAF Ed4.2.1, 24 years
October 2000 – September 2024, **1** = 26.68 Wm^{-2}

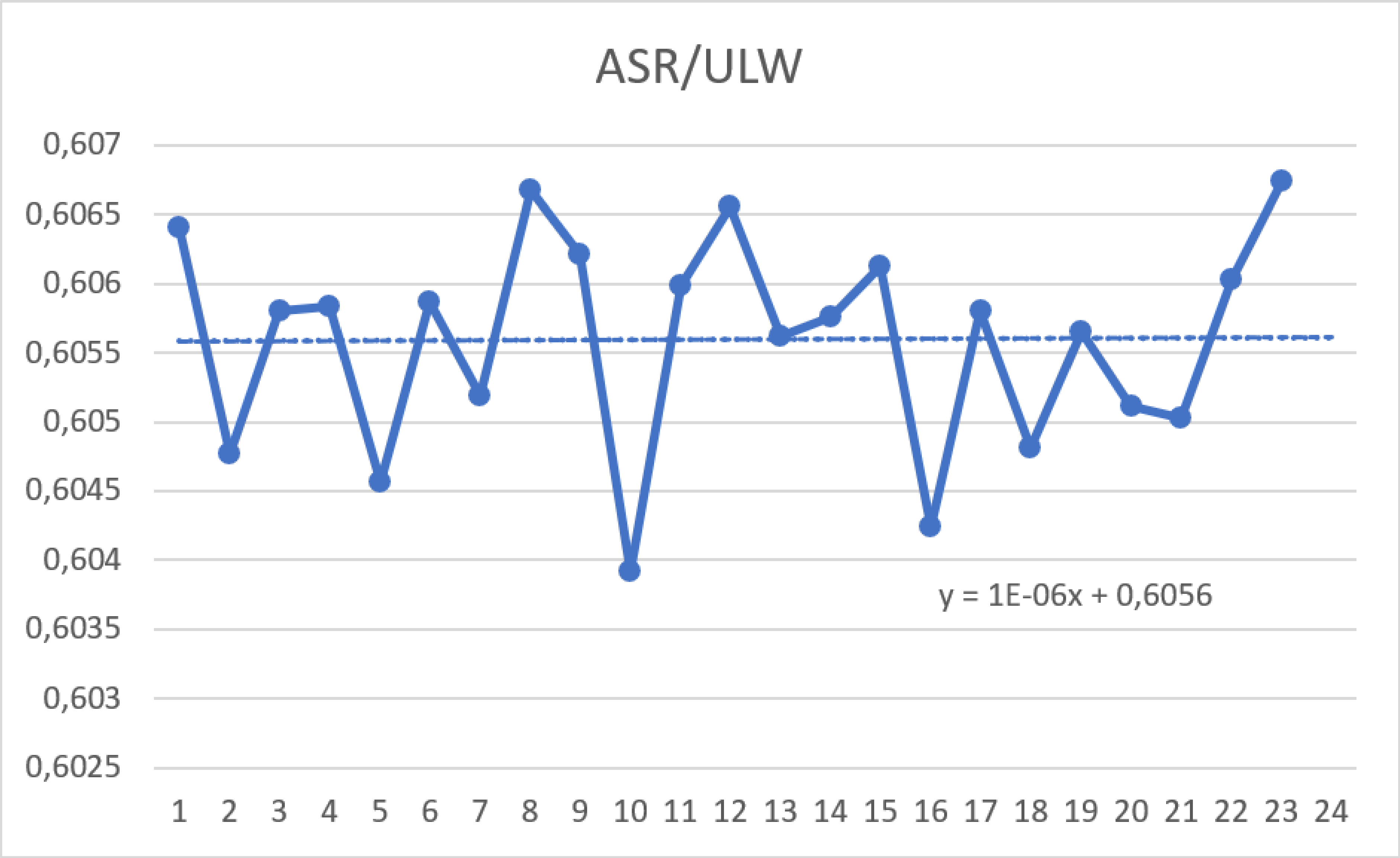
		N	**N** × Unit	EBAF Ed4.2.1	Difference (Wm^{-2})
Clear-Sky TOA	LW	**40**/4	266.80	265.98	-0.82
	SW	**8**/4	53.36	53.76	0.40
	Net	**3**/4	20.01	20.46	0.45
Clear-Sky Surface	LW down	**12**	320.16	318.25	-1.91
	LW up	**15**	400.20	398.92	-1.28
	LW net	**–3**	–79.98	–80.67	0.63
	SW down	**9**	240.12	240.97	0.85
	SW up	**1**	26.68	29.67	2.99
	SW net	**8**	213.44	211.30	-2.14
	SW + LW net	**5**	133.40	130.63	-2.77
	G	**5**	133.40	132.94	-0.46

CERES EBAF Ed4.2.1, 24 years; 1 = 26.68 Wm^{-2}

	All-sky	**N**	**N × Unit**	**EBAF Ed4.2.1**	**Data – Theory (Wm^{-2})**
TOA	SW insolation	51/4	340.17	340.21	0.04
	SW up	15/4	100.05	98.91	-1.14
	LW up	36/4	240.12	240.42	0.30
	TOT net	0	0	0.88	0.88
Surface	SW down	7	186.76	187.12	0.36
	SW up	1	26.68	23.44	-3.24
	SW net	6	160.08	163.68	3.60
	LW down	13	346.84	346.44	-0.40
	LW up	15	400.20	398.72	-1.48
	LW net	–2	–53.36	-52.28	1.08
	TOT net	4	106.72	111.40	4.68
	CRE				
TOA	SW	–7/4	–46.69	-45.15	1.54
	LW	1	26.68	25.56	-1.12
	TOT	–3/4	-20.01	-19.59	0.42

g(clear), CERES EBAF Ed4.2.1, 24-yr mean (Oct 2000 – Sep 2024)

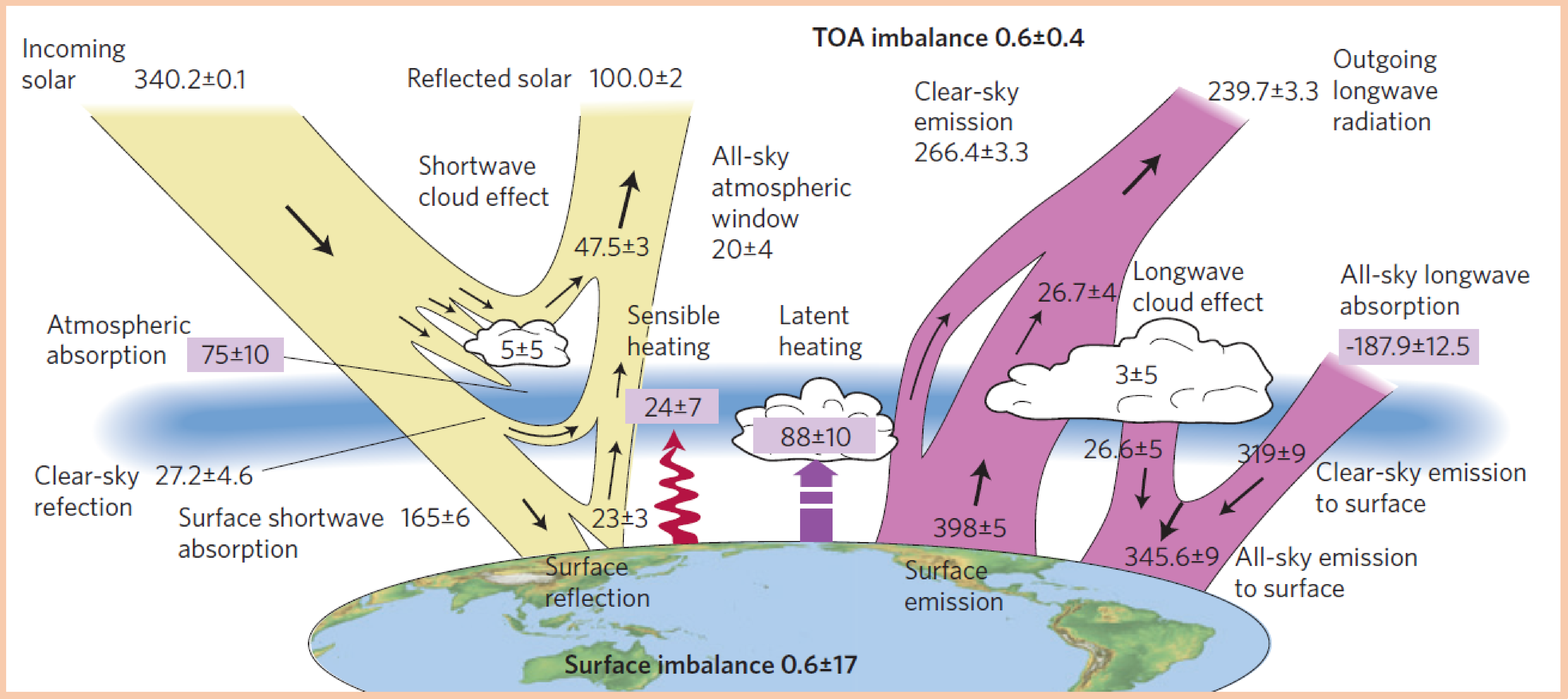
Clear-sky	N	Theory	CERES
Surface LW up	15	400.20	398.9233
TOA LW up	10	266.80	265.9823
G	5	133.40	132.9410
g	5/15	1/3	0.33325
All-sky	**N**	**Theory**	**CERES**
Surface LW up	15	400.20	398.72
TOA LW up	9	240.12	240.42
G	6	160.08	158.30
g	6/15	0.4	0.397

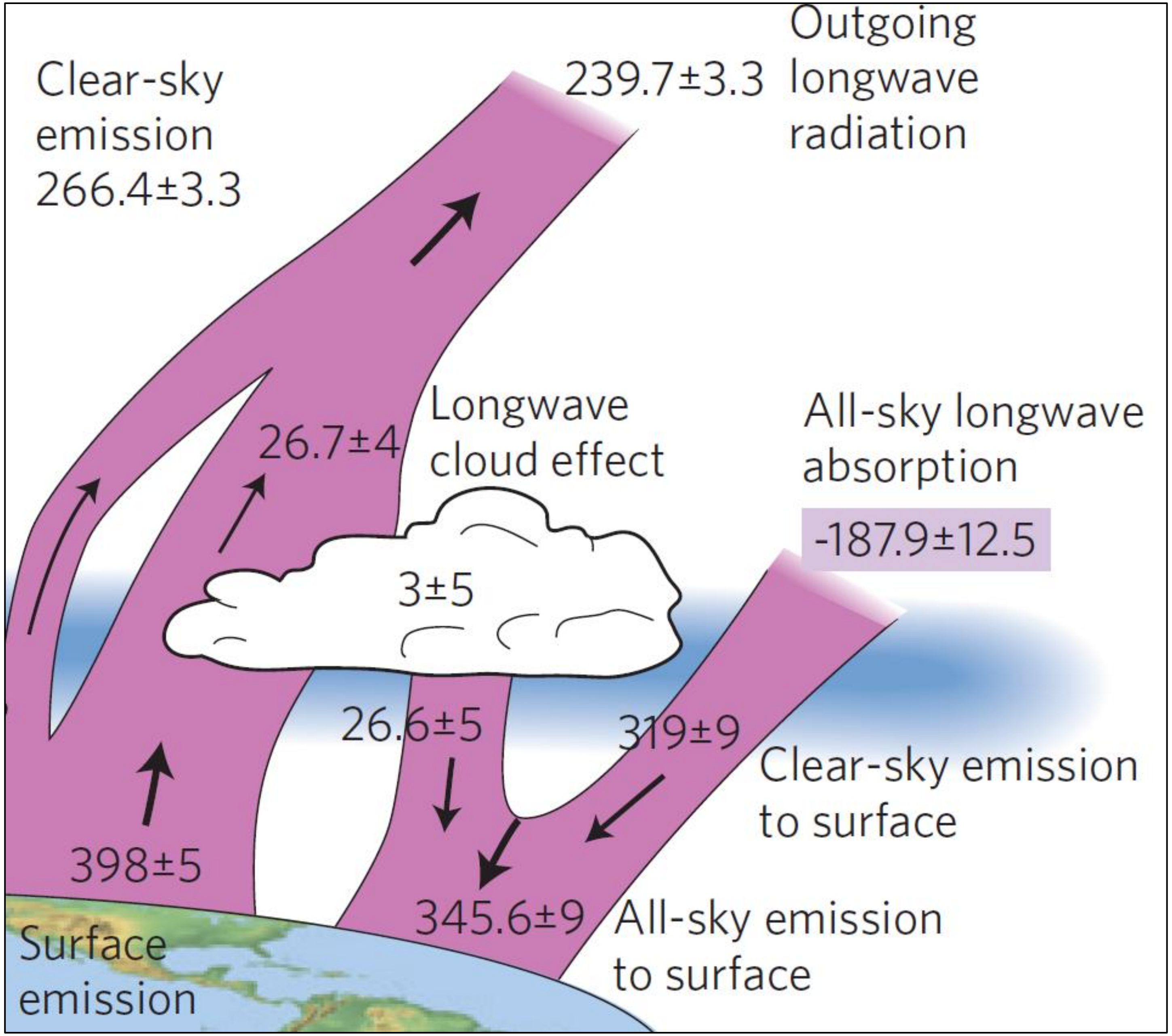

ASR/ULW
0,607
0,6065
0,606
0,6055
0,605
0,6045
0,604
0,6035
0,603
0,6025
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
y = 1E-06x + 0,6056

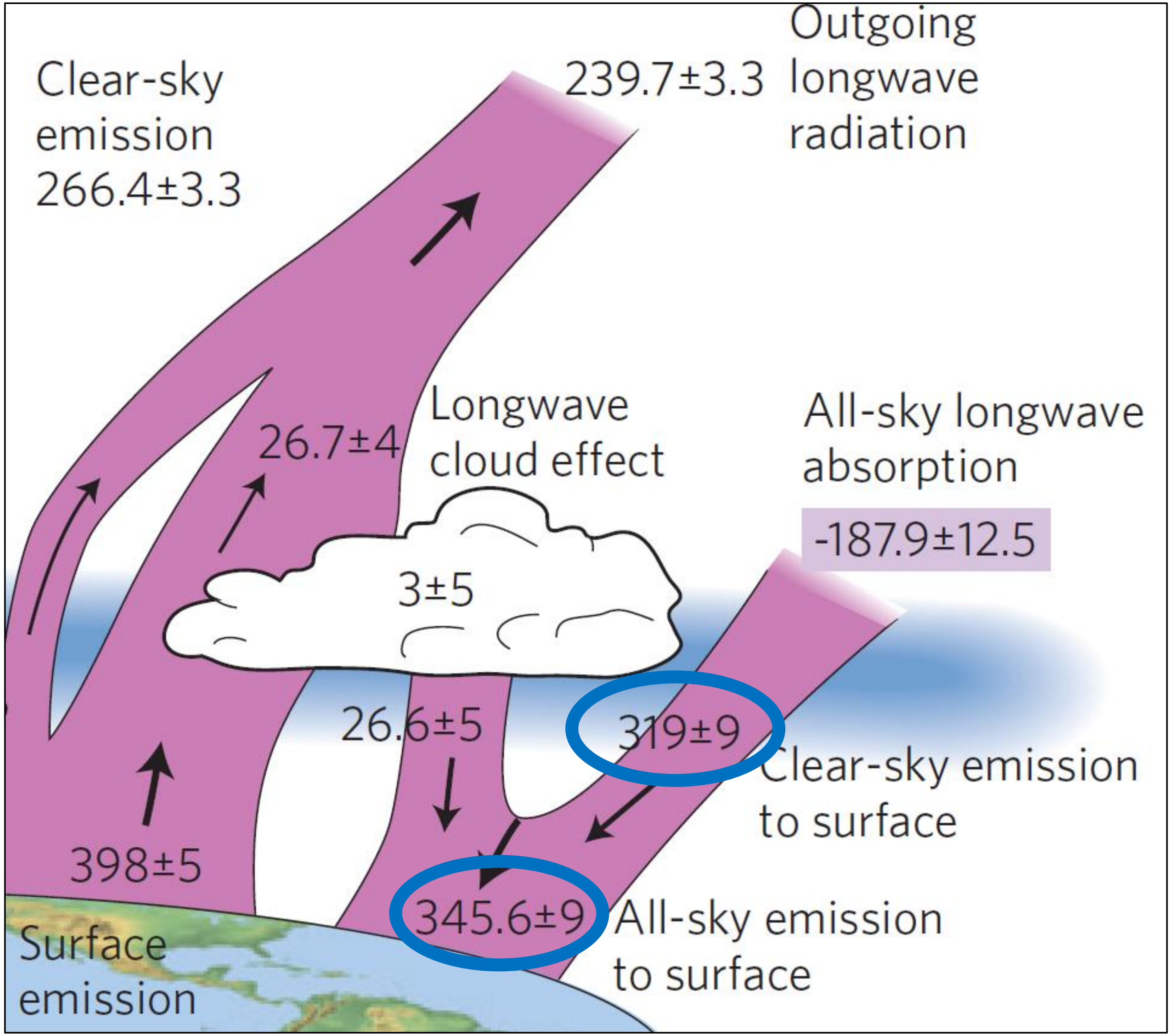
An update on Earth's energy balance in light of the latest global observations

Nature Geosci. (2012)

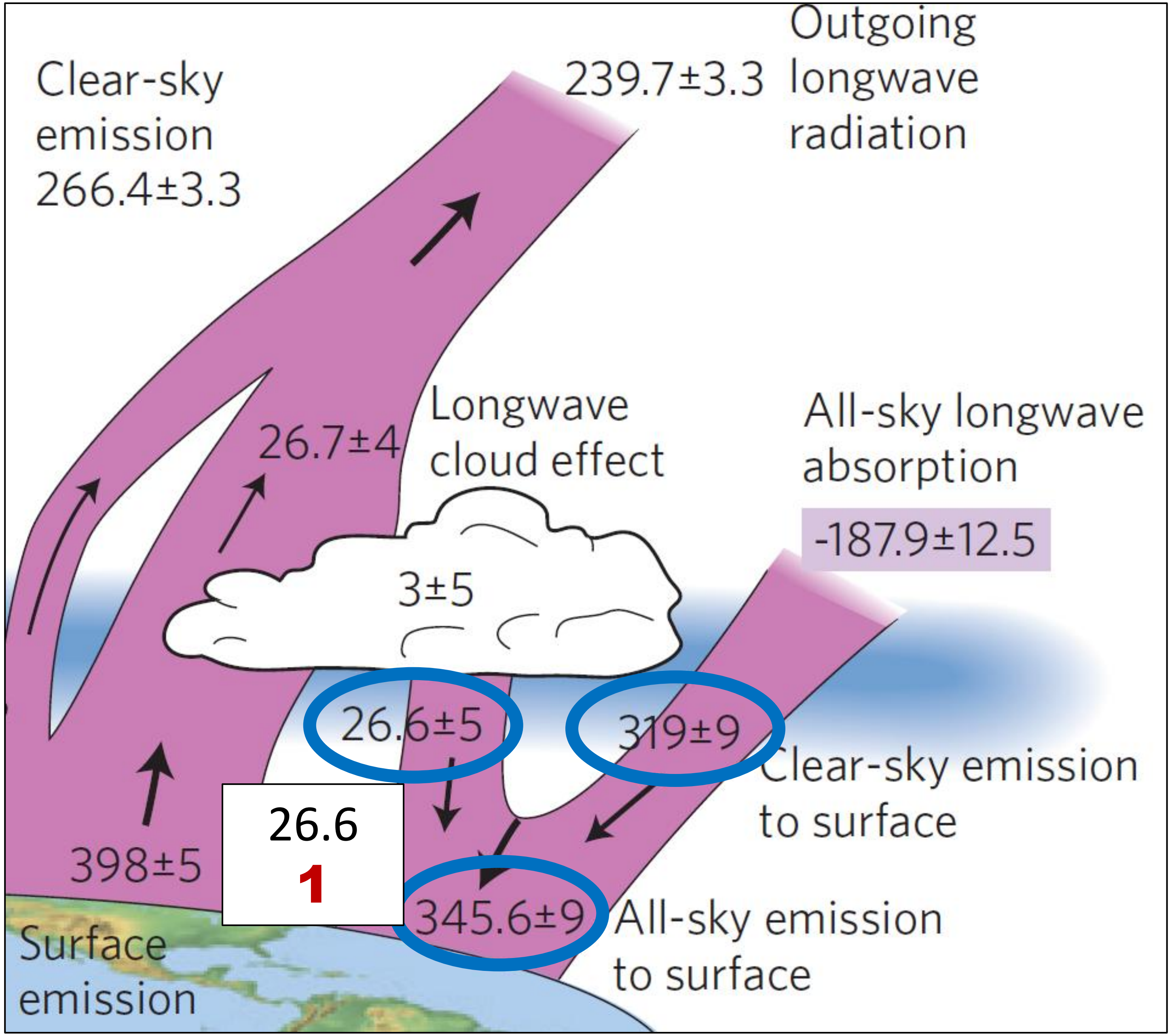
Graeme L. Stephens[1]*, Juilin Li[1], Martin Wild[2], Carol Anne Clayson[3], Norman Loeb[4], Seiji Kato[4], Tristan L'Ecuyer[5], Paul W. Stackhouse Jr[4], Matthew Lebsock[1] and Timothy Andrews[6]

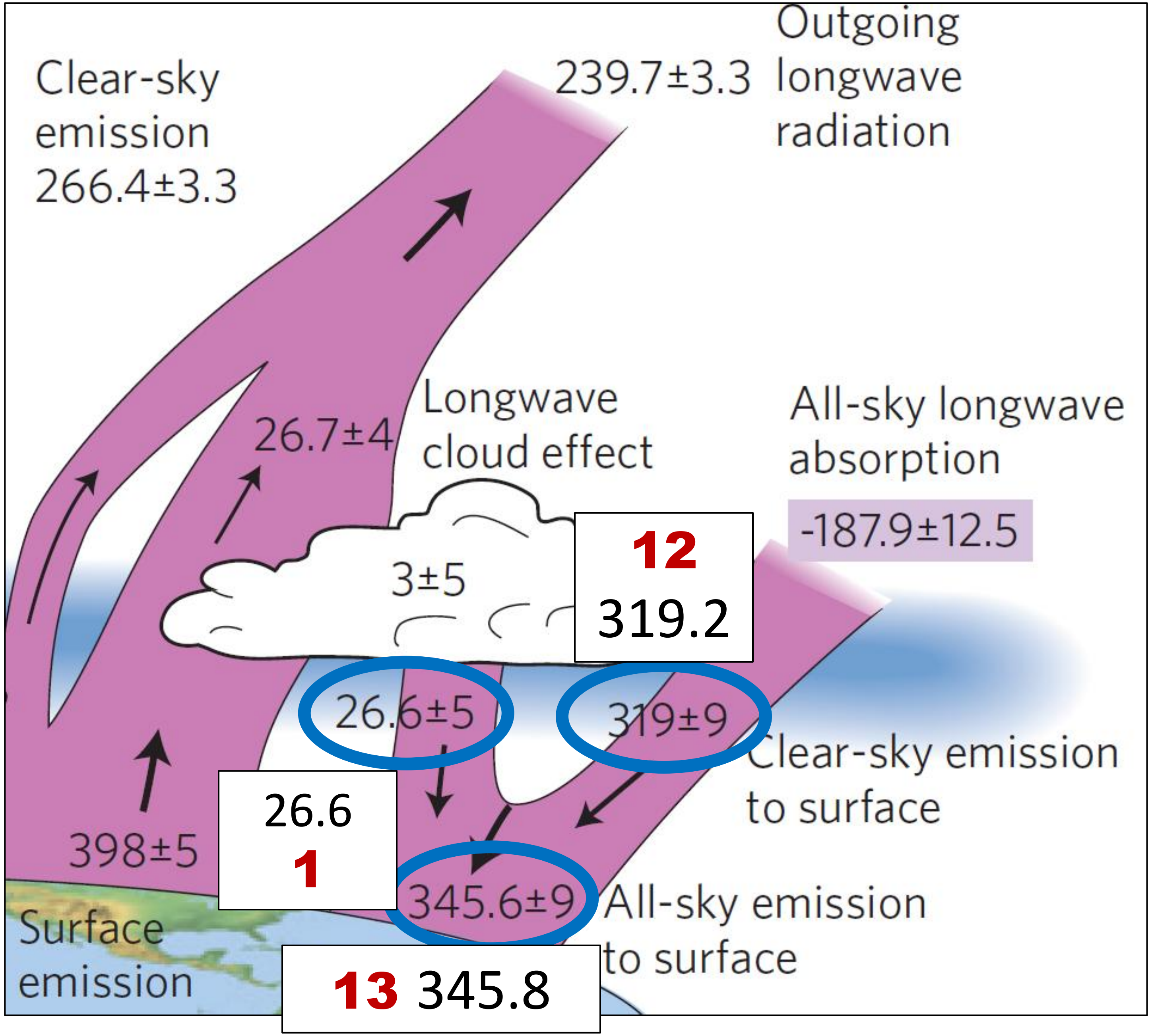



Outgoing
longwave
radiation
Clear-sky
emission
266.4±3.3
239.7±3.3
26.7±4
Longwave
cloud effect
All-sky longwave
absorption
-187.9±12.5
3±5
26.6±5
319±9
Clear-sky emission
to surface
398±5
345.6±9
All-sky emission
to surface
Surface
emission

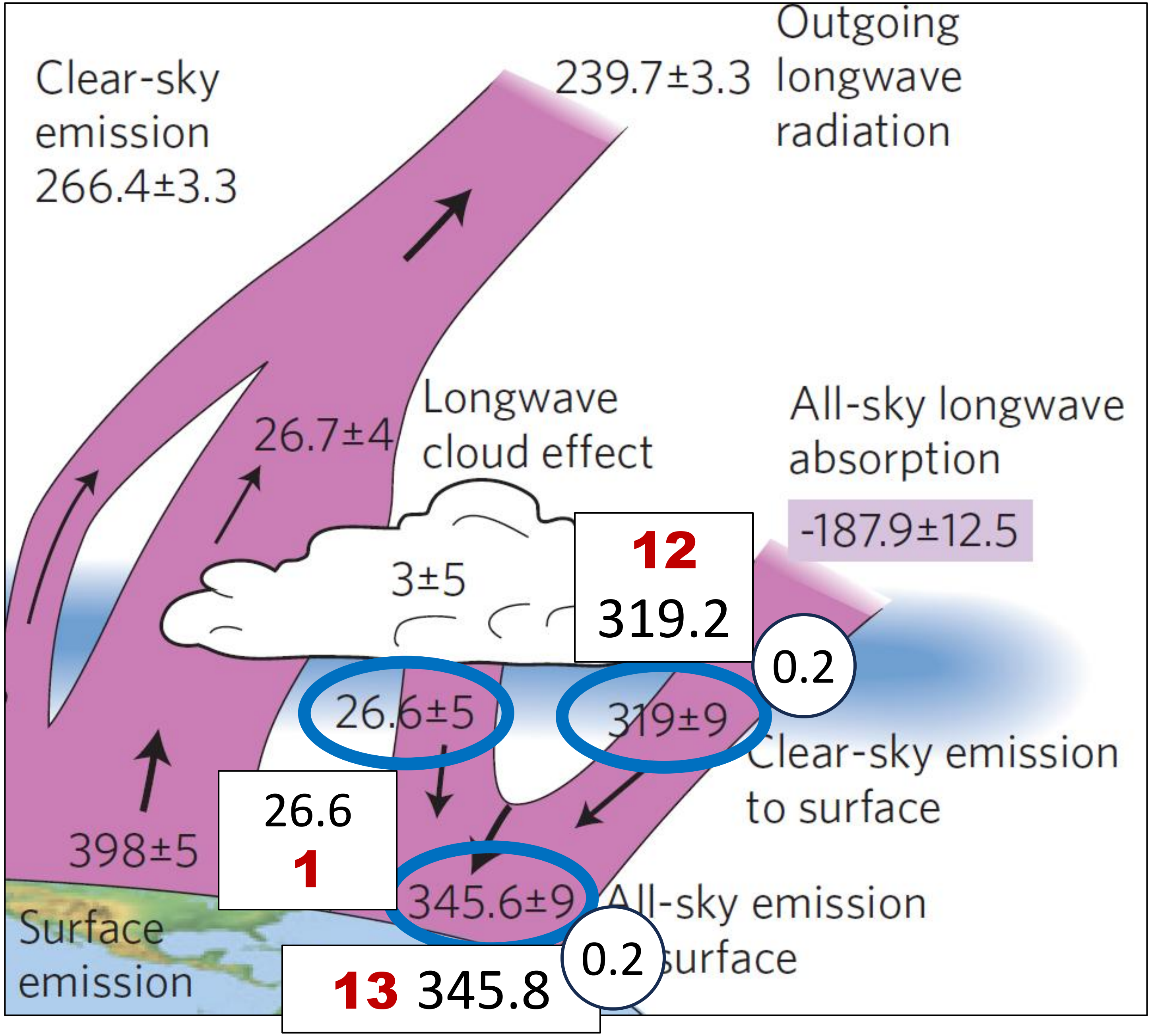


Outgoing longwave radiation
239.7±3.3
Clear-sky emission
266.4±3.3
26.7±4
Longwave cloud effect
3±5
All-sky longwave absorption
-187.9±12.5
26.6±5
319±9
Clear-sky emission to surface
398±5
345.6±9
All-sky emission to surface
Surface emission

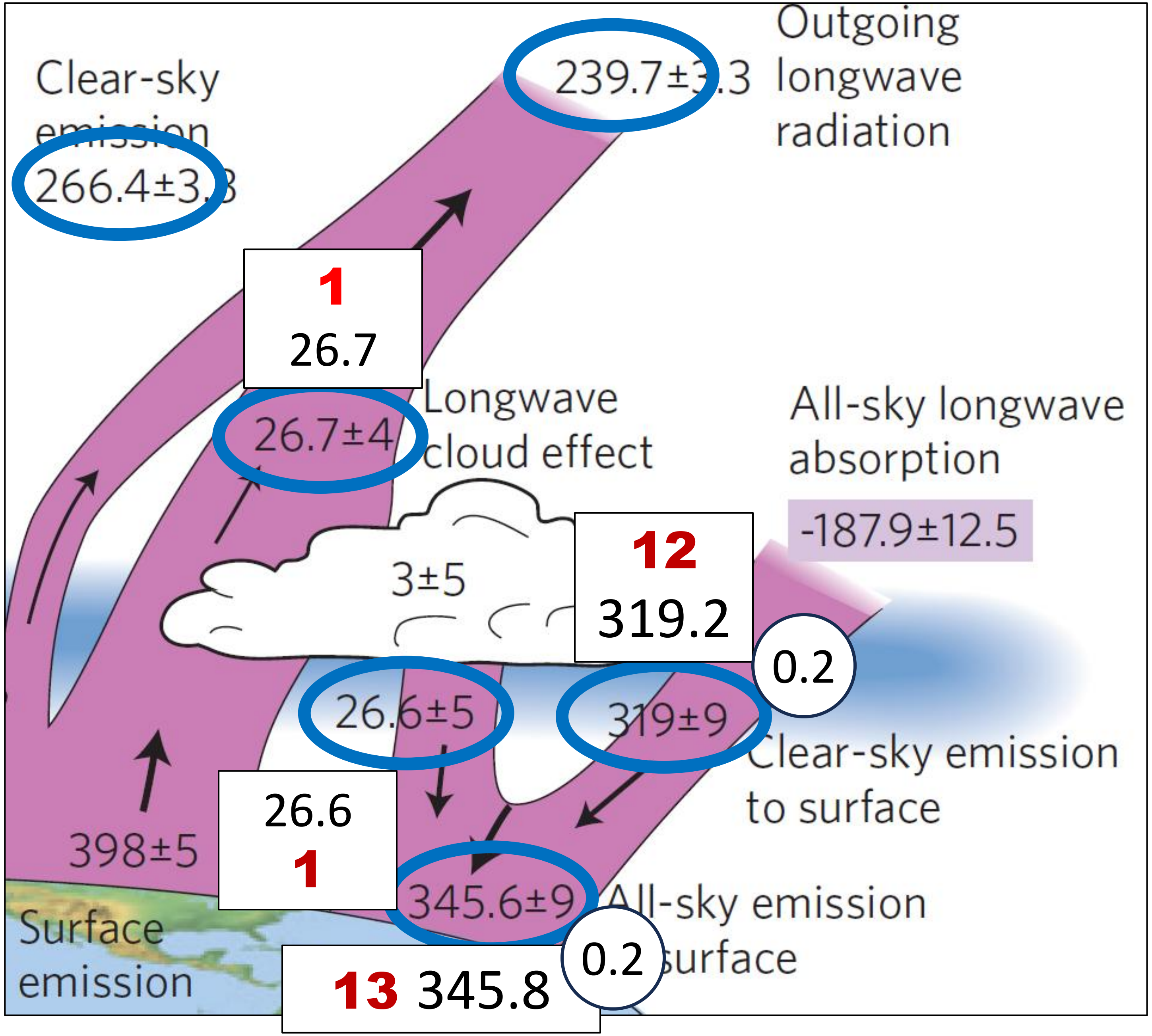

Outgoing
longwave
radiation
239.7±3.3
Clear-sky
emission
266.4±3.3
26.7±4
Longwave
cloud effect
All-sky longwave
absorption
-187.9±12.5
3±5
26.6±5
319±9
Clear-sky emission
to surface
26.6
1
398±5
345.6±9
All-sky emission
to surface
Surface
emission



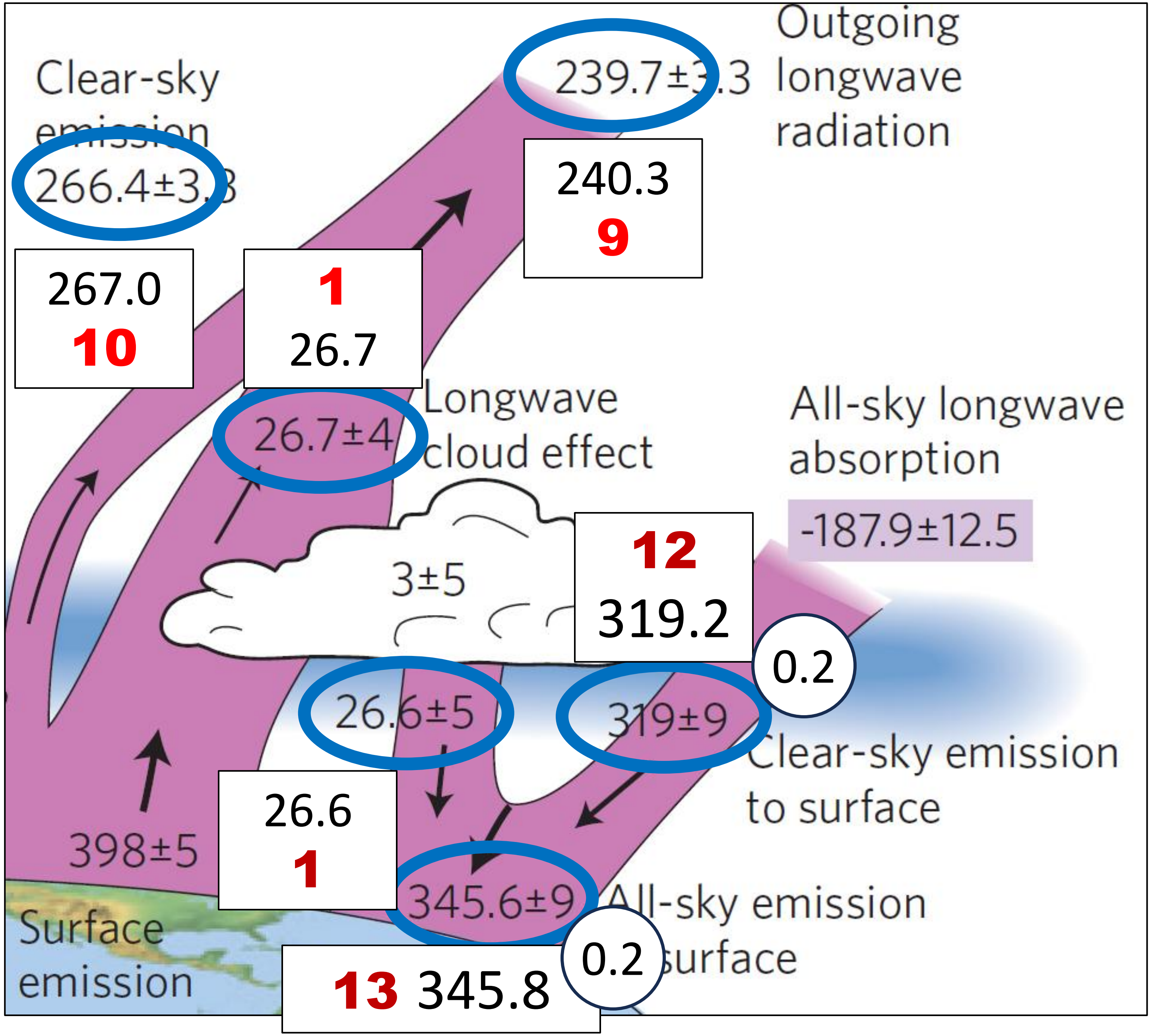
Outgoing
239.7±3.3 longwave
radiation
Clear-sky
emission
266.4±3.3
26.7±4
Longwave
cloud effect
All-sky longwave
absorption
-187.9±12.5
3±5
12
319.2
26.6±5
319±9
Clear-sky emission
to surface
26.6
1
398±5
345.6±9
All-sky emission
to surface
Surface
emission
13 345.8



Outgoing longwave radiation
239.7±3.3
Clear-sky emission 266.4±3.3
26.7±4
Longwave cloud effect
All-sky longwave absorption
-187.9±12.5
3±5
12
319.2
0.2
26.6±5
319±9
Clear-sky emission to surface
26.6
1
398±5
345.6±9
All-sky emission
surface
0.2
Surface emission
13 345.8



Clear-sky emission
266.4±3.3
239.7±3.3
Outgoing longwave radiation
1
26.7
26.7±4
Longwave cloud effect
All-sky longwave absorption
-187.9±12.5
3±5
12
319.2
0.2
26.6±5
319±9
Clear-sky emission to surface
398±5
26.6
1
345.6±9
All-sky emission to surface
Surface emission
0.2
13 345.8


Outgoing longwave radiation
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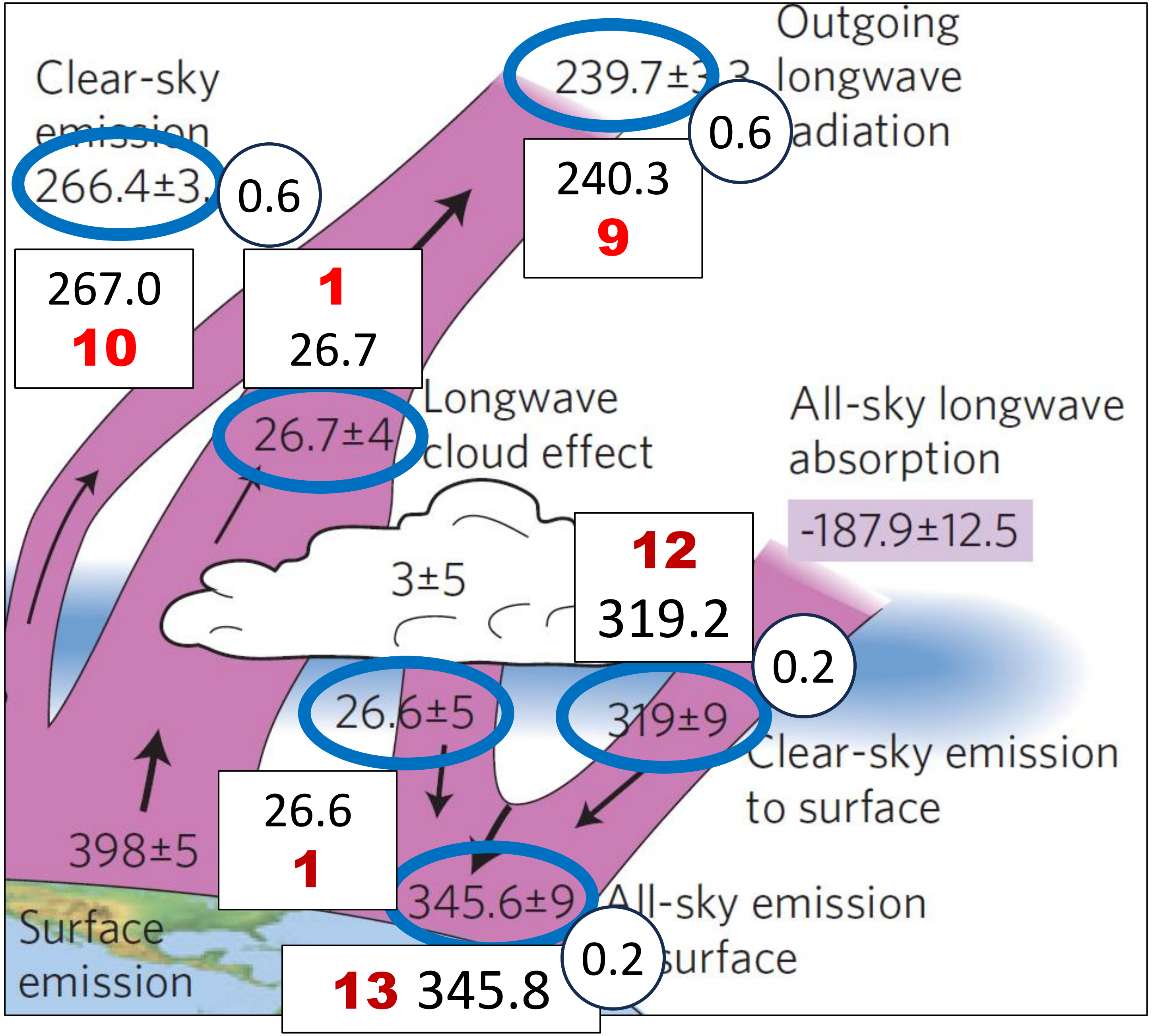
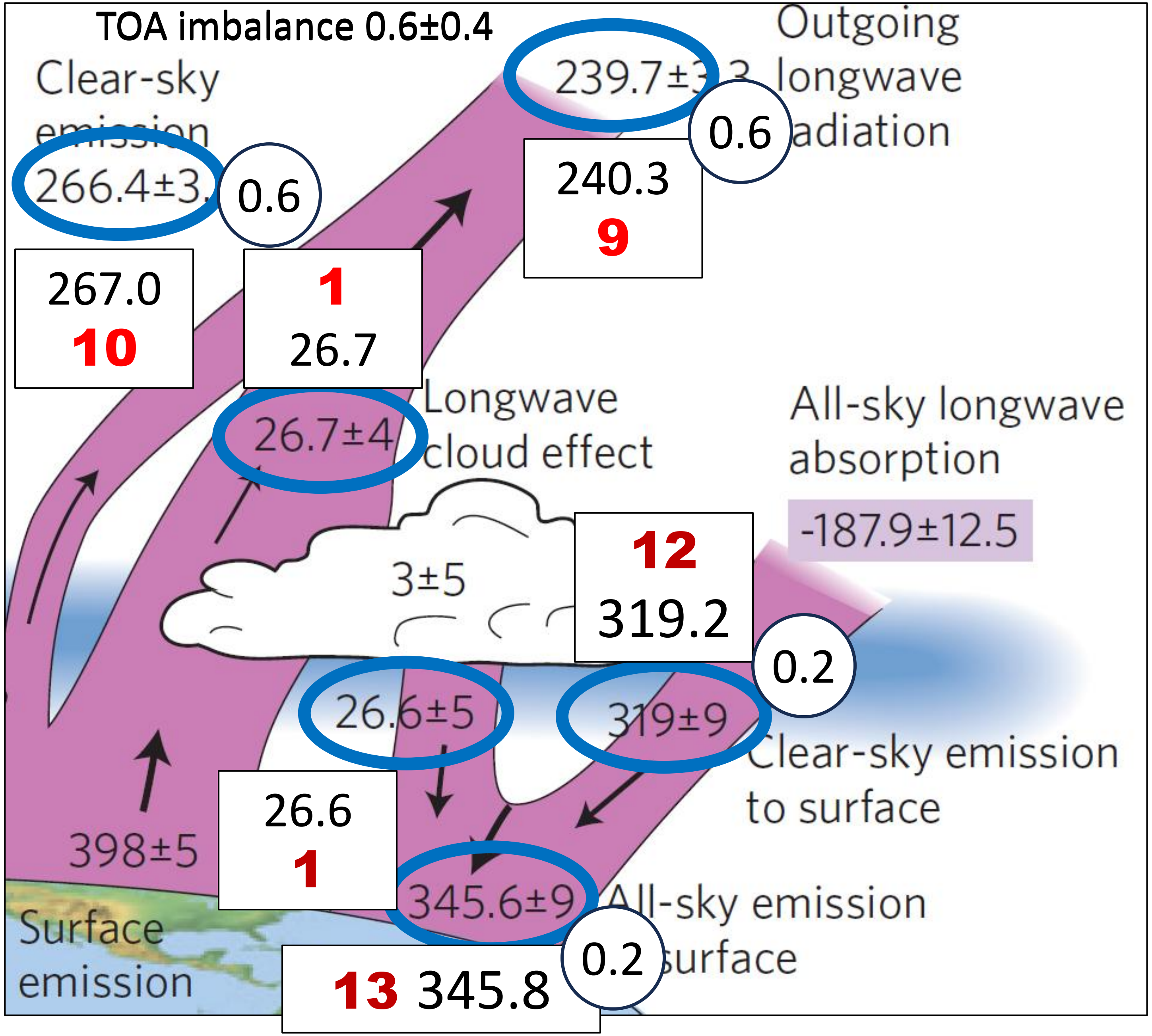
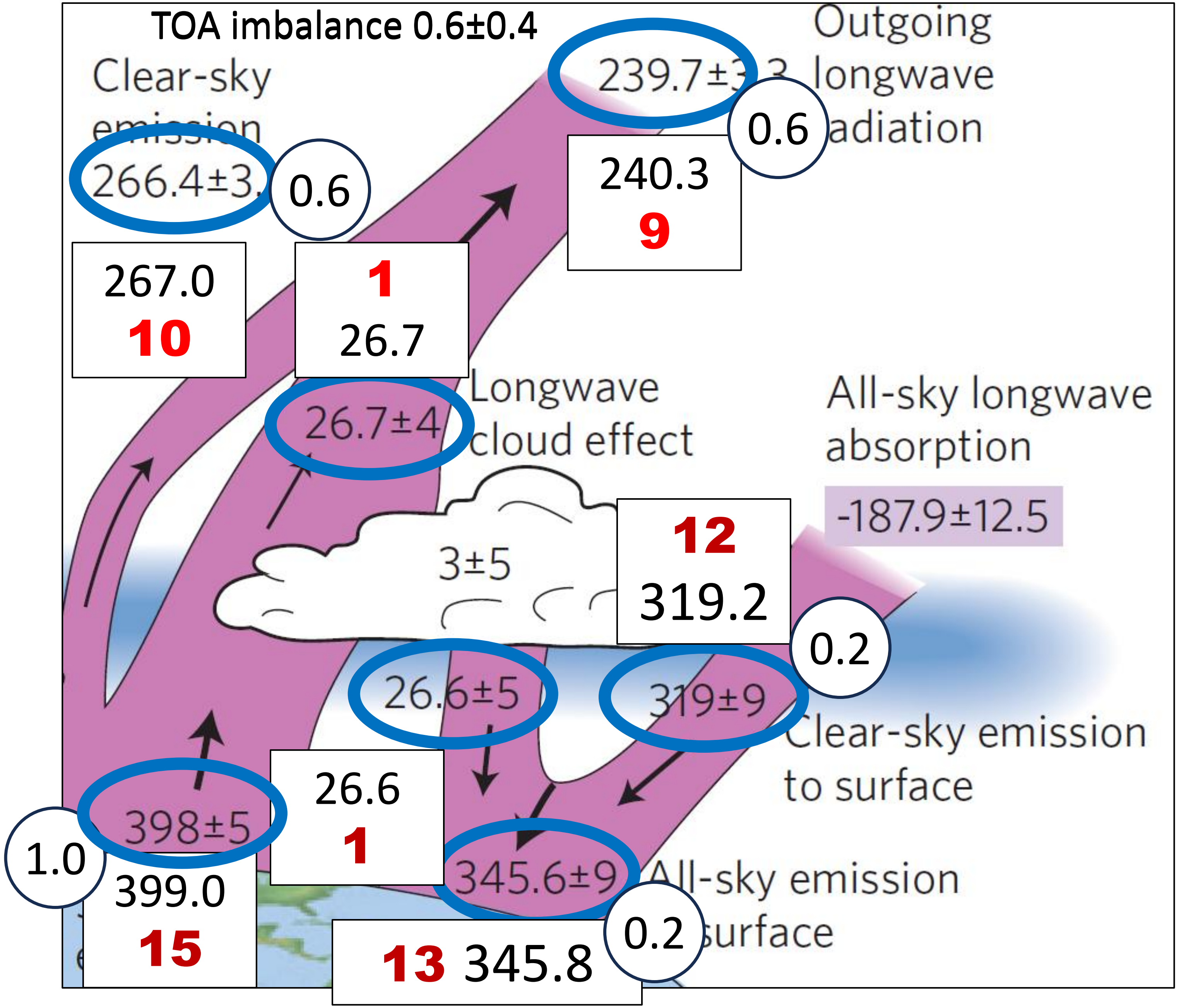
Clear-sky emission
266.4±3.3
267.0
10
1
26.7
26.7±4
Longwave cloud effect
All-sky longwave absorption
-187.9±12.5
12
319.2
0.2
3±5
26.6±5
319±9
Clear-sky emission to surface
398±5
26.6
1
345.6±9
All-sky emission to surface
0.2
Surface emission
13 345.8



Outgoing longwave radiation
239.7±3.3
0.6
240.3
9
Clear-sky emission
266.4±3.
0.6
267.0
10
1
26.7
26.7±4
Longwave cloud effect
All-sky longwave absorption
-187.9±12.5
3±5
12
319.2
0.2
26.6±5
319±9
Clear-sky emission to surface
26.6
1
398±5
345.6±9
All-sky emission surface
0.2
Surface emission
13
345.8

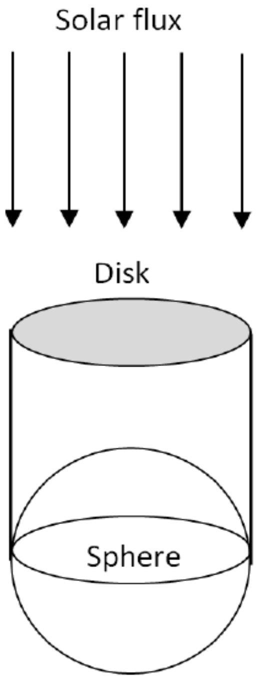


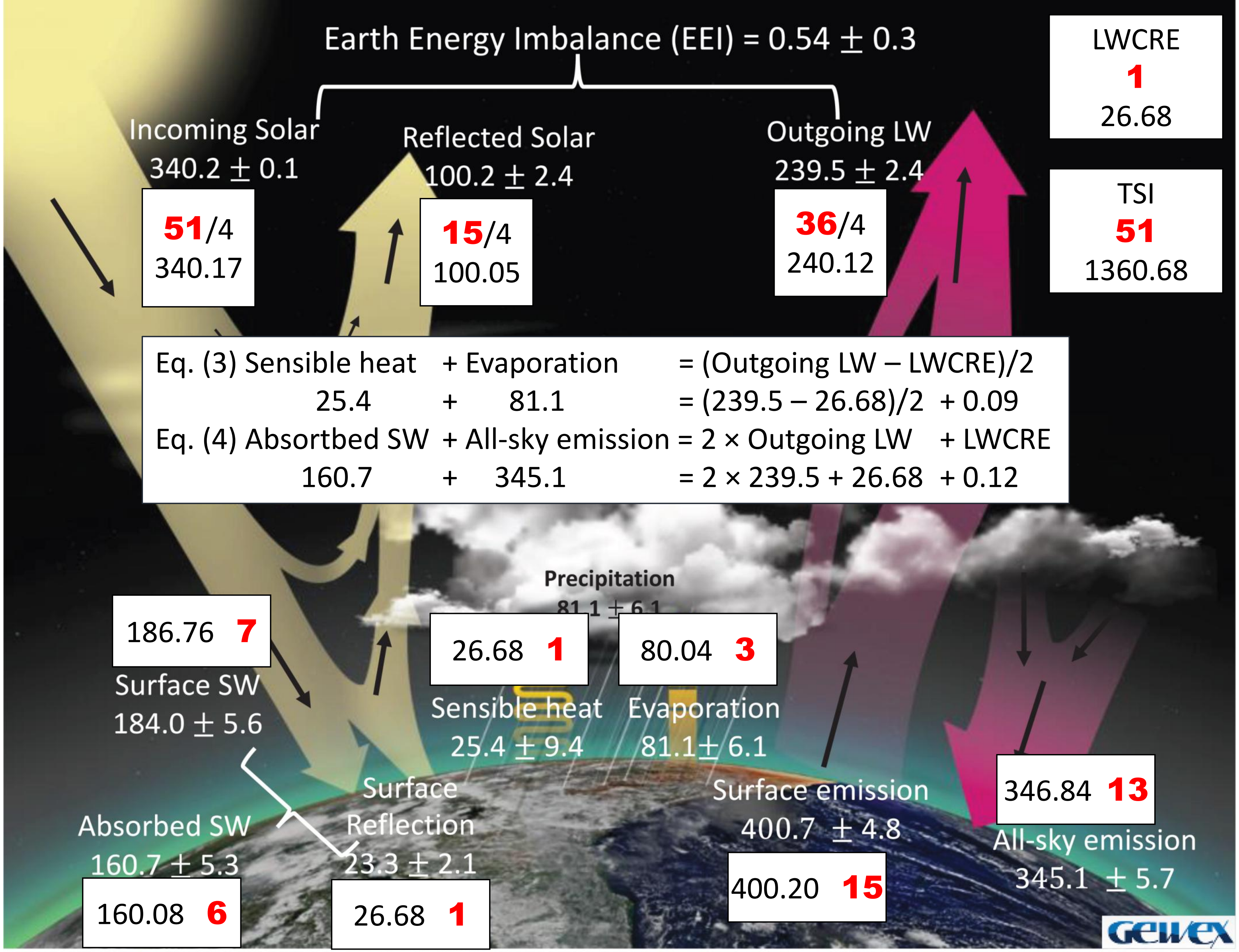
TOA imbalance 0.6±0.4
Clear-sky emission
266.4±3.
0.6
267.0
10
239.7±3.3
Outgoing longwave radiation
0.6
240.3
9
1
26.7
26.7±4
Longwave cloud effect
All-sky longwave absorption
-187.9±12.5
3±5
12
319.2
0.2
26.6±5
319±9
Clear-sky emission to surface
398±5
26.6
1
345.6±9
All-sky emission to surface
0.2
Surface emission
13 345.8


TOA imbalance 0.6±0.4
Clear-sky
emission
266.4±3.
0.6
267.0
10
1
26.7
239.7±3.3
Outgoing
longwave
radiation
0.6
240.3
9
26.7±4
Longwave
cloud effect
All-sky longwave
absorption
-187.9±12.5
3±5
12
319.2
0.2
26.6±5
319±9
Clear-sky emission
to surface
398±5
1.0
399.0
15
26.6
1
345.6±9
All-sky emission
surface
0.2
13 345.8

Stephens et al.
(2023, BAMS)

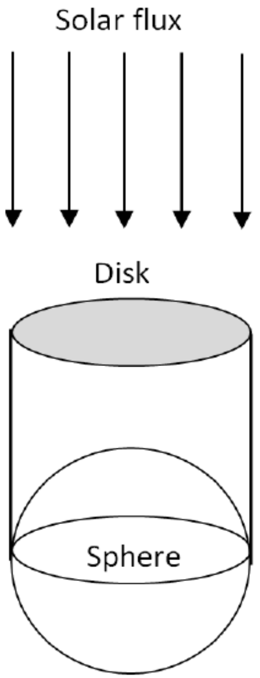
TSI
51

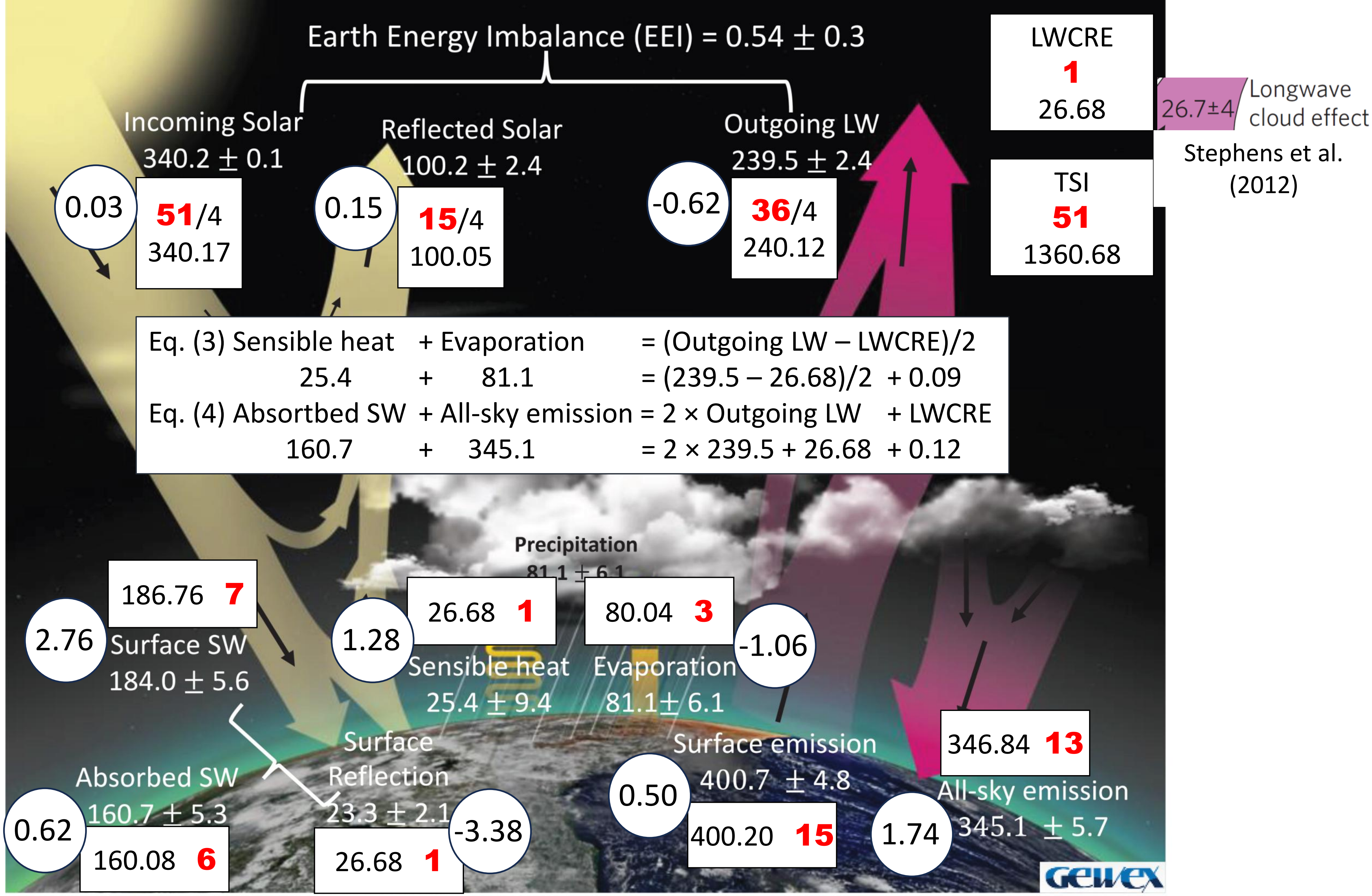




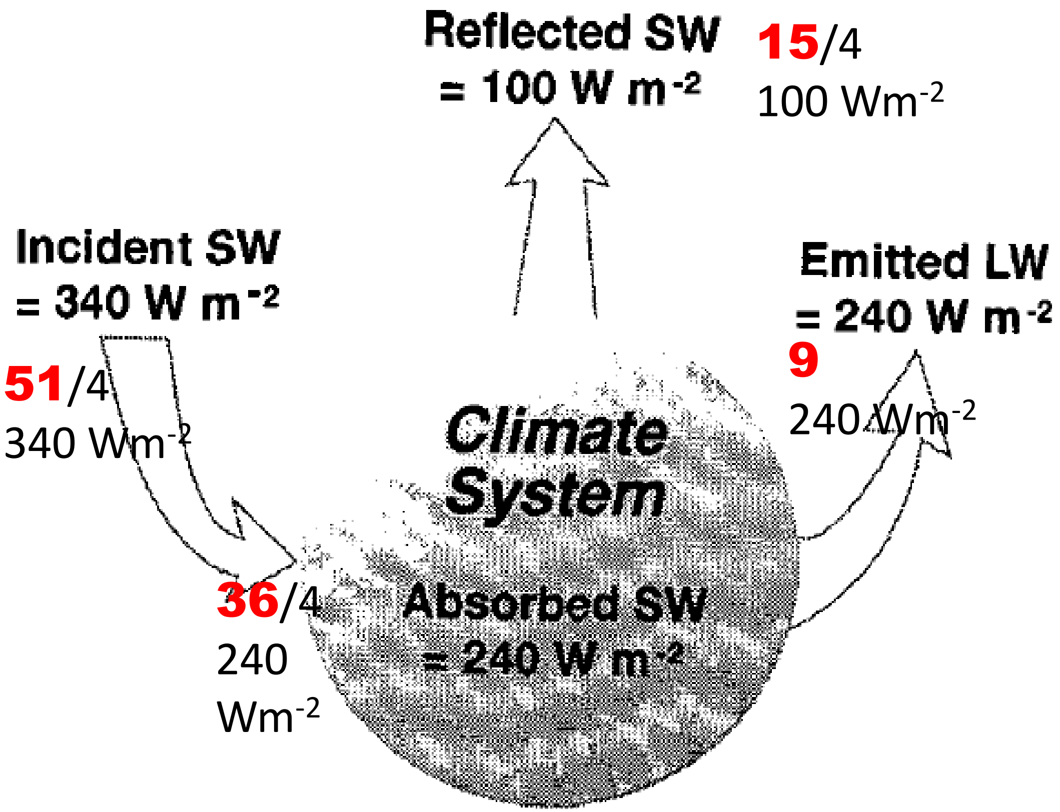
Stephens et al.
(2023, BAMS)

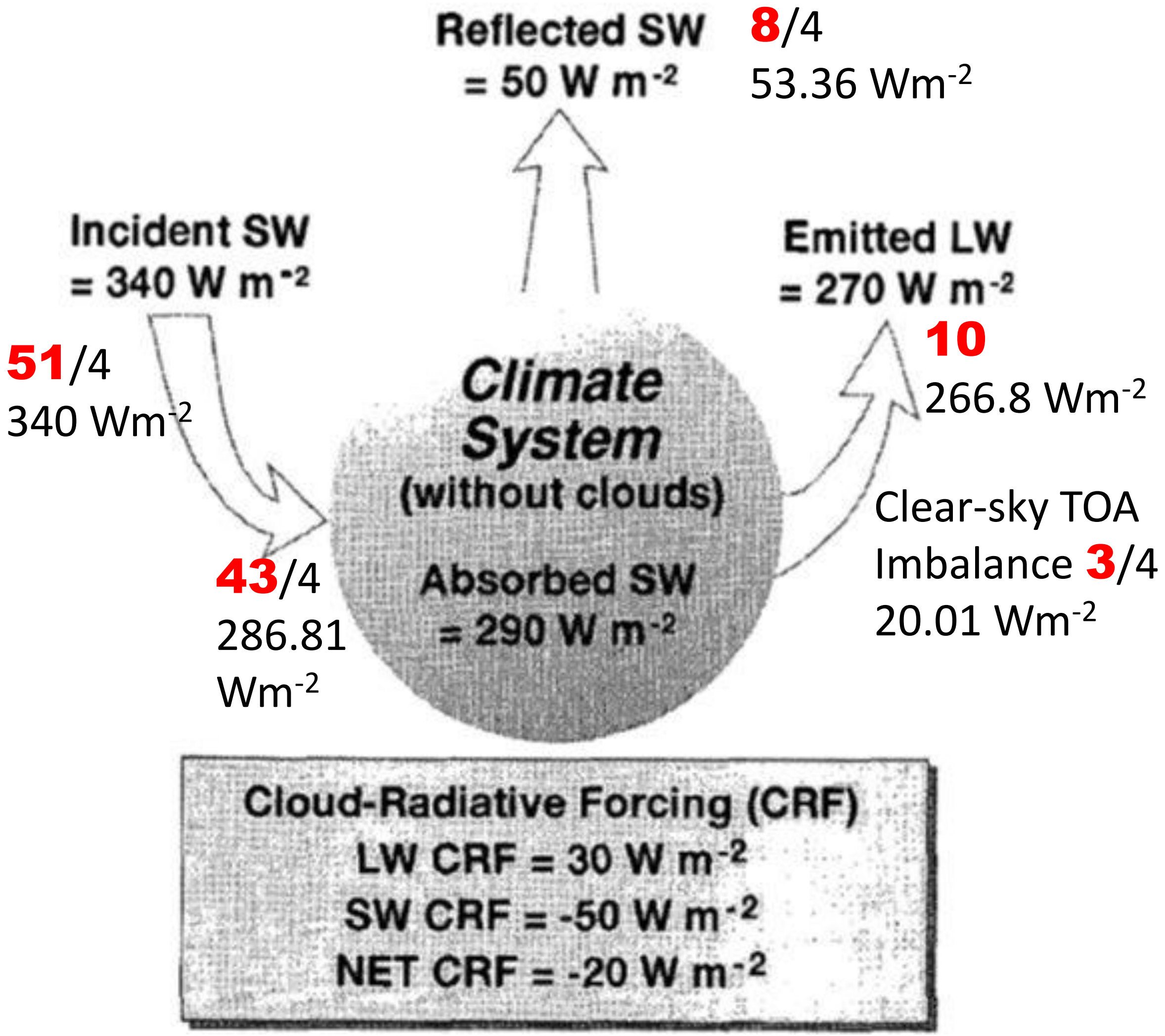
TSI
51





Wielicki et al. (1995)





Theory verified

Incident SW = **51**/4 = 340 Wm^{-2}

Reflected SW = **15**/4 = 100 Wm^{-2}

Absorbed SW = **36**/4 = 240 Wm^{-2}

Emitted LW = **36**/4 = 240 Wm^{-2}

Theory verified

LW CRF = **1** = 26.68 Wm^{-2}

SW CRF = **-7**/4 = -46.69 Wm^{-2}

Net CRF = -**3**/4 = -20.01 Wm^{-2}

Concluding

- *„CERES is an investigation to examine the role of cloud/radiation feedback in the Earth's climate system*.” (Wielicki et al. 1996). The investigation concludes in a set of four radiative transfer equations with a solution of small integer ratios related to the longwave radiative effect of clouds (LWCRE).
- CERES CDR (climate data redord) prove the validity of the four equations and justify their integer solution within uncertainty; inversely, CERES CDR prove that Earth follows accurately these four equations and implements their integer solution system.
- The four equations and the extended set of their integer solution give an explicit answer to the initial problem, by quantifing the SW, LW and net CRE in the climate system and prescribing their position in the integer ratio system related to the total solar irradiance (TSI).
- The four equatioins and their integer solution define a steady state of Earth’s global mean energy flow system. It is a greenhouse gas-independent geometric state, driven by TSI, and is based on a known radiative transfer constraint: the net radiation at the surface is half of the outgoing LW radiation at the top of the atmosphere in the clear-sky. The all-sky system is modulated by LWCRE.
- The discovery of this steady state has consequences pointing far beyond the original scope of the mission, and affects every aspect of the greenhouse theory: sensitivity studies, cloud feedback estimates, climate models, climate change assessments, and climate projections.
- CERES CDR played an indispensable role in verifying the clear-sky system. Future satellite missions with better absolute calibration are needed to monitor the deviation from the steady-state; to understand the impact of man's activities and predict their long-term consequences.